SRM2 assignment 2

Q1].

i

$$\begin{array}{c|cc} & B & O \\ \hline B & & \\ O & & \end{array} \quad \begin{bmatrix} -\lambda & \lambda \\ u & -u \end{bmatrix}$$

ii The holding times are exponentially distributed with parameter λ in ~~statement~~ B & u in state O.

iii
$$\frac{d}{dt}\, {}_tP_s^{BB} = -\lambda\, {}_tP_s^{BB} + u\, {}_tP_s^{BO}$$

$$\frac{d}{dt}\, {}_tP_s^{BO} = \lambda\, {}_tP_s^{BB} - u\, {}_tP_s^{BO}$$

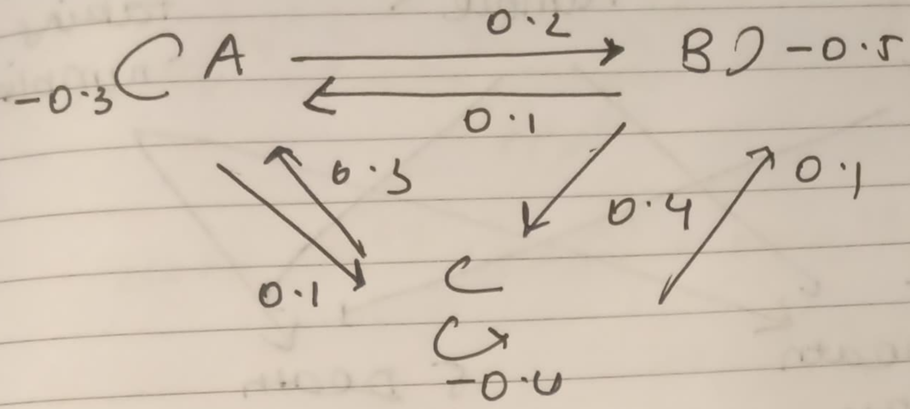
iv
$${}_tP_s^{BB} + {}_tP_s^{BO} = 1$$

$$\frac{\partial}{\partial t}\, {}_tP_s^{BB} = -\lambda \cdot {}_tP_s^{BB} + u \cdot (1 - {}_tP_s^{BB})$$

$$\frac{\partial}{\partial t}\left[\exp((\lambda+u)t)\, {}_tP_s^{BB}\right] = u \exp((\lambda+u)t)$$

$$\exp((\lambda+u)t)\, {}_tP_s^{BB} = \frac{u}{\lambda+u} \exp((\lambda+u)t) + c$$

q2



i)

$$\frac{d}{dt} P_{AA}(t) = -0.3\, P_{AA}(t) + 0.1\, P_{AB}(t) + 0.3\, P_{AC}(t)$$

$$\frac{d}{dt} P_{AB}(t) = 0.2\, P_{AA}(t) - 0.5\, P_{AB}(t) + 0.1\, P_{AC}(t)$$

$$\frac{d}{dt} P_{AC}(t) = 0.1\, P_{AA}(t) + 0.4\, P_{AB}(t) - 0.4\, P_{AC}(t)$$

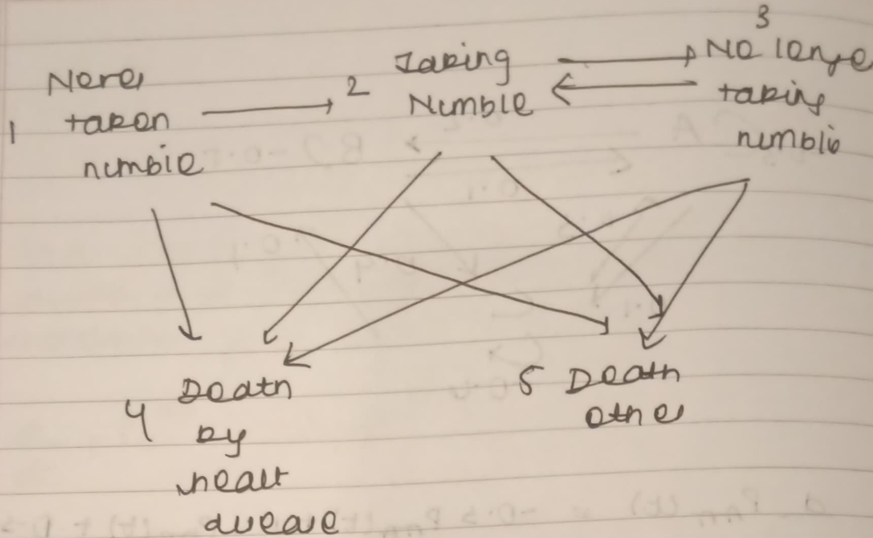
ii) $P_{\bar{A}\bar{A}}(t) = -0.3\, P_{\bar{A}\bar{A}}(t)$

$= \exp(-0.3t)$

$= t=2 \longrightarrow \exp(-0.3 \times 2) - 0.5488$

Q3.

i



ii $\frac{d}{dt}\left({}_tP_x^{34}\right) = {}_tP_x^{32}\,\mu_{x+t}^{24} + {}_tP_x^{33}\,\mu_{x+t}^{34}$

$dt{+}t\,P_x^{34} = {}_tP_x^{33}\; dt\,P_{x+t}^{34}$

$+ {}_tP_x^{31}\,\cancel{dt\,P_{x+t}^{14}}$

$+ {}_tP_x^{32}\, dt\,P_{x+t}^{24}$

$+ {}_tP_x^{34}\, dt\,P_{x+t}^{44}$

$+ {}_tP_x^{35}\,\cancel{dt\,P_{x+t}^{54}}$

$dt\,P_{x+t}^{54} = {}_tP_x^{31} = 0$

$dt\ P_{x+t}^{ij} = \mu_{x+t}^{ij}\ \text{to}(dt)$

$dt+t\ P_x^{34} = {}_tP_x^{32}\ \mu_{x+t}^{24}\ dt + {}_tP_x^{33}\ \mu_{x+t}^{34}\ dt$
$+ {}_tP_x^{34}$

$\cdot\ dt+t\ P_x^{34} - {}_tP_x^{34} = \cancel{t}\ {}_tP_x^{32}\ \mu_{x+t}^{u}$
$+ {}_tP_x^{33}\ \mu_{x+t}^{34}\ \text{to}(dt)$

Q4

i Mean = parameter = 1 calls per hr

ii 20 min

iii $Po(0.5) = \frac{e^{-1.5}\, 1.5^{0}}{0!} = 0.2231$

Q5

$$P_{HH}(x, t+dt) = P_{HH}(x,t)\, P_{HH}(t, t+dt)$$

$$+ P_{HS}(x,t)\, \cancel{P}_{SH}(t, t+dt)$$

$$+ P_{HD}(\cancel{t}\, x,t)\, \underbrace{P_{DH}(t, t+dt)}_{0}$$

$$\because P_{ij}(t, t+dt) = \lambda_{ij}(t)\, dt + o(dt)$$

$$P_{ii}(t, t+dt) = 1 + \lambda_{ii}(t)\, dt + o(dt)$$

$$P_{HH}(x, t+dt) = P_{HH}(\cancel{t}\, x, t)\,(1 - \sigma(t)\, dt - \mu(t)\, dt)$$

$$+ P_{HS}(x,t)\, \rho(t, t) + o(dt)$$

Q7

ii $P_{AA}(s,t)$

$$\frac{\partial}{\partial t} P_{AA}(s,t) = -P_{AA}(s,t)\,\mu$$

$$\frac{\partial}{\partial t} P_{AT}(s,t) = P_{AA}(s,t)\,\mu$$

$$\frac{\partial}{\partial t} P(s,t) = P(s,t)\,M$$

$$M = \begin{pmatrix} -\mu & \mu \\ 0 & 0 \end{pmatrix}$$

iii

$$\begin{array}{ccc} \text{Active} & \xrightarrow{\mu} & \text{Inept} \\ \lambda \big\downarrow & & \\ \text{Lapsed} & & \end{array}$$

$$\frac{\partial}{\partial t} P_{AA}(t) = -P_{AA}(t)(\mu+\lambda)$$

$$\frac{\partial}{\partial t} P_{AT}(t) = P_{AA}(t)\,\mu = \mu \exp(-(\mu+\lambda)t)$$

Q8. {1, 2, 3, 4}

i

ii

$$0 \xleftarrow[\mu]{} 1 \underset{\mu}{\overset{\lambda}{\rightleftarrows}} 2 \underset{\mu}{\overset{\lambda}{\rightleftarrows}} 3 \underset{\mu}{\overset{\lambda}{\rightleftarrows}} 4 \xrightarrow{\lambda}$$

iii

$$\begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \mu & -(\mu+\lambda) & \lambda & 0 & 0 \\ 0 & \mu & -(\mu+\lambda) & \lambda & 0 \\ 0 & 0 & \mu & -(\mu+\lambda) & \lambda \\ 0 & 0 & 0 & \mu & -(\mu+\lambda) \\ \cancel{0} & \cancel{0} & \cancel{0} & & \end{bmatrix} \end{array}$$

Q9

Q10