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Assignment 2:

1. $I = \int_{1/50}^{2} \frac{e^{2/\omega}}{\omega^2} d\omega$

$\frac{2}{\omega} = t$

$\frac{-2}{\omega^2} = dt$

at $w = 2$
$t = 1$

at $\omega = \frac{1}{50} \Rightarrow t = 100$

$\therefore I = \int_{100}^{1} e^t \frac{dt}{-2} = \left[\frac{e^t}{-2}\right]_{100}^{1}$

$= \boxed{\frac{e^{1} - e^{100}}{-2}}$

2. $I = \int \frac{2t^3+1}{(t^4+2t)^3} dt$

$t^4 + 2t = u$
$(4t^3 + 2)dt = du$
$2(2t^3+1)dt = du$

$\therefore I = \int \frac{du}{2} \frac{1}{u^3} = \frac{u^{-2}}{-2} \cdot \frac{1}{2} + c$

$= \frac{-1}{4} \cdot \frac{1}{u^2} + c = \boxed{-\frac{1}{4u^2} + c}$

$I = \boxed{\frac{-1}{4(t^4+2t)^2} + c}$

3. $f'(x) = x^4 + 3x - 9$

$$f(x) = \int f'(x) = \int x^4 + 3x - 9\,dx$$

$$= \boxed{\frac{x^5}{5} + \frac{3}{2}x^2 - 9x + c}$$

4. $$I = \int_{-2}^{3} f(x)\,dx = \int_{1}^{3} 6\,dx + \int_{-2}^{1} 3x^2\,dx$$

$$= [6x]_1^3 + [x^3]_{-2}^{1}$$

$$= [18 - 6] + [1 + 8]$$

$$= 12 + 9 = \boxed{21}$$

5. $$I = \int 3(8y-1)e^{4y^2-y}\,dy$$

let $4y^2 - y = t$

$(8y - 1)dy = dt$

$$I = \int 3e^t\,dt = 3e^t + c$$

$$= \boxed{3e^{4y^2-y} + c}$$

6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$ $\qquad f(1) = -\frac{5}{4}$

$\int f''(x) = f'(x)$ $\qquad f(4) = 404$

$= \int (15x^{1/2} + 5x^3 + 6)\,dx$

$= 15 \times \frac{2}{3}(x^{3/2}) + \frac{5}{4}x^4 + 6x + C_1$

$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1$ — ①

$\int f'(x) = f(x)$

$= \int 10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1$

$= 10 \times \frac{2}{5}x^{5/2} + \frac{1}{4}x^5 + 3x^2 + C_1x + C_2$

$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + C_1x + C_2$ — ②

$f(1) = -\frac{5}{4}$

$\therefore -\frac{5}{4} = 4(1)^{5/2} + \frac{(1)^5}{4} + 3(1)^2 + C_1(1) + C_2$

$-\frac{5}{4} = 7 + \frac{1}{4} + C_1 + C_2$

$-\frac{17}{2} = C_1 + C_2$

$C_2 = -C_1 - \frac{17}{2}$ — ③

$f(4) = 404$

$404 = 4(4)^{5/2} + \frac{(4)^5}{4} + 3(4)^2 + C_1(4) + C_2$

$404 = 432 + 4C_1 - C_1 - \frac{17}{2}$

$-28 + \frac{17}{2} = 3C_1 \Rightarrow \frac{-56+17}{2} = 3C_1$

$C_1 = \boxed{-6.5} \quad \therefore \boxed{C_2 = -2}$

$\therefore f(x) = \boxed{4x^{5/2} + \frac{x^5}{4} + 3x^2 - 6.5x - 2}$ //.

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8. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $\quad\quad r(1) = 2$

$\frac{1}{r^2}\, dr = \frac{1}{\theta}\, d\theta$ $\quad\quad$ integrating b.s.

$\int \frac{1}{r^2}\, dr = \int \frac{1}{\theta}\, d\theta$

$\Rightarrow \frac{-1}{r} = \log\theta + c$

$\therefore \frac{-1}{(1)} = \log(2) + c$

$\boxed{c = -3.3219280}$

$\Rightarrow \boxed{\frac{-1}{r} = \log\theta - 3.3219280}$

9. $\frac{dv}{dt} = 9.8 - 0.196v$

$$\cancel{(9.8 - 0.196v)}$$

$$\left(\frac{1}{9.8 - 0.196v}\right) dv = 1\,dt$$

$$\int \frac{1}{\left(\frac{7\sqrt{5}}{5}\right)^2 - \left(\frac{7\sqrt{10v}}{50}\right)} dv = \int 1\,dt$$

$$\Rightarrow \frac{1}{2 \times \frac{7\sqrt{5}}{\sqrt{5}}} \ln\left(\frac{7\sqrt{5}/5 + 7\sqrt{10v}/50}{7\sqrt{5}/5 - 7\sqrt{10v}/50}\right) = t + c$$

$$\Rightarrow \frac{\sqrt{5}}{14} \ln\left(\frac{\sqrt{5} + \sqrt{v/10}}{\sqrt{5} - \sqrt{v/10}}\right) = t + c$$

$$\rightarrow \frac{\sqrt{5}}{14} \ln\left(\frac{\sqrt{50} + \sqrt{v}}{\sqrt{50} - \sqrt{v}}\right) = t + c$$

$$\Rightarrow \boxed{\ln\left(\frac{\sqrt{50} + \sqrt{v}}{\sqrt{50} - \sqrt{v}}\right) = \frac{14}{\sqrt{5}} t + c}$$

10. $ty' + 2y = t^2 - t + 1 \qquad y(1) = \frac{1}{2}$

$$t\frac{dy}{dt} + 2y = t^2 - t + 1$$

$$\frac{dy}{dt} + \frac{2y}{t} = t - 1 + \frac{1}{t}$$

$$\frac{dy}{dt} = t - 1 + \frac{1}{t} - \frac{2y}{t}$$

11. $$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}} \qquad y(0) = -1$$

$$\int \frac{1}{y^3}\,dy = \int \frac{x}{\sqrt{1+x^2}}\,dx$$

$1 + x^2 = t$

$2x\,dx = dt$

$dx = \frac{1}{2}\,dt$

$$\frac{-1}{2y^2} = \frac{1}{2}\int \frac{1}{\sqrt{t}}\,dt$$

$$\frac{-1}{2} \cdot \frac{1}{y^2} = \cancel{2} \times \frac{1}{\cancel{2}} \sqrt{1+x^2} + c$$

$$-\frac{1}{2}\cdot\frac{1}{(1)^2} = \sqrt{1+(0)^2} + C$$

$$-\frac{1}{2} = 1 + c$$

$$\boxed{-\frac{3}{2}} = C$$

$$\Rightarrow \boxed{-\frac{1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}}$$

12. $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$f'(x) = 3x^2 - 20x + 0$

$f''(x) = 6x - 20$

$f'''(x) = 6 - 0$

$f''''(x) = 0$

$f''''(3) = 0$

$f(3) = 27 - 90 + 6$
$= -57$

$f'(3) = 27 - 60$
$= -33$

$f''(3) = 18 - 20$
$= -2$

$f'''(3) = 6$

Taylor series:

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) \dots$$

$$\therefore f(x) = f(3) + (x-3)f'(3) + \frac{(x-3)^2}{2!}f''(3) + \frac{(x-3)^3}{3!}f'''(3) \dots$$

$$f(x) = -57 + (x-3)(-33) + \frac{(x-3)^2}{2!}(-2) + \frac{(x-3)^3}{3!}(6) + 0$$

$$f(x) = \boxed{-57 + (x-3)(-33) - (x-3)^2 + (x-3)^3}$$

13. $f(x) = (1+x)^{\mu}$

Maclaurin series:

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) \dots$$

$f(x) = (1+x)^{\mu}$ $\quad f(0) = (1)^{\mu} = 1$

$f'(x) = \mu(1+x)^{\mu-1}$ $\quad f'(0) = \mu(1)^{\mu-1} = \mu$

$f''(x) = \mu(\mu-1)(1+x)^{\mu-2}$ $\quad f''(0) = \mu(\mu-1)(1)^{\mu-2}$

$= \mu^2 - \mu$

$f'''(x) = (\mu^2 - \mu)(\mu-2)(1+x)^{\mu-3}$

$f'''(0) = \mu^3 - 3\mu^2 + 2\mu \; (1)^{\mu-3}$

$= \mu^3 - 3\mu^2 - 2\mu$

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Series :

$$\cancel{f(x) = 1 + x(\mu) + \frac{x^2}{2!}(\mu^2-\mu) +}$$

$$f(x) = 1 + x(\mu) + \frac{x^2}{2!}(\mu)(\mu-1) + \frac{x^3}{3!}(\mu)(\mu-1)(\mu-2) \ldots\ldots$$

14. $f(x) = \sqrt{1+x}$

Maclaurin series :

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0)\ldots$$

$$f(x) = \sqrt{1+x} \Rightarrow f(0) = \sqrt{1+0} = \boxed{1}$$

$$f'(x) = \frac{1}{2\sqrt{1+x}} \Rightarrow f'(0) = \frac{1}{2\sqrt{1+0}} = \boxed{\frac{1}{2}}$$

$$f''(x) = \frac{-1}{4}.(1+x)^{-1/2-1} = \frac{-1}{4}(1+x)^{-3/2}$$

$$\Rightarrow f''(0) = \boxed{\frac{-1}{4}}$$

$$f'''(x) = \frac{3}{8}(1+x)^{-5/2} \Rightarrow f'''(0) = \boxed{\frac{3}{8}}$$

Series:

$$f(x) = 1 + x\left(\frac{1}{2}\right) + \frac{x^2}{2!}\left(\frac{-1}{4}\right) + \frac{x^3}{3!}\left(\frac{+3}{8}\right) \ldots$$

15. $f(x) = e^{-6x}$ for $x = -4$

Taylor series:

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) \ldots$$

$f(x) = e^{-6x}$ $\quad$ $f(-4) = e^{24}$

$f'(x) = -6e^{-6x}$ $\quad$ $f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x}$ $\quad$ $f''(-4) = 36e^{24}$

$f'''(x) = -216e^{-6x}$ $\quad$ $f'''(-4) = -216e^{24}$

Series:

$$f(x) = e^{24} + (x+4)(-6.e^{24}) + \frac{(x+4)^2}{2!}(36.e^{24}) + \frac{(x+4)^3}{3!}(-216.e^{24}) \ldots$$

16. $f(x) = \ln(3+4x)$ about $x=0$

taylor series :

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) \dots$$

$$f(x) = \ln(3+4x) \qquad f(0) = \ln(3)$$

$$f'(x) = \frac{1}{3+4x}\cdot(4) \qquad f'(0) = \frac{4}{3}$$

$$f''(x) = \frac{-4\cdot 1}{(3+4x)^2} \qquad f''(0) = \frac{-4}{9}$$

$$\cancel{f'''(x) = \frac{8}{3+\dots}}$$

$$f'''(x) = 8\cdot\frac{1}{(3+4x)^3} \qquad f'''(0) = \frac{8}{27}$$

Series :

$$f(x) = \ln(3) + x\cdot\frac{4}{3} + \frac{x^2}{2!}\left(-\frac{4}{9}\right) + \frac{x^3}{3!}\left(\frac{8}{27}\right) \dots$$

$$f(x) = \boxed{\ln(3) + x\cdot\frac{4}{3} + x^2\left(-\frac{2}{9}\right) + \frac{x^3}{3!}\left(\frac{4}{81}\right) \dots}$$