Unit 2 Assignment

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Q.1 $\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$

→ Let, $\frac{2}{w} = t$

$-\frac{2}{w^2}\, dw = dt$

$dw = \frac{-w^2}{2}\, dt$

~~$\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw = \int$~~

When, $w = 2$, $t = 1$

When, $w = \frac{1}{50}$, $t = 100$

$$\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw = \int_{100}^{1} -\frac{1}{2} w^2 \frac{e^t}{w^2}\, dt$$

$$= \frac{1}{2}\int_{1}^{100} e^t\, dt$$

$$= \frac{1}{2}\left[e^t\right]_1^{100}$$

$$= \frac{e^{100} - e}{2}$$

$$\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw = \frac{e(e^{99} - 1)}{2}$$

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Q2. $\int \frac{2t^3+1}{(t^4+2t)^3}\,dt$

Let $(t^4+2t) = u$

$(4t^3+2)\,dt = du$

$dt = \frac{1}{2}\frac{du}{(2t^3+1)}$

$\int \frac{2t^3+1}{(t^4+2t)^3}\,dt = \int \frac{1}{2\cancel{(2t^3+1)}} \cdot \frac{\cancel{2t^3+1}}{u^3}$

$= \frac{1}{2}\int u^{-3}$

$= \frac{u^{-2}}{-4} + C$

$= \frac{-1}{4u^2} + C$

$I = \frac{-1}{4(t^4+2t)^2} + C.$

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Q3. $f'(x) = x^4 + 3x - 9$

$\rightarrow$ $f(x) = \int f'(x)$

$= \int x^4 + 3x - 9$

$f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C.$

Q4. $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \leq 1 \end{cases}$

$\int_{-2}^{3} f(x)\,dx = \int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$

$= \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$

$= \cancel{\int_{-2}^{1} 3x} = \left[\frac{\cancel{3}x^3}{\cancel{3}}\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$

$= \left[x^3\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$

$= \left[(1)^3 - (-2)^3\right] + \left[6(3) - 6(1)\right]$

$= \left[1 - (-8)\right] + \left[18 - 6\right]$

$= 9 + 12$

$I = 21.$

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Q5. $\int 3(8y-1)e^{4y^2-y}\,dy$

Let $(4y^2-y) = t$

$(8y-1)\,dy = dt$

$dy = \dfrac{dt}{(8y-1)}$

$\int 3(8y-1)e^{4y^2-y}\,dy = 3\int \dfrac{\cancel{(8y-1)}}{\cancel{(8y-1)}} e^{t}\,dt$

$= 3\int e^{t}\,dt$

$= 3e^{t} + C$

$I = 3e^{4y^2-y} + C.$

Q6. $f''(x) = 15\sqrt{x} + 5x^3 + 6.$

$\rightarrow$ $f'(x) = \int f''(x)\,dx$

$= \int (15\sqrt{x} + 5x^3 + 6)\,dx$

$= \left[\dfrac{\cancel{15}x^{3/2}}{\cancel{3}}(2) + \dfrac{5x^4}{4} + 6x + k\right]$

$= 10x^{3/2} + \dfrac{5x^4}{4} + 6x + k$

$f(x) = \int f'(x)\,dx$

$= \int 10x^{3/2} + \dfrac{5x^4}{4} + 6x + k$

$$I.I = \frac{\cancel{10}^{\,2}x^{5/2}}{\cancel{5}/2} + \frac{\cancel{8}\times x^5}{4\ \cancel{8}} + \frac{\cancel{6}^{\,3}x^2}{2} + kx + C$$

$$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + kx + C$$

- $f(1) = 4(1)^{5/2} + \frac{(1)^5}{4} + 3(1)^2 + k(1) + C$

$$-\frac{5}{4} = 4 + \frac{1}{4} + 3 + k + C$$

$$-\frac{5}{4} = 7 + \frac{1}{4} + k + C$$

$$-\frac{5}{\cancel{4}} = \frac{28+1+4k+4C}{\cancel{4}}$$

$$-5 = 29 + 4k + 4C$$

$$4k + 4C = -34 \quad - (1)$$

- $f(4) = 404$

$$f(4) = 4(4)^2\sqrt{4} + \frac{(\cancel{4})^{\cancel{5}\,4^4}}{\cancel{4}} + 3(4)^2 + 4k + C$$

$$404 = 128 + 256 + 48 + 4k + C$$

$$404 = 432 + 4k + C$$

$$4k + C = -28 \quad - (2)$$

~~(2) = Subtracting (1) from (2)~~

~~$4k + C = -28$~~

Subtracting (2) from (1).

$$4k + 4C = -34$$
$$-4k - C = +28$$
$$3C = -6$$
$$C = -2.$$

Using eq^n ②

$$4k + C = -28$$
$$4k - 2 = -28$$
$$4k = -26$$
$$k = \cancel{-6.5}\ -\frac{13}{2}$$

$$\Rightarrow f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - \frac{13}{2}x - 2.$$

Q7. $Z = f(x+iy) + g(x-iy)$

→ Partially differentiating wrt x.
Let it be denoted by a.

~~Note: Chain rule of differentiation is followed. $f(x+iy)$ differentiated then, [illegible]~~

$$a = f'(x+iy) + g'(x-iy) \quad — ①$$ ~~chain rule different [illegible]~~

Partially differentiating w.r.t y. Let it be denoted by b.

$$b = f'(x+iy)\,i + g'(x-iy)(-i) \quad — ②$$

→ Partially differentiating ① w.r.t x. Let it be denoted by c.

$$c = f''(x+iy) + g''(x-iy)$$

Partially differentiating ② w.r.t y. Let it be denoted by d.

$$d = f''(x+iy)(i^2) + g''(x-iy)(-i)^2$$

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$$d = f''(x+iy)i^2 + g''(x-iy)i^2$$

$$d = i^2\left(f''(x+iy) + g''(x-iy)\right)$$

$$d = (\sqrt{-1})^2 (c)$$

$$d = -c.$$

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Q8. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$

$\rightarrow$ $\theta = 1,\ r = 2.$

$$\frac{dr}{d\theta} = \frac{r^2}{\theta}$$

$$\frac{dr}{r^2} = \frac{d\theta}{\theta}$$

Integrating both sides.

$$\int r^{-2}\,dr = \cancel{\frac{d\theta}{\theta}} \int \frac{d\theta}{\theta}$$

$$-\frac{1}{r} + c_1 = \log\theta + c_2$$

$$\log\theta + \frac{1}{r} = k$$

When, $\theta = 1,\ r = 2$

$$\log(1) + \frac{1}{2} = k$$

$$k = \frac{1}{2}$$

Thus, the eq^n is,

$$\log\theta + \frac{1}{r} - \frac{1}{2} = 0.$$

Q9. $$\frac{dv}{dt} = 9.8 - 0.196v$$

$$\rightarrow \quad \frac{dv}{9.8 - 0.196v} = dt$$

Integrating both sides,

$$\frac{1}{9.8}\left(\int \frac{dv}{1 - 0.02v}\right) = \int dt$$

Let $(1 - 0.02v) = u$

$$-0.02\,dv = du$$

$$dv = \frac{\cancel{1}}{\cancel{0.02}} \quad \frac{du}{-0.02}$$

$$\frac{-1}{0.196}\int \frac{du}{u} = \int dt$$

$$\frac{-1}{0.196}\log u = t + k$$

$$0.196t + \log u = \cancel{k(0.196)} - k(0.196)$$

$$0.196t + \log u + 0.196k = 0.$$

$$0.196t + \log(1 - 0.02v) + 0.196k = 0.$$

Q10. $t\frac{dy}{dt} + 2y = t^2 - t + 1$

$\frac{dy}{dt} + \frac{2}{t}y = t - 1 + t^{-1}$

$I.F = e^{\int P dt}$

$= e^{\int \frac{2}{t} dt}$

$= e^{2\int t^{-1} dt}$

$= e^{2\log t}$

$= e^{\log t^2}$

$I.F = t^2.$

$y \times I.F = \int Q \times I.F$

$yt^2 = \int t^2(t - 1 + t^{-1})$

$yt^2 = \int t^3 - t^2 + t$

$yt^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + C$

$y = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + Ct^{-2}$

When, $t = 1$, $y = \frac{1}{2}$.

$\cancel{\frac{1}{2}} = \frac{1}{4} - \frac{1}{3} + \cancel{\frac{1}{2}} + C$

$C = \frac{1}{3} - \frac{1}{4}$

$C = \frac{1}{12}.$

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$$y = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + \frac{t^{-2}}{12}$$

$$y - \frac{t^2}{4} + \frac{t}{3} - \frac{t^{-2}}{12} - \frac{1}{2} = 0$$

$$288y - 72t^2 + 96t - 24t^{-2} - 144 = 0$$

$$12y - 3t^2 + 4t - t^{-2} - 6 = 0$$

$$\Rightarrow 12y - 3t^2 + 4t - t^{-2} - 6 = 0.$$

Q11. $\dfrac{dy}{dx} = \dfrac{xy^3}{\sqrt{1+x^2}}$

$\dfrac{dy}{y^3} = \dfrac{x\,dx}{\sqrt{1+x^2}}$

Integrating both sides

$\int y^{-3}\,dy = \int \dfrac{x}{\sqrt{1+x^2}}\,dx$

Let $(1+x^2) = t$

$2x\,dx = dt$

$dx = \dfrac{dt}{2x}$

$\dfrac{-1}{\cancel{2}y^2} = \dfrac{1}{\cancel{2}} \int \dfrac{1}{\sqrt{t}}\,dt$

$-\dfrac{1}{y^2} = 2\sqrt{t} + C$

$-\dfrac{1}{y^2} = 2\sqrt{1+x^2} + C$

$x=0,\ y=-1$

$-1 = 2 + C$

$C = -3$

$2\sqrt{1+x^2} + C + \dfrac{1}{y^2} = 0$

$2y^2\sqrt{1+x^2} + Cy^2 + 1 = 0$

$2y^2\sqrt{1+x^2} - 3y^2 + 1 = 0.$

[illegible]

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$\sqrt{1+x^2}$ is continuous at all points. The interval of validity must include the value $x=0$ as well which is mentioned in the question. $\sqrt{1+x^2}$ is continuous for all real values of x.

Thus, the interval of validity $= (-\infty, \infty)$.

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Q12. $f(3) = (3)^3 - 10(3)^2 + 6$
$= -57$

$f'(x) = \cancel{3x^2 - 10}\ 3x^2 - 20x$
$f'(3) = -33$

$f''(x) = 6x - 20$
$f''(3) = -2$

$f'''(x) = 6$
$f'''(3) = 6$

$\cancel{f(x) = -57 + (x-3)}$ $f''''(x) = 0$
$f''''(3) = 0$

Taylor Series:

$$f(x) = f(3) + (x-3)f'(3) + \frac{(x-3)^2}{2!}f''(3) + \frac{(x-3)^3}{3!}f'''(3) \dots + \frac{(x-4)^4}{4!}f''''(3)$$

$$= -57 + (x-3)(-33) + \frac{(x-3)^2}{\cancel{2}}(\cancel{-2}) + \frac{(x-3)^3}{\cancel{6}}(\cancel{6}) \dots$$

$$\cancel{= -57 - 33x +} \quad + \frac{(x-4)^4}{24}(0)$$

$\cancel{f(x) = -57 + (x-3)}$

$$f(x) = -57 - 33(x-3) - (x-3)^2 + (x-3)^3.$$

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Q13 $f(0) = 1$

$f^{I}(x) = \mu(1+x)^{\mu-1}$
$f^{I}(0) = \mu$

$f^{II}(x) = \mu(\mu-1)(1+x)^{\mu-2}$
$f^{II}(0) = \mu(\mu-1)$

$f^{III}(x) = \mu(\mu-1)(\mu-2)(1+x)^{\mu-3}$

$f^{III}(0) = \mu(\mu-1)(\mu-2)$

$f^{IIII}(x) = \mu(\mu-1)(\mu-2)(\mu-3)(1+x)^{\mu-4}$
$f^{IIII}(0) = \mu(\mu-1)(\mu-2)(\mu-3)$.

$\Rightarrow$ Maclaurin series:

$$f(x) = f(0) + \frac{x^1}{1!}f^{I}(0) + \frac{x^2}{2!}f^{II}(0) + \frac{x^3}{3!}f^{III}(0) + \frac{x^4}{4!}f^{IIII}(0) \ldots$$

$$= 1 + x\mu + \frac{x^2}{2}\mu(\mu-1) + \frac{x^3}{3}\mu(\mu-1)(\mu-2) + \frac{x^4}{4}\mu(\mu-1)(\mu-2)(\mu-3) \ldots$$

~~3x²~~
~~6x~~
~~2.3.3x~~

$$f(x) = 1 + x\mu + \frac{x^2}{2}\mu(\mu-1) + \frac{x^3}{3}\mu(\mu-1)(\mu-2) + \frac{x^4}{4}\mu(\mu-1)(\mu-2)(\mu-3) + \ldots$$

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Q14. $f(0) = 1$

$f'(x) = \frac{1}{2\sqrt{1+x}}$

$f'(0) = \frac{1}{2}$

$f''(x) = \frac{-1}{4(1+x)^{3/2}}$

$f''(0) = \frac{-1}{4}$

$f'''(x) = \frac{3}{8(1+x)^{5/2}}$

$f'''(0) = \frac{3}{8}$

$f''''(x) = \frac{-15}{16(1+x)^{7/2}}$

$f''''(0) = \frac{-15}{16}$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f''''(0) + \ldots$$

$$= 1 + \frac{x}{2} + \frac{x^2}{2}\left(\frac{-1}{4}\right) + \frac{x^3}{\cancel{6}_2}\left(\frac{\cancel{3}^1}{8}\right) + \frac{x^4}{\cancel{24}_8}\left(\frac{-\cancel{15}^5}{16}\right) + \ldots$$

$$f(x) = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \ldots\ldots$$

Q15. $f(x) = e^{-6x}$

$f(-4) = e^{24}$

$f'(x) = -6e^{-6x}$

$f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$

$f''(-4) = 36e^{24}$

$f'''(x) = -216e^{-6x}$

$f'''(-4) = -216e^{24}$

$f''''(x) = 1296e^{-6x}$

$f''''(-4) = 1296e^{24}$

Taylor Series

$$f(x) = f(-4) + (x+4)f'(-4) + \frac{(x+4)^2}{2!}f''(-4) + \frac{(x+4)^3}{3!}f'''(-4) + \frac{(x+4)^4}{4!}f''''(-4) + \dots$$

$$\cancel{= e^{24} + (x+4)(-6e^{24}) + \frac{(x+4)^2}{2!}}$$

$$= e^{24} + (x+4)(-6e^{24}) + \frac{(x+4)^2}{2!}(\overset{18}{\cancel{36}}e^{24})$$

$$+ \frac{(x+4)^3}{6!}(-\overset{36}{\cancel{216}}e^{24}) + \frac{(x+4)^4}{24!}(\overset{54}{\cancel{1296}}e^{24}) + \cdots$$

$$f(x) = e^{24} - 6e^{24}(x+4) + 18e^{24}(x+4)^2 - 36e^{24}(x+4)^3$$

$$+ 54e^{24}(x+4)^4 \cdots\cdots$$

Q16. $f(x) = \ln(4x+3)$

$f(0) = \ln(3)$

$f'(x) = \frac{1}{4x+3}$

$f'(0) = \frac{1}{3}$

$f''(x) = \frac{-1}{(4x+3)^2}$

$f''(0) = -\frac{1}{9}$

$f'''(x) = \frac{2}{(4x+3)^3}$

$f'''(0) = \frac{2}{27}$

$f''''(x) = \frac{-6}{(4x+3)^4}$

$f''''(0) = \frac{-\cancel{6}\,2}{\cancel{81}\,27}$

$f''''(0) = -\frac{2}{27}$

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$$f(x) = f(0) + (x-0)f'(0) + \frac{(x-0)^2}{2!}f''(0) + \frac{(x-0)^3}{3!}f'''(0) + \frac{(x-0)^4}{4!}f''''(0) \ldots$$

$$= \ln(3) + \frac{x}{3} + \frac{x^2}{2}\left(\frac{-1}{9}\right) + \frac{x^3}{\cancel{6}\,3}\left(\frac{\cancel{2}\,1}{27}\right) + \frac{x^4}{\cancel{24}\,12}\left(-\frac{\cancel{2}\,1}{27}\right)$$

$$f(x) = \ln(3) + \frac{x}{3} - \frac{x^2}{9} + \frac{x^3}{81} - \frac{x^4}{324} \ldots$$