

Assingment - 1

DATE

Q1) calculate mean median mode standard deviation interquartile range of the following data

i] 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

→ a] Mean : $\frac{4 + 7 + 9 + 9 + 11 + 15 + 18 + 18 + 18 + 25}{10}$

$$\text{Mean} = \frac{134}{10} = \boxed{13.4}$$

b] median : $\frac{n^{\text{th}} \text{ observation} + (n+1)^{\text{th}} \text{ observation}}{2}$

$$\text{median} = \frac{5 + 6}{2} = \frac{11}{2} = \boxed{5.5}$$

c] mode : 18

d] standard deviation :

Here $n = 10$ mean : 13.4

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
4	$4 - 13.4 :$	$(-9)^2 = 81$
7	$7 - 13.4 :$	$(-6)^2 = 36$
9	$9 - 13.4 :$	$(-4)^2 = 16$
9	$9 - 13.4 :$	$(-4)^2 = 16$
11	$11 - 13.4 :$	$(-2)^2 = 4$
15	$15 - 13.4 :$	$(2)^2 = 4$
18	$18 - 13.4 :$	$(5)^2 = 25$
18	$18 - 13.4 :$	$(5)^2 = 25$
18	$18 - 13.4 :$	$(5)^2 = 25$
25	$25 - 13.4 :$	$(12)^2 = 144$

classmate

PAGE 144

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
134	.	376

$$= \frac{376}{10} = 37.6$$

$$\text{Standard Deviation } \sigma = \sqrt{\text{Var}(x)} = \sqrt{37.6}$$

e] Interquartile Range

$$Q_1 : \left(\frac{n+1}{4} \right)^{\text{th}} = \left(\frac{10+1}{4} \right) = 2.75^{\text{th}} \text{ observation}$$

$$Q_2 : \left[2 \left(\frac{n+1}{4} \right) \right]^{\text{th}} = \left[2 \left(\frac{10+1}{4} \right) \right] = 5.5^{\text{th}} \text{ observation}$$

$$Q_3 : \left[3 \left(\frac{n+1}{4} \right) \right]^{\text{th}} = \left[3 \left(\frac{10+1}{4} \right) \right] = 8.25^{\text{th}} \text{ observation}$$

ii	No. of households	0	1	2	3	4
	No. of people	5	11	26	15	3

mean : No. of households	class	No. of people	cummulative frequency
0	0-1	5	5
1	1-2	11-5 = 6	11
2	2-3	26-11 = 15	26
3	3-4	15-26 = 11	15
4		3-15 = 12	3

median class : $\left(\frac{N}{2}\right)^{\text{th}}$ observation

$$= \frac{3}{2} = 1.5^{\text{th}} \text{ observation}$$

median class is 1 - 2

Here $L = 1$ $h = 0$ $f = 11$ $c.f = 11$

$$\text{Median} = L + \frac{h}{f} \left(\frac{N}{2} - c.f \right)$$

$$= 1 + \frac{0}{11} \left(\frac{3}{2} - 11 \right)$$

$$\text{median} = 1.$$

$$\text{mean} = \frac{5 + 11 + 26 + 15 + 3}{5}$$

$$\text{mean} = \frac{70}{5} = 14$$

standard deviation :

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
5	$5 - 14 = -9$	$(-9)^2 = 81$
11	$11 - 14 = -3$	$(-3)^2 = 9$
26	$26 - 14 = 12$	$(12)^2 = 144$
15	$15 - 14 = 1$	$(1)^2 = 1$
3	$3 - 14 = -11$	$(-11)^2 = 121$
<u>60</u>		

$$= \frac{356}{5} = 71.2$$

$$\text{standard deviation } \sigma = \sqrt{\text{Var}(x)} = \sqrt{71.2}$$

Interquartile range

$$Q_1 = \left(\frac{n+1}{4} \right)^{th} : \frac{5+1}{4} = 1.5$$

$$Q_2 = \left[2 \left(\frac{n+1}{4} \right) \right]^{th} : \left[2 \left(\frac{5+1}{4} \right) \right] = 3$$

$$Q_3 = \left[3 \left(\frac{n+1}{4} \right) \right]^{th} : \left[3 \left(\frac{5+1}{4} \right) \right] = 4.5$$

Q2) ~~It is assumed that claims arising on an industrial policy can be modelled as Poisson process at a rate of 0.5 per year.~~

i) Determine that probability that no claims arise in a single year.

Q3] An analyst interested in using gamma distribution with parameters $\alpha = 2$ $\lambda = 1/2$ the range of x is defined as $0 < x < \infty$

i) State the mean and standard deviation of this distribution

$$\text{mean} : \frac{\alpha}{\lambda} = \frac{2}{1/2} = 4$$

$$\text{standard deviation} = \sqrt{4} = 2$$

ii) comment briefly on its shape

→ The gamma distribution is the member of the general exponential family of distributions. The gamma distribution with shape parameters $k \in (0, \infty)$ and scale parameter $b \in (0, \infty)$ is a two parameter exponential family with natural parameters $(k-1, -\frac{1}{b})$ and natural statistics $(\ln x, x)$.

iii) Calculate the cumulative distribution function

Q4) In an actuarial office there are 20 males and 8 female staff 6 of the males and 7 of the females are fully qualified actuaries calculate the probability that the team of 2 workers randomly selected consist of 2 qualified females given that it contains at least one female

→ Total females : 8
qualified females : 7

∴ probability that 2 qualified females are selected = $\frac{8}{7} = 1.14$.

Q6) If $X \sim U(1, 2)$ and $Y = \frac{3}{6-X}$ determine the of Y

→

Q7) A Discrete Random variable X has the following probability distribution

X	1	2	3	4	5
$P(X=x)$	0.05	0.15	0.35	0.4	0

Calculate

1] $F(3) = \cancel{1.05} 1.05$

2] $E(X) = 15$

3] Coefficient of skewness : 0.203

Q8) Insurance claim amounts are independent and exceed 1000 with probability 0.2 in sample of 15 claims calculate the probability that fewer than 3 of them exceed 1000

Q9) A man enters a raffle each week where the probability of winning prize is 0.01 calculate the probability that he takes 7 weeks up to and including the week where he wins his 3rd prize

~~0.01~~ = probability that he takes 7 weeks is 0.014

Q10] The number of claims submitted to an insurance office averages two per working weeks (of 5 days) calculate the probability that more than 2 claims arrive in a day

→ Probability that more than 2 claims arrive more than 2 claims per day = $\frac{2}{5} = 0.4$

Q11] If the probability of having male & female child is equal and a woman has 5 boys and 2 girls calculate the probability that her next three children are boys.

→ The probability that next three children are boys is 0.50 because every woman has a kid there is equal chance of them having a girl or a boy

Q12) Claims follows a poisson process with rate 10 per day calculate the probability that the time between two claims is less than 0.1 days

Q13) If X is following uniform distribution with parameters $a = 75$ $b = 120$ calculate the probability that X lies between 80 and 100.

$$\frac{1}{(b-a)} = \frac{1}{120 - 75} = 45$$

Q14) If $X \sim \text{gamma}(5, 0.4)$ calculate $P(X < 22.89)$

Q15) A continuous random variable has the following probability density function

$$\frac{k}{(x+5)^4} \quad x > 0$$

$$= k(x+5)^{-4}$$

$$f(10) = k \cdot (10+5)^{-4}$$

$$f(10) = k \cdot (15)^{-4}$$

$$f(10) = k \cdot \underline{\underline{0.19}}$$