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Q1 $\lim_{h\to 0} \frac{2(-3+h)^2 - 18}{h}$

$\lim_{h\to 0} \frac{2(h^2+9-6h)-18}{h}$

$\lim_{h\to 0} \frac{18+2h^2-12h-18}{h}$

$\lim_{h\to 0} \frac{h(2h-12)}{h}$

$= -12$

Q2) $g(x) = \begin{cases} 2x & x<6 \\ x-1 & x \geq 6 \end{cases}$

a) $x=4$ $\quad \lim_{x\to 4^+} 2(4+h)$

$= 8$

$\lim_{x\to 4^-} 2(4-h)$

$= 8$

$\lim_{x\to 4} 2(4)$

$= 8$

hence continuous.

b) $\quad \lim_{x\to 6^+} 6+h-1$

$= 5$

$$\lim_{x \to 6^-} 2(6-h)$$

$$= 12$$

Hence discontinuous.

Q3 $$\lim_{x \to 9} \frac{x-9}{3-\sqrt{x}}$$

$$\lim_{x \to 9} \frac{(\sqrt{x})^2 - (3)^2}{3-\sqrt{x}}$$

$$\lim_{x \to 9} \frac{(\sqrt{x}+3)\cancel{(\sqrt{x}-3)}}{\cancel{(\sqrt{x}-3)}}$$

$$\lim_{x \to 9} -(\sqrt{x}+3)$$

$$= -(3+3) = -6$$

$$-(-3+3) = 0$$

Q4) $$f(x) = \frac{4x+5}{9-3x}$$

a) $$\lim_{x \to -1} \frac{4(x)+5}{9-3f(x)}$$

$$= \frac{-4+5}{9+3}$$

$$= \frac{1}{12}$$

b) $\lim_{x \to 0} \frac{4(x)+5}{9-3x}$

$= \frac{0+5}{9-0}$

$= \frac{5}{9}$

c) $\lim_{x \to 0} \frac{4x+5}{9-3x}$

$\frac{4(3)+5}{9-9}$

Infinite Discontinuous.

Q5) $\lim_{x \to \infty} \frac{6e^{4x} - e^{-3x}}{8e^{4x} - e^{2x} + 3e^{-x}}$

$\lim_{x \to \infty} \frac{e^{4x} \; (6 - e^{-6x})}{e^{4x} \; (8 - e^{-2x} + 3e^{-5x})}$

$\frac{6}{8} = \frac{3}{4}$

Q6) $g(y) = \begin{cases} y^2+5 & \text{if } y < -2 \\ 1-3y & \text{if } y \geq -2 \end{cases}$

a) $\lim_{y \to 6} g(y)$

$\lim_{y \to 6} 1 - 3(\cancel{x}y)$

$= 1-3(6) = -17$

b) $\lim_{x \to -2} g(y)$

$$\lim_{x \to -2^+} 1 - 3(-x + h)$$

$$= 1 - 3(-2 + 0)$$
$$= 7$$

$$\lim_{x \to -2^-} (x - h)^2 + 5$$

$$= 4 + 5$$
$$= 9$$

Not continuous.

Q7) $f(t) = (4t^2 - t)(t^3 - 8t^2 + 12)$

diff w.r.t. t.

$$f'(t) = (4t^2 - t)(3t^2 - 16t) + (t^3 - 8t^2 + 12)(8t - 1)$$

$$= 12t^4 - 64t^3 - 3t^3 + 16t^2 + 8t^4 - 64t^3 + 96t + 8t^2 - 12$$

$$= 20t^4 - 132t^3 + 24t^2 + 96t - 12$$

Q8) $f(w) = \dfrac{3w + w^4}{2w^2 + 1}$

diff. w.r.t. w

$$= \frac{(2w^2+1)(3+4w^3)-(3w+w^4)(4w)}{(2w^2+1)^2}$$

$$= \frac{6w^2+6w^5+3+4w^3-12w^2+4w^5}{(2w^2+1)^2}$$

$$= \frac{2w^5-6w^2+4w^3+3}{(2w^2+1)^2}$$

$$= \frac{2w^5-6w^2+4w^3+3}{4w^4+1+4w^2}$$

Q9) $f(x) = 2e^x - 8^x$

diff. w.r.t. x

$f'(x) = 2e^x - 8^x \log 8$

Q10) $y = z^5 - e^z m(z)$

diff. w.r.t. z

$y' = 5z^4 - e^z m(z) - \frac{e^z}{z}$

Q11) a) $f(x) = (6x^2+7x)^4$

diff w.r.t. x

$f'(x) = 4(6x^2+7x).(12x+7)$

b) $f(t) = 5+e^{4t+t^2}$

diff. w.r.t. t

$f'(t) = 0 + e^{4t+t^2}(4+2t)$

Q 12) a) $Z = \ln(7 - x^3)$

diff w.r.t. x

$$Z' = \frac{-3x^2}{7 - x^3}$$

$$= \frac{(7-x^3)(-6x) - (-3x^2)\cdot(0-3x^2)}{(7-x^3)^2}$$

$$Z'' = \frac{-42x + 6x^4 - 9x^4}{(7-x^3)^2}$$

$$= \cancel{\frac{-42x - 3x^4}{(7x - 3x^3}} = \frac{-42x - 3x^4}{(7 - x^3)^2}$$

b) $g(v) = \frac{2}{(6 + 2x - x^2)^4}$

$$g'(v) = \frac{(6+2x-x^2)^4 \cdot (0) - 2 \cdot 4(6+2x-x^2) \cdot (2-2x)}{(6+2x-x^2)^8}$$

$$= -16(1-v)(6 + 2v - v^2)^{-5}$$

diff w.r.t. x

$$g''(v) = -16\Big((1-v)(-5)(6+2v-v^2)^{-6}$$

$$(2-2v) + (-1)(6+2v-v^2)^{-5}\Big)$$

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$= (16)(6+2v+v^2)^{-5}((1-v)(2-2v)(5)+1)$

$g''(v) = (16)(6+2v+v^2)^{-5}((1-v)(2-2v)(5)+1)$

c) $f(t) = \ln(1+t^2)$

diff. w.r.t.t.

$$f'(t) = \frac{2t}{1+t^2}$$

diff. w.r.t.t.

$\cancel{f''(t)} = \cancel{2} \; \cancel{(1+t^2)} -$

$$f''(t) = \frac{(1+t^2)(2)-(2t)(2t)}{(1+t^2)^2}$$

$$f''(t) = \frac{2+2t-4t}{(1+t^2)^2}$$

Q13) $f(x) = x^3 - 3x^2 - 9x + 12$

diff. w.r.t. x.

$f'(x) = 3x^2 - 6x - 9$

$= 3(x^2 - 2x - 3)$

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(-3+x) + 1(x-3) = 0$

$(x-3)(x+1) = 0$

$\boxed{x = -1}$ $\boxed{x = 3}$

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$f''(x) = 6x - 6$

$f''(-1) = 6(-1) - 6 = -12 \rightarrow$ max

$f''(3) = 6(3) - 6 = 12 \rightarrow$ min

Q 14) $f(x) = 4x^3 - 18x^2 + 24x - 7$

Diff w.r.t. x

$f'(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2) = 0$

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2)$

$(x-2) \quad (x-1)$

$x = 2 \quad x = 1$

$f''(x) = 12(2x - 3)$

$= 12(2(1) - 3) = -12$

$f''(2) = 12(2(2) - 3) = 12$

$f(1) = 3$

$f(2) = 1$

Q15) $f(x) = x^2 (x+3)$

$= x^3 + 3x^2$

diff. w.r.t. x

$f'(x) = 3x^2 + 6x$

let $f'(x) = 0$

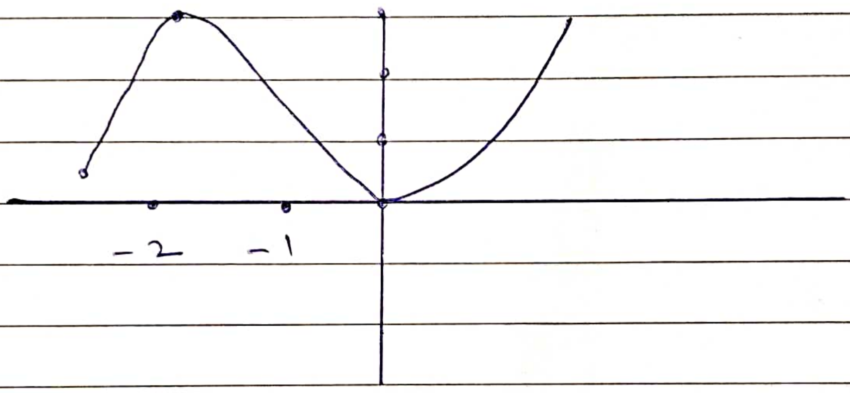
$3x(x+2) = 0$

$x = -2$ $\quad$ $x = 0$

$f''(x) = 6x + 6$

$f''(0) = 6$ (Minimum)

$f''(-2) = -6$ (Maximum)



Q16) $f(x) = 3x^2 - 6x$ $\quad$ $f(1) = 3 - 6 = -3$

$f(0) = 0$

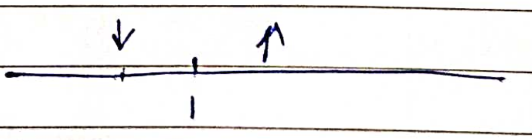
diff. w.r.t. x.

$f'(x) = 6x - 6$

$f'(x) = 0$

$6(x-1)$

$x = 1$



$f''(x) = 6$ (minimum)