SRM-3 Assignment-1 Atharva Padwal

Q1.

AY	DY 0	1	2	3
2001	1245	999	610	356
2002	1455	1088	723	
2003	1567	1079		
2004	1491			

Inflation index

2002	2001	$(1.021)\times(1.012)\times(0.992) = 1.0249$
2003	2002	$1.012 \times 0.992 = 1.003904$
2004	2003	0.992
	2004	1

Incremental claims mid 2004 prices

AY	DY 0	1	2	3
2001	1276.11	1002.9	605.15	356
2002	1460.68	1079.30	723	
2003	1554.46	1079		
2004	1491			

Cumulative claims mid 2004 prices

AY/DY	0	1	2	3
2001	1276.11	2279.01	2884.13	3240.13
2002	1460.68	2539.98	3262.98	
2003	1554.46	2633.46		
2004	1491			

	0	1	2	~~3~~ 3
Total Claims	5782.25	7452.45	6147.11	3240.13
(Last claim)	(1491)	(2633.46)	(3268.98)	(0)
	4291.25	4818.99	2884.13	3240.13

DF   1.73866   1.2756   1.12343

	DY			
AY	0	1	2	3
2001	1276.11	2279.01	2884.13	3240.13
2002	1460.68	2539.98	3262.98	3665.74
2003	1554.46	2633.46	3359.245	3773.89
2004	1491	2589.36	3302.99	3710.69

Incremental claims ~~for the yo~~ mid 2004 prices

	DY			
AY	0	1	2	3
2001				
2002				
2003				
2004		1098.36	712.64	407.7
Inflation		× 1.025	× 1.050625	× 1.076891
Inflation adjusted =		1125.819	748.72	439.05

Total claims = 2313.5875 (figure in £000s)

(ii) Ultimate amount for 2004 policies = 5250000 × 0.75
= 3937500

~~Cumulative DF~~

Cumulative DFs	—	2.48871	1.43304	1.12343
Reverse	1	1.12343	1.43304	2.48871
1/f	1	0.89013	0.6978	0.4018
1 − 1/f	0	0.109837	0.3022	0.5982

~~0.5982 × 3937500~~

~~Total emerging liability using BFM = 2355.4125~~ (figure in '000)

For 2004 AY,

Dy	Emerging liability		Inflation	
1	$3937.5 \times \left(\frac{1}{1.43304} - \frac{1}{2.4887}\right)$	= 1165.51 ×	1.025	= 1194.65
2	$3937.5 \times \left(\frac{1}{1.12343} - \frac{1}{1.433}\right)$	= 757.24 ×	1.050625	= 795.57
3	$3937.5 \times \left(1 - \frac{1}{1.12343}\right)$	= 432.61 ×	1.076891	= 465.87

Total emerging liability = 2456.09 (figure in '000s)

2. U/W ~~AY~~

DY	0	1	2	3
2005		3512	7210	9563
2006		2889	6723	
2007		3876		

Loss ratio = 87%

	Premium income	Total claims paid
2005	11041	20103000
2006	11314	
2007	12549	

r =	2.1767	1.3264	1
f =	2.8872	1.3264	1
$1-1/f$ =	0.6536	0.24608	0

AY	2007	2006	2005
EP	12549	11314	11041
IUL	10917.63	9843.18	9605.67
EL	7135.76	2422.21	0
RL	3876	6723	9563
UL	11011.76	9145.21	9563

Total UL = ~~29713~~ 29719.97 × 1000

Reserves = 9616970

Q4

i ~~AoF~~ cumulative no. of claims

	DY		
AY	0	1	2
2005	82	121	156
2006	128	163	
2007	98		

2 Total claim amts.

	DY		
AY	0	1	2
2005	43512	87210	129663
2006	62889	116723	
2007	73876		

ACPC table

	DY			
AY	0	1	2	Ult
2005	530.634	720.744	830.532 →	830.53
~~2005~~	(63.891%)	(86.781%)	(100%)	
2006	491.320	716.092	→	825.1714
	(59.542%)	(86.781%)		
2007	753.836	→	→	1221.459
	(61.716%)			

Ultimate for claim nos.

AY	DY 0	1	2	Ult
2005	82 (52.561%)	121 (77.564%)	156 (100%)	→ 156
2006	128 (60.909%)	163 (77.564%)		→ 210.149
2007	98 (56.738%)			→ 172.7237

Project ultimate loss

2005 129562.99

~~2006 = 1221.4595 × 172.7237 = 2~~

2006 = 825.1714 × 210.149 = 173408.9445

2007 = 1221.4595 × 172.7237 = 210975.0042

O/S claims = 129563 + 173408.9445 + 210975.0042 − [73876 + 116723 + 129563]

= 193784.9384

Q.5.

$X \sim \exp(\lambda)$, Show that $Y = kX$

Now,

$$P(Y = y) = P(kX < y)$$
$$= P(X < y/k)$$
$$= \int_0^{y/k} \lambda e^{-\lambda z} \, dz$$
$$= \int_0^{y} \lambda e^{-\frac{\lambda}{k} x} \frac{dx}{k}$$
$$= \int_0^{y} \frac{\lambda}{k} e^{-\frac{\lambda}{k} x} \, dx$$

which is the distn function of exp distn with parameter λ/k. So Y is exponentially distributed with parameter λ/k.

(ii) ~~First note~~ $P[\text{Loss in } 2006 > M] = e^{-\lambda M}$

$$L = C \times e^{-4\lambda M} \times \prod_i \left(\lambda e^{-\lambda x_i}\right) \times e^{-6\lambda M/k} \times \prod_j \left(\lambda/k \; e^{-\lambda y/k}\right)$$

where x_i represents the claims in 2006 and y_i represents the claim in 2007.

$$\log L = C' - 4\lambda M + 10 \log \lambda - \lambda \sum x_i - 6\lambda M/k + 12 \log \lambda - \lambda/k \sum y_i$$

$$= C' - 4\lambda M + 22 \log \lambda - 13500\lambda - 6\frac{\lambda M}{k} - 17000\frac{\lambda}{k}$$

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Differentiating ~~wrt~~

$$\ell' = -4M + \frac{22}{\lambda} - 13500 - \frac{6M}{k} - \frac{17000}{k}$$

$$-4M + \frac{22}{\hat{\lambda}} - 13500 - \frac{6M}{k} - \frac{17000}{k} = 0$$

$$\frac{22}{\hat{\lambda}} = 13500 + \frac{17000}{k} + 4M + \frac{6M}{k}$$

$$\hat{\lambda} = \frac{22}{13500 + \frac{17000}{k} + 4M + \frac{6M}{k}}$$

$$\ell'' = \frac{-22}{\lambda^2} < 0$$

(iii) a) $\hat{\lambda} = \dfrac{22}{13500 + \frac{17000}{1.1} + 4\times1600 + \frac{6\times1600}{1.1}}$

$= 0.000499$

b) Expected payment per claim for 2006 =

$$\int_0^M x\lambda e^{-\lambda x}\,dx + M\int_M^\infty \lambda e^{-\lambda x}\,dx$$

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$$= \left[-x e^{-\lambda x}\right]_0^M + \int_0^M e^{-\lambda x}\,dx + M\left[e^{-\lambda x}\right]_M^{\infty}$$

$$= \frac{1}{\lambda}\left(1 - e^{-\lambda M}\right)$$

$$= \frac{1}{0.000499}\left(1 - e^{-0.000499 \times 1600}\right)$$

$$= 1102.107$$

$$2008 \sim \exp\left(\mu = \frac{\lambda}{1.05 \times 1.1}\right)$$

$$1102.107 = \int_0^R x \mu e^{-\mu x}\,dx + R\int_R^{\infty} \mu e^{-\mu x}\,dx$$

$$= \frac{1}{\mu}\left(1 - e^{-\mu R}\right)$$

$$= \frac{1.05 \times 1.1}{0.0004999}\left(1 - e^{\frac{0.000499 \cdot R}{1.05 \times 1.1}}\right)$$

$$\therefore R = 1496.52$$

6.

i) Heterogenous group

(ii) a. $E(\lambda_i) = 0.5 \times 0.1 + \frac{1}{3} \times 0.3 + \frac{1}{6} \times 0.9$

$= 0.3$

$E[\lambda_i^2] = 0.5 \times 0.1^2 + \frac{1}{3} \times 0.3^2 + \frac{1}{6} \times 0.9^2$

$= 0.17$

$\therefore Var(\lambda_i) = 0.08$

b. $E(S_i | \lambda_i) = [\lambda_i m_1]$ & $Var[S_i | \lambda_i] = [\lambda_i m_2]$

c. $E[S_i] = E[E[S_i | \lambda_i]] = E[\lambda_i m_1]$

$= 0.3 m_1 = 0.3 \times 250 = 75$

d. For the whole portfolio, since the variables are ~~independent~~ & iid, the mean $= 100 \times 75$

$= 7500$

var $= 100^2 \times 23810$

$= 2381000$

(iii) Now homogenous

So $Var(\lambda_i) = 0.5 * (0.2^2 + 0.4^2) - 0.3^2 =$

0.01

$$E\left[\sum_{i=1}^{N} S_i\right] = nE(S) = 0.3 \times 100 \times 250 = 7500$$

$$Var\left[\sum_{i=1}^{N} S_i\right] = E\left[Var\left(\sum_{i=1}^{N} S_i / \lambda\right)\right] + Var\left[E\left(\sum_{i=1}^{n} S_i / \lambda\right)\right]$$

$$= E[n\lambda m_2] + var[n\lambda m_1]$$

$$= 0.3 \times 100 \times 200 + 0.1 \times 100^2 \times 250^2$$

$$= 6257000$$

iv) The mean is the same, but variance is very high. This makes sense since parameter uncertainty is at the overall level ~~rater that~~ rather than at the individual level, so affects aggregate claim vairance much more.

Q7. Claim $\sim$ Poi(50)

Claim amt $\approx 100\,X$

$$f(x) = \frac{3}{32}\, x\,(6x - x^2 - 5)$$

(i)

$$E[100\,X] = 100\,E[X]$$

$$= 100 \times \frac{3}{32} \int_1^5 x.\,(6x - x^2 - 5)\,.dx$$

$$= 100 \times \frac{3}{32} \int_1^5 6x^2 - x^3 - 5x \;.dx$$

$$= \frac{300}{32} \times \left[2x^3 - \frac{x^4}{4} - \frac{5x^2}{2}\right]_1^5$$

$$= \frac{300}{32} \times [31.25 + 0.75]$$

$$= 300$$

$$Var[100\,X] = E[(100X)^2] - (E[100X])^2$$

$$100^2 \times \frac{3}{32} \int_1^5 x^2 .\,(6x - x^2 - 5)\,.dx$$

$$= 98000$$

$$Var[100X] = 98000 - (300)^2$$

$$= 8000$$

Q7. contd.

$\cancel{E(x) = 0}$ $E(Y) = 0.3 \times 200 = 60$

$Var(Y) = 0.3 \times 200^2 - (60)^2$

$= 8400$

$E[Z] = E[X] + E[Y]$

$= 300 + 60$

$= 360$

$Var[Z] = Var(X+Y)$

$= Var(X) + Var(Y)$

$= 8000 + 8400 = 16400$

$E[S] = E[N] . E[Z]$

$Var[S] = E[N] \cdot Var[Z] + Var[N] \cdot E[Z]^2$

$E[S] = 18000$

$V[S] = 50 \times 16400 + 50 \times 300^2$

$= 7300000$