FM ASSIGNMENT →

2) $d^{(4)} = 8\%$

$$\cancel{1+0}\quad 1-d = \left(1-\frac{d^{4}}{4}\right)^{4} \ldots \left[(1-d) = \left(1-\frac{d^{p}}{p}\right)^{p}\right]$$

$$\therefore d = 1-\left(1-\frac{d^{4}}{4}\right)^{4}$$

$$= 1-(1-2\%)^{4}$$

$$d = 7.763184\%$$

(i) equivalent force of interest $= \delta$

$$\therefore \delta = \ln(1+i) = \ln\left(\frac{1}{1-d}\right) = -\ln(1-d)$$

$$\therefore \delta = -\ln(1-d)$$

$$\therefore \delta = 8.0810829277\%$$

(ii) equivalent effective rate of interest per annum $= i$

$$\therefore (1+i)(1-d) = 1$$

$$\therefore i = \frac{d}{1-d} = 8.416578\%$$

(iii) equivalent nominal rate of discount per annum convertible monthly $= d^{12}$

$$d^{12} = 12 \times \left(1-(1-d)^{1/12}\right)$$

$$= 12 \times \left(1-(1-7.763184\%)^{1/12}\right)$$

$$d^{12} = 8.053934\%$$

2) $\delta(t) = 0.004t + 0.0002t^2$, for all t.

(i) Amount due $= C = ₹1000$

$$PV_0 = C \times \frac{1}{a(t)} = C \times v(t) \quad \dots \left(a(t) = \frac{1}{v(t)} \right)$$

$$= \frac{C}{e^{\int_0^{10} 0.004t + 0.0002t^2 dt}}$$

$$\therefore PV_0 = 1000 \times e^{-\int_0^{10} 0.004t + 0.0002t^2 dt}$$

$$= 1000 \times e^{-\left(0.002t^2 - \frac{0.0002}{3}t^3\right)_0^{10}}$$

$$= 1000 \times \text{expo}(-0.26666667)$$

$$\therefore PV_0 = 765.93$$

(ii) $(1+i)^{10} = \cancel{e^{0.}}\ \text{expo}\left(\int_0^{10} 0.0004t + 0.0002t^2\, dt\right)$

$$(1+i)^{10} = \text{expo}(0.26666667)$$

$$i = 2.70254037\%$$

~~(iii)~~

3) (i) The relationship $d = iv \rightarrow$

* Interest earned on an investment paid at the beginning of the period is d. Interest earned on an investment paid at the end of the period is i. Therefore, if we discount i from the end of the period with

the discounting factor ν, we obtain d.

we know that $\nu = \frac{1}{1+i}$ and $(1+i)(1-d) = 1$

$$\therefore (1+i)(1-d) = 1$$

$$1-d = \frac{1}{1+i}$$

$$\therefore d = 1 - \frac{1}{1+i} = \frac{1+i-1}{1+i} = \frac{i}{1+i} = i\nu$$

$$\therefore d = i\nu$$

(ii) a) $\frac{d^{(4)}}{4} = 2.25\%$

$$(1-d) = \left(1 - \frac{d^{p}}{p}\right)^{p}$$

$$d = 1 - \left(1 - \frac{d^{p}}{p}\right)^{p}$$

$\therefore$ ~~$d = 1 - (1 - d$~~

$$\therefore \left(1 - \frac{d^{m}}{m}\right)^{m} = \left(1 - \frac{d^{p}}{p}\right)^{p}$$

$$\therefore \left(1 - \frac{d^{2}}{2}\right)^{2} = \left(1 - \frac{d^{4}}{4}\right)^{4}$$

$$\therefore d^{2} = 2 \times \left(1 - \left(1 - \frac{d^{4}}{4}\right)^{4/2}\right)$$

$$= 2 \times \left(1 - \left(1 - \frac{d^{4}}{4}\right)^{2}\right)$$

$$d^{2} = 8.89875\%$$

(iii)

(b)

(iii) ν

4) (i)

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(~~iii~~) (b) $d^{(1/2)} = 5\%$.

$$\therefore \left(1 - \frac{d^2}{2}\right)^2 = \left(1 - \frac{d^{1/2}}{1/2}\right)^{1/2}$$

$$\therefore d^2 = 2 \times \left(1 - \left(1 - 2d^{1/2}\right)^{1/4}\right)$$

$$\therefore d^2 = \cancel{2\times}\, 5.199250715\%.$$

(iii) $v\left(0, \frac{91}{365}\right) = \frac{1}{(1+i)^{91/365}} = \left(1 - \frac{91}{365} d\right)$

$$\frac{1}{(1.05)^{91/365}} = \left(1 - \frac{91}{365} d\right)$$

$$d = \frac{365}{91}\left(1 - \frac{1}{(1.05)^{91/365}}\right)$$

$$d = 4.8494627\%.$$

4) (i) <u>The relationship</u> $d = iv$,

Interest earned on an investment paid at the beginning of the period is d. Interest earned on an investment, paid at the end of the period is i. Therefore, if we ~~deer~~ discount i from the end of the period with the discounting factor v, we obtain d.

We know that $v = \frac{1}{1+i}$ and $(1+i)(1-d) = 1$

$$\therefore (1+i)(1-d) = 1$$

$$\therefore d = \frac{i}{1+i} = iv$$

(ii) $\cancel{(a)}\ \cancel{\frac{d^{4}}{4}}\ i = 0.08 = 8\%$

(a) $d = \frac{i}{1+i} = 7.407407407\%$

$d^{12} = 12 \times \left(1 - (1-d)^{1/12}\right)$

$d^{12} = 7.671478\%$

(b) $i^{365} =$ ..... $\left(i^{p} = p \times \left((1+i)^{1/p} - 1\right)\right)$

$i^{365} = 365 \times \left[(1.08)^{1/365} - 1\right]$

$i^{365} = 7.696915\%$

(c) $\delta = \ln(1+i)$

$= \ln(1.08) = 7.696104\%$

$\therefore \delta = 7.696104\%$

(d) $i^{1/2}$

$i^{p} = p \times \left((1+i)^{1/p} - 1\right)$

$i^{1/2} = \frac{1}{2}\left((1.08)^{2} - 1\right) = 8.32\%$

$i^{1/2} = 8.32\%$

5)

(ii)

5) (i) $d^4 = 6\%$

$$1-d = \left(1-\frac{d^p}{p}\right)^p$$

$$\therefore\ d = 1-\left(1-\frac{d^p}{p}\right)^p$$

$$d = 1-\left(1-\frac{d^4}{4}\right)^4$$

$$\therefore\ d = 1-(1-1.5\%)^4 = 5.86634493\%$$

$$\therefore\ d = 5.86634493\%$$

(a) $i = \frac{d}{1-d} = 6.23193153 8\%$

$$i^p = p \times \left((1+i)^{1/p}-1\right)$$

$$i^2 = 2 \times \left((1+i)^{1/2}-1\right) = 6.137752\%$$

$$\therefore\ i^2 = 6.137752\%$$

(b) d^{12} ~~$\$$~~ $\qquad d^p = p \times \left(1-(1-d^p)^{1/p}\right)$

$$d^{12} = 12 \times \left(1-(1-d)^{1/12}\right) = 6.030253\%$$

$$\therefore\ d^{12} = 6.030253\%$$

(ii) a) $i = \frac{20,000}{2,00,000} \times 100 = 10\%$

number of ~~dates~~ days = 93 (excluding end date)

$$AV_{\frac{93}{365}} = 2,00,000\,(1+i)^{93/365}$$

$\therefore AV_{\frac{93}{365}} = 200000 \times (1 \cdot 1)^{93/365}$

$\therefore AV_{93/365} = 2,04,916 \cdot 3564$

Q.6) Let the retail price be x.

(i) if retailler pays cash $= \frac{70}{100}(x)$

a) then $PV = 0.7x$

b) if retailler uses 6 month credit then he has to pay 75% of x which is $0.75x$ in 6 months time.

time 6 months $= \frac{1}{2}$ years.

$$PV_0 = AV(1-d)^n$$

$$0.7\cancel{x} \; \cancel{0.7x} = 0.75\cancel{x}(1-d)^{1/2}$$

$$\left(\frac{0.7}{0.75}\right)^2 - 1 = -d$$

$$\therefore d = 1 - \left(\frac{0.7}{0.75}\right)^2 = 12.88889\%$$

(ii) a) $d^{12} = 12 \times \left(1 - (1-d)^{1/12}\right) \cdots \left[d^p = p \times \left(1 - (1-d)^{1/p}\right) \right]$

$$d^{12} = 12 \times \left(1 - (1 - 12.88889\%)^{1/12}\right)$$

$$d^{12} = 13.719544\%$$

b)

c)

Q.7) Q

AV

(i)

(ii)

b) $i = \frac{d}{1-d}$ .... $\left((1+i)(1-d)=1\right)$

$\therefore i = 14.79591837\%$.

$i^{2} = 2 \times \left((1+i)^{1/2} - 1\right)$ ... $\left(i^{p} = p \times \left((1+i)^{1/p} - 1\right)\right)$

$i^{2} = 14.285714\%$.

c) $\delta = \ln(1+i) = 13.7985742\%$.

$\therefore \delta = 13.7985742\%$.

Q.7) amount invested (C) = 20,000

$AV_{17 \, months} = AV_{1.41666667} = 26,000$

$AV_{n} = C \times (1+i)^{n}$

$26000 = 20000\,(1+i)^{1.41666667}$

$i = 20.34570672\%$.

(i) $i^{4} = 4 \times \left((1+i)^{1/4} - 1\right)$ .... $\left(i^{p} = p \times \left((1+i)^{1/p} - 1\right)\right)$

$i^{4} = 18.955255\%$.

(ii) $\left(1 - \frac{d^{p}}{p}\right)^{p} = (1-d) = \frac{1}{(1+i)}$

$d^{p} = p\left(1 - (1-d)^{1/p}\right) = p \times \left(1 - (1+i)^{-1/p}\right)$

~~d^{p}~~ $d^{2} = 2 \times \left(1 - (1.2034570672)^{-1/2}\right)$

$d^{2} = 17.6882353\%$.

(iii) $26000 = 20000\left(1 + 1.41666667\, i_s\right)$

$i_s =$ ~~21.7~~ 21.17647061%

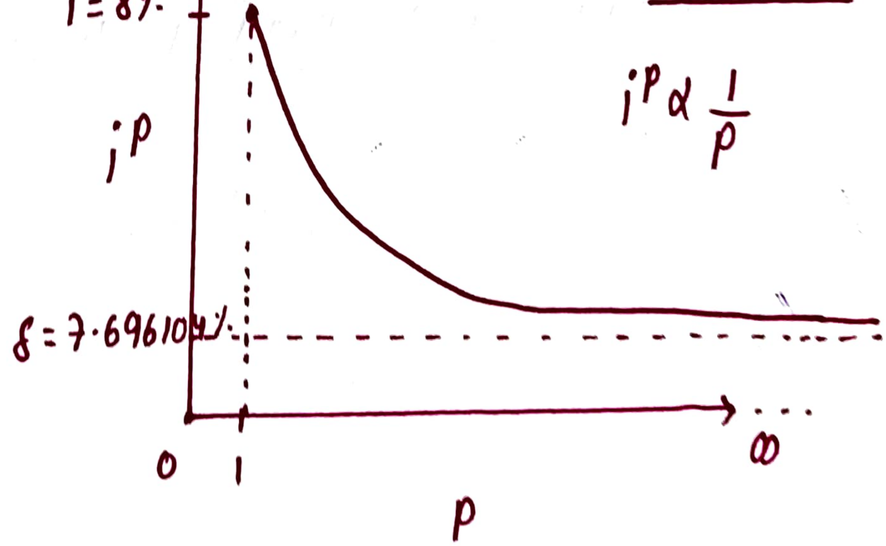
(iv) Numerically, in terms of compounding, i has the highest numerical value and d has the lowest numerical value. This is just numerical, as both yields the same ~~accumulating no~~ accumulate and present value i.e. (AV and PV). The simple interest, i_s is greater than i because, since there is no interest on interest accummulation in simple interest, it has to compensate for the compounding effect and hence it has greater value than i.

8) $i = 8\%$

$$i^{p} = p \times \left((1+i)^{1/p} - 1\right)$$
$$= p \times \left((1.08)^{1/p} - 1\right)$$

p	i^{p}
1	8%
2	7.84609%
3	7.79567%
4	7.77062%
5	7.755639%
6	7.745674%
12	7.72084%
100	7.69907%
365	7.69691%
∞	7.696104%

graph of i^{p} v/s p



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more frequently the compounding takes place (i.e. as p increases), the smaller is the equivalent nominal annual rate

$$i^{p} \propto \frac{1}{p}$$

when continuous compounding takes place i.e. when p approaches to ∞ then i^{p} has a limiting value which is known as the force of interest or δ.

9) (i) Force of interest → the measure of interest at individual moments of time or at any instant of time is known as force of interest.

(ii) $i^{(2)} = 0.0775$

a) $i = \left(1 + \frac{i^{2}}{2}\right)^{2} - 1$ .... $\left((1+i) = \left(1 + \frac{i^{p}}{p}\right)^{p} \right)$

$i = 7.900156\%$

b) $\delta = \ln(1+i)$

$\delta = 7.603613\%$

c) $d = \frac{i}{1+i}$ ..... $\left((1+i)(1-d) = 1 \right)$

$= 7.321728276\%$

$d^{12} = 12\left(1 - (1-d)^{1/12}\right)$ .... $\left((1-d) = \left(1 - \frac{d^{p}}{p}\right)^{p} \right)$

$d^{12} = 7.579575\%$

d) $i^{(1/2)} = \frac{1}{2}\left((1+i)^{2} - 1\right)$ .... $\left(i^{p} = p \times \left((1+i)^{1/p} - 1\right) \right)$

$i^{1/2} = 8.212219\%$

10)

10)

$$\delta(t) = \begin{cases} 0.08 & 0 \leq t < 5 \\ 0.06 & 5 \leq t < 10 \\ 0.04 & t \geq 10 \end{cases}$$

as $a(t) = e^{\int \delta dt}$

$v(t) = e^{-\int \delta dt}$

as $a(t) = \frac{1}{v(t)}$

and $c = 1$.

c is amount due.

when, $0 \leq t < 5$

$$v(t) = \text{expo}\left(-\int_0^t 0.08\, ds\right)$$

$$= \text{expo}(-0.08t)$$

$$v(t) = e^{-0.08t} \qquad PV = c \times v(t) = e^{-0.08t}$$

when, $5 \leq t < 10$

$$v(t) = \text{expo}\left(-\int_0^5 0.08\, dt - \int_5^t 0.06\, ds\right)$$

$$= \text{expo}\left(-[0.08t]_0^5 - [0.06t]_5^t\right)$$

$$= \text{expo}(-0.06t - 0.1)$$

$$v(t) = e^{-0.06t - 0.1} \qquad PV = c \times v(t) = e^{-0.06t - 0.1}$$

when $t \geq 10$

$$v(t) = \text{expo}\left(-\int_0^5 0.08\, dt - \int_5^{10} 0.06\, dt - \int_{10}^t 0.04\, ds\right)$$

$$= \text{expo}\left(-0.06 - 0.1 - [0.04]_{10}^{t}\right)$$

$$= \text{expo}(-0.04t - 0.3)$$

$$= e^{-0.04t - 0.3} \qquad PV = c \times v(t) = e^{-0.04t - 0.3}$$

11) (a) $n = \frac{9}{12} = \frac{3}{4}$ years

amount due (C) = 50,000 , $d = 18\%$

$$PV_0 = C(1-d)^n$$

$$= 50{,}000(1-18\%)^{3/4}$$

$$PV_0 = 43{,}085.43$$

(b) (i) $\delta = 6\%$

$$\delta = \ln(1+i) = -\ln(1-d) = -\ln\left(1-\frac{d^{12}}{12}\right)^{12}$$

$$e^{-\delta} = \left(1-\frac{d^{12}}{12}\right)^{12}$$

$$\frac{d^{12}}{12} = 1 - e^{-\delta/12}$$

$$d^{12} = 12 \times \left(1 - e^{-\delta/12}\right)$$

$$d^{12} = 5.985025\%$$

(ii) $d^4 = 6\%$

$$d = 1 - \left(1 - \frac{d^4}{4}\right)^4 = 5.866344937\%$$

$$i = \frac{d}{1-d} = 6.231931537\%$$

$$i^4 = 4\left((1+i)^{1/4} - 1\right) = 6.09137057\%$$

i^{12} =

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$$i^{12} = 12\left[(1+i)^{1/12} - 1\right]$$

$$i^{12} = 6.0607097\%$$

c) ascending order: $d < \delta < i^{(4)}$

d will always be the smallest numerical value as it is charged in the beginning of the term. so due to TVM (time value of money) $d < \delta$ and $d < i^4$.

δ refers to continuous compounding i.e. more frequently than quaterly compounding. Due to more compounding than $(i)^{(4)}$, δ will yield higher ~~value~~ accumulated value than $i^{(4)}$ if $\delta = i^{(4)}$. thus δ is smaller than i^4 to yield the same accumulated value, thus $\delta < i^4$ and.

$$d < \delta < i^4.$$

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d) The relationship $d = iv$.

Interest earned on an investment, paid at the beginning of the period is d. Interest earned on an investment, paid at the end of the period is i. Therefore, if we discount i from the end of the period with the discounting factor v, we obtain d.

as we know that $v = \frac{1}{(1+i)} = \frac{1}{a}$, $(1+i)(1-d) = 1$

$$1 - d = \frac{1}{1+i}$$

$$\therefore d = iv$$

$$d = \frac{i}{1+i} = iv$$

12) amount invested (C) = 7049

$$AV_6 = C \cdot A(0,6)$$

$$= 7049 \cdot A(0,2) \times A(2,4.5) \times A(4.5,6)$$

(by principle of consistency)

For A(0,2)

$$(i)^{12} = 6\%$$

$$\frac{(i)^{12}}{12} = 0.5\%$$

$$\therefore A(0,2) = (1.005)^{2\times 12}$$

For A(2,4.5)

simple interest per quater = 1.5%

$$A(2,4.5) = (1 + (4.5-2) \times 4 (0.015))$$

$$= (1 + 10(0.015))$$

$$= (1.15)$$

For A(4.5,6)

$$d^{12} = 6\%$$

$$\frac{d^{12}}{12} = 0.5\%$$

$$\therefore A(4.5,6) = \frac{1}{(1-0.005)^{12\times 1.5}} = \frac{1}{(1-0.005)^{18}}$$

b)

$AV_6 = 7049 \cdot A(0,2) \times A(2,4.5) \times A(4.5,6)$

$= 7049 \times (1.005)^{24} \times (1.15) \times \frac{1}{(1-0.005)^{18}}$

$AV_6 = 9999.8945.$

Q.13) amount invested = $C = y$

$AV_n = 2y$

time = n years

(i) $AV_n = C(1+i)^n$

$i = 10\%$

$2\cancel{y} = \cancel{y}(1.10)^n$

$\ln(2) = n \ln(1.10)$

$n = 7.27$ years

(ii) $AV_n = \frac{C}{(1-d)^n} = \frac{C}{\left(1-\frac{d^{12}}{12}\right)^{12n}}$

$2\cancel{y} = \frac{\cancel{y}}{\left(1-\frac{10\%}{12}\right)^{12n}} \quad \ldots \left(d^{12} = 10\%\right)$

$2\left(1-\frac{d^{12}}{12}\right)^{12n} = 1$

18

$$\left(1-\frac{d^{12}}{12}\right)^{n\times 12} = \frac{1}{2}$$

$$12n \ln\left(1-\frac{d^{12}}{12}\right) = \ln\left(\frac{1}{2}\right)$$

$n = 6.902$ years.