Numerical Methods of Algebra Assignment

Q.1. Wrong → 12.65 Right: 12.5 Area $= \pi r^2$

i.) Absolute Error → $A_R - \pi \times (12.5)^2 = 491.0714$

$A_W = \pi \times (12.65)^2 = 502.92778$ Error → 11.8564578

ii.) Relative Error → (Power of 'x') × (% error in R)

~~12.6~~ $0.15 = \frac{x}{100} \times 12.5$ → $x = 1.2\%$

∴ Relative Error: $2 \times 1.2\% = 2.4\%$ Error in Area.

iii.) Percentage Error: $\frac{11.8564578}{491.0714} \times 100 = \underline{2.41\%}$

Q.2

i.) $x = 4$ | $\log x = 0.60206$

$x = 5$ | ↓(↑1) | $\log x = ?$ | x ↓ | 0.1760913

$x = 6$ | $\log x = 0.7781513$ ↓

$x+2$

↑2 → ↑ 0.1760913.

↑1 → x $x = 0.088045$.

∴ $\log 5 = \log 4 + 0.088045 = \underline{0.69010565}$

ii.) $x = 4.5$ | $\log x \rightarrow 0.6532125$ | ↑1 → 0.0871502

$x = 5$ | $\log x \rightarrow x$: 0.0871502 | 0.5 → x

$x = 5.5$ ↓1 | $\log x \rightarrow 0.7403627$ ↓ | $x = 0.04435751$

∴ $\log 5 - \log 4.5 + 0.04435751 = 0.69679876$

Q.3 $x^4 - x - 10 = 0$

$x_1 = \frac{a+b}{2}$

a	b	y_1	y_2	x_1	y_1
1.5	2	-6.4375	4	1.75	-2.37109

x	y	y
1.75	-2.3710	$x_2 = 1.875$
2	4	$y_2 = 0.484619$

x	y	x_3	y_3
1.75	-2.3710	1.8125	-1.0202
1.875	0.4848		

x	y	x_4	y_4
1.8125	-1.0202	1.84375	-0.26473
1.875	0.4848		

x	y	x_5	y_5
1.84375	-0.2847	1.859375	0.093378
1.875	0.4844		

∴ Root: 1.859375

8.4 $f(x) = x^3 - x - 1$ $\quad f'(x) = 3x^2 - 1$

x	$f(x)$
0	-1
2	5

$x_0 = 0$

$f(x_0) = -1$

$f'(x_0) = -1$

$x_1 = 0 - \frac{(-1)}{(-1)} = 1 = -1 - x_1$

$f(x_1) = -3$ $\quad f'(x_1) = 2$

$\rightarrow x_2 = (-1) - \left(\frac{-3}{2}\right) = 0.5$ (1.5)

$f(x_2) = -1.375$, $\quad f'(x_2) = -0.25$

$x_3 = 0.5 + \left(\frac{1.375}{-0.25}\right) = -5.$

$f(x_3) = -121$

Q.5 $A^2 = A$

$\rightarrow 7A - (1+A)^3 = 7A - [I^3 + A^3 + 3(A \cdot I)[I + A]]$

$= 7A - I^3 - A^3 - 3(A)(I+A)$

$= 7A - I^3 - (A^2 \cdot A) - 3[AI + A^2]$

$= 7A - I^3 - A^2 - 3A - 3(A^2) \rightarrow 7A - 3A - 3A - A - I^3 = -I^3$

ANS $= -I$

Q.6 $\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix} \times A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\rightarrow R_2 - 4R_1$

$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 4 & -2 \\ 0 & 1 & -1 \end{bmatrix} \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\rightarrow R_2 \rightarrow R_2 - 3R_3 \;||\; R_3 \rightarrow R_3 - R_2$

$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & -3 \\ 4 & -1 & 4 \end{bmatrix}$

$R_1 \rightarrow R_1 + R_2$

$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -3 & 1 & -3 \\ -4 & 1 & -3 \\ 4 & -1 & 4 \end{bmatrix}$

$R_2 \rightarrow R_2 - R_3 \;||\; R_1 \rightarrow R_1 - 3R_3$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} -15 & 4 & -15 \\ -8 & 2 & -7 \\ 4 & -1 & 4 \end{bmatrix}$ $\quad \therefore A^{-1} \times I = A^{-1} = \begin{bmatrix} -15 & 4 & -15 \\ -8 & 2 & -7 \\ 4 & -1 & 4 \end{bmatrix}$

$\sin 90° = 1$

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Q.12

Q.7 $\bar{A} = 8i - 9j \quad \bar{B} = 7i - 9j$

Dot Prod $= \bar{A} \cdot \bar{B} = 8(7) + (-9 \times -9) = 56 + 81 = 147$

$\bar{A} \cdot \bar{B} = |\bar{A}| \times |\bar{B}| \times \cos\theta$

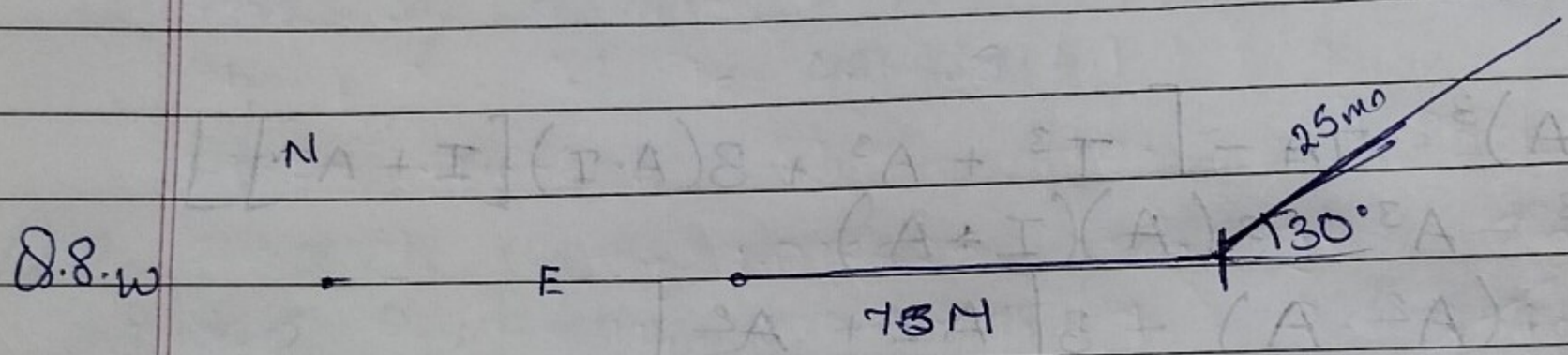
$|\bar{A}| = \sqrt{8^2 + (-9)^2} = \sqrt{145}$

$|\bar{B}| = \sqrt{7^2 + (-9)^2} = \sqrt{130}$

$\therefore 147 = \sqrt{145} \times \sqrt{130} \times \cos\theta$

$\boxed{\cos\theta = 1.07068.}$

Q.8.



$\rightarrow \bar{S} = 75 \times 75i + 25j \times \cos 30°$

$|\bar{S}| = \sqrt{75^2 + 25^2 \times \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{5625 + \frac{1875}{4}} = \sqrt{5625 + 468.75}$

$\boxed{\bar{S} = 78.0624}$

Q.9 $a = 101 \quad l = 998 \quad d = 3$

$l = T_n = 998 = 101 + (n-1)3 \quad \therefore \frac{897}{3} = n - 1 \quad \boxed{n = 300}$

$S_n = \frac{300}{2}\left[101 * 998\right] = (150 \times 1099)$

$= \boxed{164850 \rightarrow S_n}$

Q.10 $T_6 = ar^5 = 32 \quad T_8 = ar^7 = 128$

$\frac{T_8}{T_6} \quad \frac{ar^7}{ar^5} = \frac{128}{32} = 4 \quad \therefore \boxed{r = \pm 2}$

Q.11 $(4 + 3x)^5 \rightarrow a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$= (4)^5 + 5(4)^4(3x) + 10(4)^3 9x^2 + 10(16)(27x^3) + 5(4)(81x^4) + 243x^5$

$\rightarrow (1024) + (3840x) + (5760x^2) + (4320x^3) + 1620x^4 + 243x^5$

Q.12 $\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow \lambda^3 - 0(2-5+3)\lambda^2 + (-15+6-4)\lambda - [2(-15)+3(6)] = 0$

$= \lambda^3 - 13\lambda + 12 = 0$

$\lambda = 1$ & ~~$\lambda = -1$~~

	1	0	−13	12
1	↓ (+)1	(+) 1	(+) −12	
	1	1	−12	0

$= x^2 + x - 12 = 0$

$x^2 + 4x - 3x - 12 = 0$

$(x+4)(x-3) = 0$

$\therefore$ $x = 3$ or $x = -4$ or $x = 1$

$\lambda = 3$ or $\lambda = -4$ or $\lambda = 1$

Q.13 $f(x) = x^3 - 7x^2 + 8x - 3$

$f'(x) = 3x^2 - 14x + 8$, $x_0 = 5$

~~$x_1 = 5 \rightarrow f(x_0) = -13$~~ $f'(x_0) = -13$

$x_1 = 5 - \left(\frac{-13}{-13}\right)$ ~~$x_1 = 4.40909$~~

$x_1 = 6$

$f(x_1) = -18.095$

$f'(x_1) = 32$

$x_2 = 4.409 - \frac{-18.095}{32}$

~~$x_2 = 3.742022$~~ $\quad -26.2711$

$x_2 = 5.71875$

Q.14 $(x^2 - x + 1)^4 \rightarrow [x^2 - (x-1)]^4$ $\quad x - 1 = y$

$(x^2 - y)^4 = (x^2)^4 - 4(x^2)^3 y + 6(x^2)^2 y^2 - 4x^2 y^3 + y^4$

$\rightarrow x^8 - 4x^6 y + 6x^4 y^2 - 4x^2 y^3 + y^4$

$\rightarrow x^8 - 4[x^6(x-1)] + 6x^4(x-1)^2 - 4x^2(x-1)^3 + (x-1)^4$

$= x^8 - 4x^7 + 4x^6 + 6x^6 + 6x^4 - 12x^5 - 4x^2[x^3 - 1 - 3x^2 + 3x] + \{x^4 - 4x^3 + 6x^2 - 4x + 1\}$

$= (x^8 - 4x^7 + 4x^6 + 6x^6 + 6x^4 - 12x^5) - 4x^5 + 4x^2 + 12x^4 - 12x^3 + x^4 - 4x^3 + 6x^2 - 4x + 1$

$\rightarrow x^8 - 4x^7 + 10x^6 - 16x^5 + 19x^4 - 16x^3 + 10x^2 - 4x + 1$

Q.15 $e^{-x} = 3\log x \quad \therefore 0 = f(x) = 3\log x \times e^{x}$

$= e^{x} \cdot \log x^{3}$

x	y
~~0.5~~ ~~0.5~~	0.5
1	1.5

$x = 0.5 \quad -1.488$

$1.5 \quad 2.8675$

$x_2 = \frac{2}{2} = 1$

x	y
~~0.5~~ 1	0
1.5	2.3675

$y_2 = 0$

$x_3 = \frac{2.5}{2} = ~~0.75~~ \; 1.25$

x	y
1	0
1.25	1.014

$y_3 = 1.01474$

$x_4 = 1.125$

$y_4 = 0.47268.$

x	y
1	0
1.125	0.47268

$x_5 = 1.0625$

$y_5 = 0.2285$

x	y
1	0
1.0625	0.2285

$x_6 = 1.03125$

$y_6 = 0.11244$

x	y
1	0
1.03125	0.1124

$x_7 = 1.015625$

$y_7 = 0.05577$

x	y
1	0
1.015	0.055

$x_8 = 1.0078125$

$y_8 = \underline{0.0277}$

$\therefore$ **Root: 1.0078**

Q.4 $f(x) = x^3 + x - 1 \qquad f'(x) = 3x^2 + 1$

$x = -1 \quad f(-1) = -3 \quad f(0) = -1 \quad f(1) = 1$

$x_{old} = 0 \quad f(0) = -1 \quad f'(-1) = 4$

$x_1 = 0 - \left(\frac{-1}{4}\right) = 0.25 = x_1 \quad f(x_1) = -0.7343$

$f'(x_1) = 1.1875$

$x_2 = 0.25 - \left(\frac{-0.7343}{1.1875}\right) = 0.8635 \quad f(x_2) = 0.5073$

$f'(x_2) = 3.23$

$f(1) = 1 \quad x_0 = 1 \quad f'(1) = 4$

$x_1 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{1}{4} = 0.75 \quad f(x_1) = 0.171875$

$f'(x_1) = 2.6875.$

$x_2 = 0.75 - \frac{0.171875}{2.6875} = 0.6860465 \quad f(x_2) = 0.003925$

$f'(x_2) = 2.41195$

$x_3 = 0.6860465 - 0.0037003 = 0.68234$

$f(x_3) = 0.00002923$

$\therefore$ **Root = 0.68234**