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Homework - 1

Lagrange's Interpolation

x	7	8	9	10	9.5
$f(x)$	3	1	1	9	?

$$\begin{aligned}
 f(9.5) &= \frac{(9.5-8)(9.5-9)(9.5-10)(3)}{(7-8)(7-9)(7-10)} + \\
 &\quad \frac{(9.5-7)(9.5-9)(9.5-10)(1)}{(8-7)(8-9)(8-10)} + \\
 &\quad \frac{(9.5-7)(9.5-8)(9.5-10)(1)}{(9-7)(9-8)(9-10)} + \\
 &\quad \frac{(9.5-7)(9.5-8)(9.5-9)(9)}{(10-7)(10-8)(10-9)} \\
 &= \frac{(1.5)(0.5)(-0.5)(3)}{(-1)(-2)(-3)} + \frac{(2.5)(0.5)(-0.5)(1)}{(1)(-1)(-2)} + \\
 &\quad + \frac{(-2.5)(1.5)(-0.5)(1)}{(2)(1)(-1)} + \frac{(2.5)(1.5)(0.5)(9)}{(3)(2)(1)}
 \end{aligned}$$

$$= 0.1875 - 0.3125 + 0.9375 + 2.1875$$

$$\therefore f(9.5) = \underline{\underline{3.625}}$$

2.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
3	168			

-48

7 120

0

-48

39

9 72

39

 $= \Delta^3 f(x_n)$

-9

 $= \Delta^2 f(x_n)$

10 63

 $= \Delta f(x_n)$ $= f(x_n)$

$$f(6) = \frac{(6-7)(6-3)(6-9)(6-10)(168)}{(3-7)(3-9)(3-10)} + \frac{(6-3)(6-9)(6-10)(120)}{(7-3)(7-9)(7-10)} \\ + \frac{(6-3)(6-7)(6-10)(72)}{(9-3)(9-7)(9-10)} + \frac{(6-3)(6-7)(6-9)(63)}{(10-3)(10-7)(10-9)}$$

$$= \frac{(-1)(-4)(-7)(-10)(168)}{(-4)(-6)(-8)} + \frac{(3)(-3)(-4)(120)}{(4)(-2)(-3)} \\ + \frac{(3)(-1)(-4)(72)}{(6)(2)(-1)} + \frac{(3)(-1)(-3)(63)}{(7)(3)(1)}$$

$$= \frac{-2016}{-168} + \frac{4320}{24} + \frac{864}{-12} + \frac{567}{21}$$

$$= 12 + 180 - 72 + 27$$

$$\therefore \underline{\underline{f(6) = 147}}$$

3.

x	1	2	7	8	6
y	4	5	5	4	?

$$f(6) = \frac{(6-2)(6-7)(6-8)(4)}{(1-2)(1-7)(1-8)} + \frac{(6-1)(6-7)(6-8)(5)}{(2-1)(2-7)(2-8)} \\ + \frac{(6-1)(6-2)(6-8)(5)}{(7-1)(7-2)(7-8)} + \frac{(6-1)(6-2)(6-7)(4)}{(8-1)(8-2)(8-7)}$$

$$= \frac{(-4)(-1)(-2)(4)}{(-1)(-6)(-7)} + \frac{(5)(-1)(-2)(5)}{(1)(-5)(-6)} \\ + \frac{(5)(4)(-2)(5)}{(6)(5)(-1)} + \frac{(5)(4)(-1)(4)}{(7)(6)(1)}$$

$$= \frac{\cancel{42} 32}{-42} + \frac{50}{30} + \frac{-200}{-30} + \frac{-80}{42}$$

$$= -0.7619 + 1.666 + 6.666 - 1.90476$$

$$\therefore \underline{\underline{f(6) = 5.66594}}$$

4.	x	1	2	3	4	7
	y	2	4	3	16	128

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)(2)}{(1-2)(1-3)(1-4)(1-7)} + \frac{(5-1)(5-3)(5-4)(5-7)(4)}{(2-1)(2-3)(2-4)(2-7)} \\ + \frac{(5-1)(5-2)(5-4)(5-7)(3)}{(3-1)(3-2)(3-4)(3-7)} + \frac{(5-1)(5-2)(5-3)(5-7)(16)}{(4-1)(4-2)(4-3)(4-7)} \\ + \frac{(5-1)(5-2)(5-3)(5-4)(128)}{(7-1)(7-2)(7-3)(7-4)}$$

$$= \frac{\cancel{3}(2)(1)(-2)\cancel{2}}{(1)\cancel{2}\cancel{3}(6)} + \frac{(4)(2)(1)\cancel{2}(4)}{(1)(1)\cancel{2}(5)} \\ + \frac{\cancel{4}(3)(1)\cancel{2}(8)}{(2)(1)(1)\cancel{4}} + \frac{(4)(3)\cancel{2}(2)(16)}{\cancel{3}\cancel{2}(1)(3)} \\ + \frac{\cancel{4}\cancel{3}\cancel{2}(1)(128)}{\cancel{6}(5)\cancel{4}(3)}$$

$$= \frac{-4}{6} + \frac{32}{5} + \frac{-24}{2} + 42.666 + 8.5333$$

$$\therefore \underline{\underline{f(5) = 32.9\bar{3}}}$$

5.

x	10	20	14	16	15
y	0.25	0.2	0.15	0.1	?

$$f(15) = \frac{(15-20)(15-14)(15-16)(0.25)}{(10-20)(10-14)(10-16)} + \frac{(15-10)(15-14)(15-16)(0.2)}{(20-10)(20-14)(20-16)} \\ + \frac{(15-10)(15-20)(15-16)(0.15)}{(14-10)(14-20)(14-16)} + \frac{(15-10)(15-20)(15-14)(0.1)}{(16-10)(16-20)(16-14)}$$

$$= \frac{(-5)(1)(-1)(0.25)}{(-10)(-4)(-6)} + \frac{(5)(1)(-1)(0.2)}{(10)(6)(4)} \\ + \frac{(5)(-5)(-1)(0.15)}{(4)(-6)(-2)} + \frac{(5)(-5)(1)(0.1)}{(6)(2)(-4)}$$

$$= -0.005083 - 0.0041667 + 0.078125 \\ + 0.0520833$$

$$\therefore \underline{\underline{f(15) = 0.120833}}$$

Homework - 2.

Regula Falsi

$$1. \quad f(x) = x^3 + x - 1 = 0 \quad 0.5 < x < 1$$

	x	y
	0.5	-0.375
$x_0 \leftarrow$	0.6	-0.184 $\rightarrow y_0$
$x_1 \leftarrow$	0.7	0.043 $\rightarrow y_1$

$$x_2 = \frac{(0.6 \times 0.043) - (0.7 \times -0.184)}{0.043 - (-0.184)}$$

$$= \underline{\underline{0.681057}}$$

$$y_2 = \underline{\underline{-0.00304180}} \text{ — replaces } y_0$$

$$x_3 = \frac{(0.681057 \times 0.043) - (0.7 \times \overset{-0.0030418}{\cancel{-0.184}})}{0.043 - (-0.0030418)}$$

$$= \underline{\underline{0.682304848}}$$

$$y_3 = \underline{\underline{-0.0000550366}}$$

$$\therefore \text{root is } x = \underline{\underline{0.682304848}}$$

2. $f(x) = x^3 - 2x + 0.5 = 0 \quad 0 < x < 1$

	x	y
	0.1	0.801
$x_0 \leftarrow$	0.2	0.108 $\rightarrow y_0$
$x_1 \leftarrow$	0.3	-0.0703 $\rightarrow y_1$

$$x_2 = \frac{(0.2 \times -0.0703) - (0.3 \times 0.108)}{-0.0703 - 0.108}$$

$$= \underline{\underline{0.2596685083}}$$

$$y_2 = \underline{\underline{-0.0018281574}} \quad \text{--- replaces } y_1$$

$$x_3 = \frac{(0.2 \times -0.0018281574) - (0.2596685083 \times 0.108)}{-0.0018281574 - 0.108}$$

$$= \underline{\underline{0.2586748734}}$$

$$y_3 = \underline{\underline{-0.0000411152}}$$

$\therefore$ root is $x = \underline{\underline{0.2586748734}}$

$$3. \quad f(x) = x^3 + 2x - 20 = 0 \quad 2 < x < 2.5$$

x	y
2.3	-3.233
$x_0 \leftarrow 2.4$	-1.376 $\rightarrow y_0$
$x_1 \leftarrow 2.5$	0.625 $\rightarrow y_1$

$$x_2 = \frac{(2.4 \times 0.625) - (2.5 \times -1.376)}{0.625 - (-1.376)}$$

$$= \underline{\underline{2.4687656172}}$$

$$y_2 = \underline{\underline{-0.0158270149}} \quad \text{--- replaces } y_0$$

~~$$x_3 = \frac{(2.4687656172 \times -0.0158270149) - (2.5 \times -1.376)}{-0.0158270149 - (-1.376)}$$~~

$$x_3 = \frac{(2.4687656172 \times 0.625) - (2.5 \times -0.0158270)}{0.625 - (-0.0158270)}$$

$$= \underline{\underline{2.469537038}}$$

$$y_3 = \underline{\underline{-0.0001747905}}$$

$$\therefore \text{root is } x = \underline{\underline{2.469537038}}$$

4. $f(x) = 3x - \cos x - 1 = 0 \quad 0 < x < 1$

x	y
0.5	- 0.4999619231
$x_0 \leftarrow 0.6$	- 0.1999451694 $\rightarrow y_0$
$x_1 \leftarrow 0.7$	0.1000746303 $\rightarrow y_1$

$$x_2 = \frac{(0.6 \times 0.1000746303) - (0.7 \times -0.1999451694)}{0.1000746303 - (-0.1999451694)}$$

$$= 0.6666439914$$

$$y_2 = -0.0000003384$$

$$\therefore \text{root is } x = 0.6666439914$$

PTO

5. $f(x) = x^3 - 3x - 5 = 0$ $2 < x < 2.5$

x	y
2.1	-2.039
$x_0 \leftarrow 2.2$	$-0.952 \rightarrow y_0$
$x_1 \leftarrow 2.3$	$0.267 \rightarrow y_1$

$$x_2 = \frac{(2.2 \times 0.267) - (2.3 \times -0.952)}{0.267 - (-0.952)}$$

$$= \underline{\underline{2.2780968007}}$$

$$y_2 = \underline{\underline{-0.0115944071}} \text{ — replaces } y_0$$

$$x_3 = \frac{(2.2780968007 \times 0.267) - (2.3 \times -0.0115944071)}{0.267 \times -0.0115944071}$$

$$= \underline{\underline{2.279011107}}$$

$$y_3 = \underline{\underline{-0.00009661}}$$

$$\therefore \text{root is } x = \underline{\underline{2.279011107}}$$

$$6. f(x) = x \log_{10} x - 1.2 = 0 \quad 2 < x < 3$$

	x	y	
	2.6	-0.1210692953	
$x_0 \leftarrow$	2.7	-0.0353178	$\rightarrow y_0$
$x_1 \leftarrow$	2.8	0.0520424	$\rightarrow y_1$

$$x_2 = \frac{(2.7 \times 0.0520424) - (2.8)(-0.0353178)}{0.0520424 - (-0.0353178)}$$

$$= \underline{\underline{2.792154119}}$$

$$y_2 = \underline{\underline{0.04513149}} \quad \rightarrow \text{replacing } y,$$

$$x_3 = \frac{(2.7 \times 0.04513149) - (\cancel{2.8} 2.792154119 \times -0.0353178)}{0.04513149 + 0.0353178}$$

$$= \underline{\underline{2.740456302}}$$

$$y_3 = \underline{\underline{-0.00016552.}}$$

$$\therefore \text{root is } \underline{\underline{x = 2.740456302}}$$

$$7. f(x) = x^3 - 4x + 1 = 0$$

	x	y	
	1.7	-0.887	
$x_0 \rightarrow$	1.8	-0.368	y_0
$x_1 \rightarrow$	1.9	0.259	y_1

$$x_2 = \frac{(1.8 \times 0.259) - (1.9 \times -0.368)}{0.259 - (-0.368)}$$

$$= \underline{\underline{1.858692185}}$$

$$y_2 = \underline{\underline{-0.013476748}} \rightarrow \text{replacing } y_0$$

$$x_3 = \frac{(1.858692185 \times 0.259) - (1.9 \times -0.013476748)}{0.259 - (-0.013476748)}$$

$$= \underline{\underline{1.860735277}}$$

$$y_3 = \underline{\underline{-0.0004507}}$$

$$\therefore \text{root is } x = \underline{\underline{1.860735277}}$$

8. $f(x) = x^3 + 2x^2 - 8 = 0$

	x	y
	1.4	-1.336
$x_0 \leftarrow$	1.5	-0.125 $\rightarrow y_0$
$x_1 \leftarrow$	1.6	1.216 $\rightarrow y_1$

$$x_2 = \frac{(1.5 \times 1.216) - (1.6 \times -0.125)}{1.216 - (-0.125)}$$

$$= \underline{\underline{1.5093214019}}$$

$$y_2 = \underline{\underline{-0.0055865404}} \rightarrow \text{replaces } y_0$$

$$x_3 = \frac{(1.5093214019 \times 1.216) - (1.6 \times -0.0055865404)}{1.216 - (-0.0055865404)}$$

$$= \underline{\underline{1.509736089}}$$

$$y_3 = \underline{\underline{-0.0002477}}$$

$$\therefore \text{root is } x = \underline{\underline{1.509736089}}$$

9. $f(x) = x^3 - 9x + 1 = 0$

x	y
0	1

$$x_0 \leftarrow 0.1 \quad 0.101 \rightarrow y_0$$

$$x_1 \leftarrow 0.2 \quad -0.792 \rightarrow y_1$$

$$x_2 = \frac{(0.1 \times -0.792) - (0.2 \times 0.101)}{-0.792 - 0.101}$$

$$= 0.1113101904$$

$$y_3 = -0.000412585$$

$$\therefore \text{root is } x = 0.1113101904$$

10. $f(x) = x^3 - x - 4 = 0$

	x	y	
	1.6	-1.504	
$x_0 \leftarrow$	1.7	-0.787	$\rightarrow y_0$
$x_1 \leftarrow$	1.8	0.032	$\rightarrow y_1$

$$x_2 = \frac{(1.7 \times 0.032) - (1.8 \times -0.787)}{0.032 - (-0.787)}$$

$$= 1.7960927961$$

$$y_2 = -0.0019884399 \text{ — replaces } y_0.$$

$$x_3 = \frac{(1.7960927961 \times 0.032) - (1.8 \times -0.0019884399)}{0.032 - (-0.0019884399)}$$

$$= 1.796321381$$

$$y_3 = -0.00000453$$

$$\therefore \text{root is } x = 1.796321381$$