

PRLI Assignment-1 Atharva Padwal

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Q1

$$i \text{ } (Ia)_{\overline{n}|i} = v + 2v^2 + 3v^3 + \dots + nv^n \quad \text{--- (A)}$$

$$(1+i) (Ia)_{\overline{n}|i} = 1 + 2v + 3v^2 + \dots + nv^{n-1} \quad \text{--- (B)}$$

$$= i (Ia)_{\overline{n}|i} = 1 + v + v^2 + v^3 + \dots + v^{n-1} - nv^n$$

$$= i (Ia)_{\overline{n}|i} = \ddot{a}_{\overline{n}|i} - nv^n$$

$$= Ia_{\overline{n}|i} = \frac{\ddot{a}_{\overline{n}|i} - nv^n}{i}$$

or

$$ii) \text{ Cost of education} = 300000 \\ \text{inflation} = 6\% \text{ pa}$$

$$\text{Contribution} = 200000 \text{ pa}$$

$$\text{interest} = 10\%$$

→ PV of contribution.

$$200000 \ddot{a}_{\overline{10}|10\%} + X (Ia)_{\overline{10}|10\%} \quad \text{at } i=10\%$$

$$= 200000 \times 6.759026 + 2 \times 25.18046$$

$$= 1351805.20 + 25.180462$$

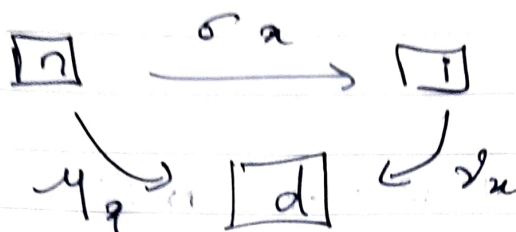
P.V of higher education :

$$3000000 \times 1.06^{10} \times v^{10} @ 10\%$$

$$= 2071360$$

$$X = 29575$$

Q2.



$$i = 5\%$$

$$\mu_n = 0.004$$

$$\gamma_x = 0.005$$

$$\sigma_x = 0.002$$

$$x = 40$$

$$n = 20$$

$$S = 100,000 \text{ (immediately)}$$

$$EPV = 100,000 \int_0^{20} v^t \cdot \frac{h}{100} (\sigma_{y_{0+t}} + \gamma_{y_{0+t}}) dt$$

$$= 100,000 \int_0^{20} e^{-\ln(1.05)t} \cdot \frac{40}{100} \cdot e^{-0.006t} \cdot 0.006 dt$$

$$= 600 \int_0^{20} e^{-\ln(1.05)t} \cdot e^{-0.006t} \cdot dt$$

$$= \frac{600}{0.5497} [e^{-0.5497t}]_0^{20}$$

$$EPV = 7290.3$$

Q3. $X = 40$

$n = 20$

$i = 4\%$

AM92 select - disability

AM92 ult = illness

i.)

$$\text{EPV of prem} = P \ddot{a}_{40:\overline{20}|}^{(12)}$$

$$= P \left[\ddot{a}_{40:\overline{20}|} - \frac{11}{24} \left(1 - \frac{D_{60}}{P_{40}} \right) \right]$$

$$= P \left[13.927 - \frac{11}{24} \times \left(1 - \frac{882.85}{2052.96} \right) \right]$$

$$= 13.66576724 P \quad \text{--- (1)}$$

EPV of claims :-

$$7500000 A_{40:\overline{20}|}^1 + 50000 A_{40:\overline{20}|}^1$$

$$\downarrow$$

$$A_{40:\overline{20}|} - \frac{D_{60}}{P_{40}}$$

$$\downarrow$$

$$A_{40:\overline{20}|} - \frac{D_{60}}{P_{40}}$$

$$= 427714.965824 \quad \text{--- (2)}$$

EPV of expenses

$$0.03 \times 427714.965824 + 0.35 P \cdot \ddot{a}_{40:\overline{20}|}^{(12)}$$

$$+ 0.05 P \left[\ddot{a}_{40:\overline{20}|}^{(12)} - 1 \right]$$

$$+ 0.1 P \times \ddot{a}_{40:\overline{20}|} \times 1.04^{0.5} + 500 \quad \text{--- (3)}$$

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~~Solve~~

$$\textcircled{1} - \textcircled{2} + \textcircled{3}$$

$$\text{Solving for } P \Rightarrow 39009.7$$

$$\therefore \text{Monthly } P = \underline{\underline{3250.8}}$$

ii. Policies with only accidental claims

$$7500000 A_{45:\overline{5}|}^1 - 39009.7 \ddot{a}_{45:\overline{5}|} \quad (12)$$

$$= 269387.1975 - 434750.6352$$

$$5V_1 = 165363.4377$$

Policies with only illness claims

$$5000000 A_{45:\overline{15}|}^1 - 320097 \ddot{a}_{45:\overline{15}|} \quad (12)$$

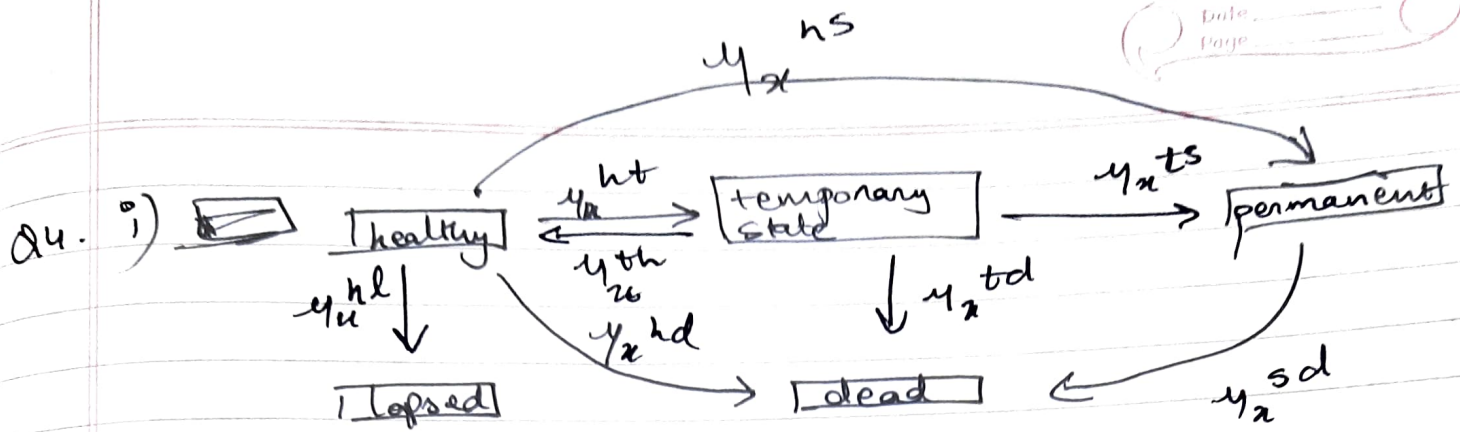
$$= 5000000 \times 0.055918293 - 434750.6352$$

$$5V_2 = 255139.1302$$

Policies with no claims

$$= 7500000 + 5000000 \times A_{45:\overline{15}|}^1 - 390097 \ddot{a}_{45:\overline{15}|}$$

$$5V_3 = 14228.0273$$



ii) $n = 15$

$x = 50$

$EPV(\text{prem}) = EPV(\text{benefit})$

$$\begin{aligned}
 EPV \text{ benefit} = & 10000 \int_0^{15} v^u \cdot u p_{50}^{hs} \cdot du \\
 & + 20000 \int_0^{15} v^u \cdot u p_{50}^{ht} \cdot du \\
 & + 100000 \int_0^{15} v^u \left(u p_{50}^{hd} \cdot y_{50+u}^{hd} \right. \\
 & \quad \left. + u p_{50}^{ht} \cdot y_{50+u}^{td} \right. \\
 & \quad \left. + u p_{50}^{hs} \cdot y_{50+u}^{sd} \right) \cdot du
 \end{aligned}$$

$EPV \text{ of premium} = P \times \int_0^{15} v^u \cdot u p_{50}^{nn} \cdot du$

$$\begin{aligned}
 \therefore P = & \left[100000 \int_0^{15} v^u \left[u p_{50}^{hd} + y_{50+u}^{hd} + u p_{50}^{ht} \cdot y_{50+u}^{td} \right. \right. \\
 & \quad \left. \left. + u p_{50}^{hs} \cdot y_{50+u}^{sd} \right] \cdot du \right. \\
 & \left. + 10000 \int_0^{15} v^u \cdot u p_{50}^{ht} \cdot du + 20000 \int_0^{15} v^u \cdot u p_{50}^{hs} \cdot du \right]
 \end{aligned}$$

$$\int_0^{15} v^u \cdot u p_{50}^{hs} \cdot du$$

Q5

i) Gross premium for the Policy:

$$\text{EPV premiums} = P \ddot{a}_{[40]:25} = 15.887P$$

$$\begin{aligned} \text{EPV expenses} &= 1200 + 500 \times \bar{A}_{[40]} + 2\% P \times (\ddot{a}_{[40]:25}) \\ &= 1200 + 500 \times (1.04)^{0.5} \times 0.2304 \\ &\quad + 0.02 \times P \times (15.887) \\ &= 1317.487 + 0.2977P \end{aligned}$$

$$\text{EPV of Benefits} = 200000 (v^{0.5})_{[40]}$$

$$+ v^{1.5} (1.04)_{1|9[40]}$$

$$+ v^{2.5} (1.04)^2_{2|9[40]} + \dots$$

$$= \frac{200000}{1.04^{0.5}} = 196116.14$$

Using equivalence principle,

$$15.88P = 196116.14 + 1317.487 + 0.29774P$$

$$P = 12664$$

ii. Revised bonus rates :-

$$\begin{aligned} \text{EPV of benefits} &= 2,00,000 \times x\% \cdot (IA)_{[40]} + 200,000 \times (1-x\%) \bar{A}_{[40]} \\ &= (1.04)^{0.5} \left[200,000 \times x\% \cdot (IA)_{[40]} + 200,000 \times (1-x\%) \bar{A}_{[40]} \right] \end{aligned}$$

$$\text{EPV of benefits} = 196115.14$$

Equating both, $x\% = 9\%$

iii. $P \ddot{a}_{40:25} = 200,000 \bar{A}_{40}$

$$P = 2960.54$$

Net premium reserve at time 10

$${}_{10}V = 200,000 \bar{A}_{50} - P \ddot{a}_{50:15}$$

$$\begin{aligned} &= 200,000 \times 1.04^{0.5} \times 0.32907 - 2960.54 \times 11.25 \\ &= 67361.10 \end{aligned}$$

Q6. Let force of decrement be μ_x ; force of decrement be δ_x

$$V = 100000 \int_0^{20} v^t + p_{45}^{hh} (1.45 + t + e_{45+t}^{hh}) dt$$

$$100000 \int_0^{20} e^{-\ln(1.05)t} \cdot t p_{45+t}^{hh} \cdot 0.008 dt$$

$$+ p^{hh} = \exp\left(-\int_0^{45} 0.008 ds\right)$$

$$= e^{-0.008t}$$

$$V = 800 \times \int_0^{20} e^{-\ln(1.05)t} \cdot e^{-0.008t} dt$$

$$= 800 \int_0^{20} e^{-0.05679t} dt$$

$$= 800 \left[\frac{-e^{-0.05679t}}{0.05679} \right]_0^{20}$$

$$= 800 (-5.65531 + 17.60873) = 9562.8$$

Value of RoP

$$p \times e_{45}^{65} (1.006 + 0.002) dt \cdot \int_0^{20}$$

$$p \times e^{-0.01t} \times 0.37189$$

Premium = Value of Benefits on Death & Cancer + value of RoI + Commission + Expenses + Profit Margin

$$P = 9562.8 + 0.32164 \times P + 0.02 \times P + 0.02 \times P + 0.1 \times P$$

$$\underline{P = 17747.15}$$

Q7. PV of net cashflows = PV of Premium - PV of benefits - PV of expenses - PV of commission

Let max for initial commission be I

$$P.V. \text{ of Premium} = 30000 \cdot \ddot{a}_{[45]:20}$$

$$= 30000 \times 11.888$$

$$= 356640$$

$$P.V. \text{ of benefits} = 1000000 \times A_{[45]:20}$$

$$= 327110$$

$$P.V. \text{ of expenses} = 5000 + 2.5\% \text{ of } 30000 \times (\ddot{a}_{[45]:20} - 1)$$

$$+ 500 \times P_{[45]} \times 1.06^{-1} \times (1 + 1.019231 \times v$$

$$\times P_{[46]} + 1.019231 \times v^2 \times 2P_{[46]})$$

$$\dots + 1.019231 \times v^{18} \times P_{[46]} \times P_{[46]}$$

$$= 5000 + 2.5\% \cdot 30000 (\ddot{a}_{[45]:20} - 1) + 500 \times P_{[45]} \times 1.06 \times \ddot{a}_{[46]:19} @ 4\%$$

$$= 19488$$

$$\begin{aligned} \text{P.V of renewal commission} &= 2\% \text{ of } 30000 \times (\ddot{a}_{[45]:20} - 1) \\ &= 6000 \times 11.888 - 1 \\ &= 6533 \end{aligned}$$

$$\begin{aligned} \text{PV of net CF} &= 3557 - 1356640 - 327110 - 19128 - 6533 \\ &= 3559 - 1 \end{aligned}$$

$$\text{Margin} = \frac{\text{PV of net CF}}{\text{Annual Premium}}$$

$$= \frac{3559 - 1}{30000} = 10\%$$

$$1 = \text{Rs. } 559$$

Q8. i) The assumption of independence of decrements is used to derive single decrement table from a multiple decrement table.

Mathematically:

$$(q_y)_x^j = q_x^j \text{ for all } j \text{ and all } x$$

When we look at transition intensities (forces of decrement) we are looking at infinitesimally small time interval for which there is only time for one decrement. Thus it is reasonable to assume that the independent and dependent forces of decrement are equal.

ii. In case of term assurance, it is likely that people who lapse their policies have a lower than average mortality.

iii.

$$a) 1000 \int_0^5 e^{-\delta t} (tP_{45}^{PI} + tP_{45}^{PP} v_{45+t}) dt$$

$$b) 250 \int_0^5 e^{-\delta t} tP_{45}^{IP} v_{45+t} dt$$

$$c) 150 \int_0^5 e^{-\delta t} tP_{45}^{IP} dt$$

$$d) 250 \int_0^5 e^{-\delta t} tP_{45}^{PP} v_{45+t} dt$$

$$iv. (aq)_{45}^C = \frac{0.05}{0.18} (1 - e^{-0.18}) = 0.045758$$

$$(aq)_{45}^S = \frac{0.13}{0.18} (1 - e^{-0.18}) = 0.118972$$

$$(ap)_{45} = 1 - 0.045758 - 0.118972 = 0.835270$$

$$(aq)_{46}^C = \frac{0.06}{0.16} (1 - e^{-0.16}) = 0.055446$$

$$(aq)_{46}^S = \frac{0.1}{0.16} (1 - e^{-0.16}) = 0.092410$$

$$(ap)_{46} = 0.852114$$

$$\text{Possibility of being in force at age 47} = 0.835270 \times 0.852114 = \underline{\underline{0.711770}}$$

Q9.

- i. Under with profit policies the contract between policyholder and the company is such that, whatever is the surplus arising on the underlying fund, would be shared between policyholders and shareholders in a predetermined manner.

Now, as the surplus arises over future years the same is shared with policyholders in form of bonuses and hence the need of bonuses under with profit policies.

- ii. Investment surplus
Expense surplus
Mortality surplus
Lapse/surrender surplus
Reinsurance surplus

- iii. Terminal bonus would be a strategy to build lower guarantees over the lifetime of with profit policies.

If the proportion of RB and TB is skewed towards TB ~~to~~ then it would mean lower guarantees to policyholders and hence companies would have greater freedom in terms of choosing investment assets.

By doing this, they can take risk of investing in real assets like equities and real estate which can lead to better final results to policyholders, which.

would be redistributed through higher TB.

810. Let monthly premium be P

$$\text{EPV of premium} = 12 \times P \times \ddot{a}_{50:10}^{(12)} \text{ at } 6\%$$

$$\ddot{a}_{50:10}^{(12)} = \ddot{a}_{50:10} - \frac{11}{24} \times (1 - 10p_{50} \times v^{10})$$

$\uparrow$ $\downarrow$
 7.694 0.728

$$\ddot{a}_{50:10}^{(12)} = 7.694 - \frac{11}{24} \times (1 - 10.06 \times \frac{9287-2164}{9712-0728})$$

$$= 7.4804$$

$$\text{EPV of premiums} = 89.7648 \times P$$

$$\text{EPV of benefits} = 10,000,000 \times (q_{50} \times v + 1.019231 \times v^2 \times 1/9_{50}$$

$$+ \dots + 1.019231^{14} \times v^{15} \times 14/9_{50}$$

$$+ 1.019231^{15} \times v^{15} \times 15/9_{50})$$

$$= \frac{10,000,000}{1.019231} \times (q_{50} \times v \times 1.019231 + 1.019231^2 \times v^2 \times 1/9_{50} + \dots$$

$$\dots + 1.01931 v^{15} \times 15/9_{50}$$

$$+ 1.01931^{15} \times v^{15} \times 15/9_{50})$$

→

Converting $\Rightarrow$ to 4%.

$$\begin{aligned} \text{EPV of benefits} &= 1000000 \times \left(\frac{1}{1.019231} \right) \times A_{60:15}^{\text{4\%}} \\ &+ 1000000 \times v^{15} \text{ at } 6\% \times {}_{15}p_{60} \\ &\times (1.019231^{15} - 1.019231) \end{aligned}$$

$$\text{EPV} = 566004.1$$

EPV of expense & commission

$$\begin{aligned} &= \frac{20+35}{100} \times 12 \times P + \left(\frac{2+15}{100} \right) \times 100 \times 1.06^{-41} \\ &\times 12 P \times \ddot{a}_{51:\overline{9}|}^{(12)} \end{aligned}$$

$$+ 2500 + 400 \times 100 \times 1.06^{-1} \times \ddot{a}_{51:\overline{14}|}^{\text{at } 6\%}$$

$$= 9.3333P + 6115.06$$

$\therefore \text{EPV of premiums} = \text{EPV of benefits} + \text{EPV of expense \& commission}$

$$89.7648 \times P = 566004.1 + 9.3333P + 6115.06$$

$$80.4315 \times P = 572119.16$$

$$P = \underline{\underline{7113.12}}$$

ii. Sum Assured at end of 10th year + Accrued bonuses

$$X = 1000000 \times 1.02^{10}$$

$$X = 1218994$$

PV of future benefits at end of 10th year

$$= \frac{X}{1.05} \times 9\% + \frac{X}{1.05} \times 1.04 \times 11\% + \frac{X}{1.05^2} \times 1.04^2 \times 12\% + \frac{X}{1.05^3} \times 1.04^3 \times 13\% + \frac{X}{1.05^4} \times 1.04^4 \times 14\% + \frac{X}{1.05^5} \times 1.04^5 \times 15\%$$

$$= 0.952948 \times X$$

$$= 1160774$$

PV of expenses = $400 \times \ddot{a}_{60:\overline{5}|} @ 5\%$

$$= 400 \times \left(1 + 1.05^{-1} \times \frac{l_{61}}{l_{60}} + 1.05^{-2} \times \frac{l_{62}}{l_{60}} + 1.05^{-3} \times \frac{l_{63}}{l_{60}} + 1.05^{-4} \times \frac{l_{64}}{l_{60}} \right)$$

$$= 1787.39$$

Reserves required = PV of benefits + PV of expenses

$$= \underline{\underline{1162561}}$$

11.

$$i = 6\%$$

$$SA = 400000$$

$$\text{Simple Bonus} = 40 \text{ per } 1000 \text{ SA}$$

Net premium ~~is~~ prospective reserve at 7

$$\text{EPV of Premium} = P \ddot{a}_{30:\overline{20}|}$$

$$\text{EPV of benefit} = \cancel{P \ddot{a}_{30}} 400000 A_{30:\overline{20}|}$$

$$\therefore P \ddot{a}_{30:\overline{20}|} = 400000 A_{30:\overline{20}|}$$

$$P = 400000 A_{30:\overline{20}|} (0.315817) / 2.08705$$

$$P = 10451.42$$

$$\begin{aligned} \text{Total bonus at time 7} &= 7 \times 0.04 \times 400000 \\ &= 112000 \end{aligned}$$

$$SA \text{ at time 7} = 400000 + 112000 = 512000$$

Reserve at time 7

$$= 512000 A_{37:\overline{13}|} - 10451.42 \ddot{a}_{57:\overline{13}|}$$

$$= 512000 (0.47128) - 10451.42 (9.335835)$$

$$= 143926.89$$