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04/03/22 PRLI Assignment - 1 Kareena Tated
Roll no - 709.

Q, 1.

i. $(Ia)_{\overline{n}|} = v + 2v^2 + 3v^3 + \dots + nv^n$ — (A)

$(1+i)(Ia)_{\overline{n}|} = 1 + 2v + 3v^2 + \dots + nv^{n-1}$ — (B)

(B) − (A)

$= i(Ia)_{\overline{n}|} = 1 + v + v^2 + v^3 + \dots + v^{n-1} - nv^n$

$i \cdot (Ia)_{\overline{n}|} = \ddot{a}_{\overline{n}|} - nv^n$

$(Ia)_{\overline{n}|} = \boxed{\dfrac{\ddot{a}_{\overline{n}|} - nv^n}{i}}$

h/p

ii cost of education = 30,00000
inflation = 6% p.a
contribution = 2,00,000 p.a
inc = x.
i on savings = 10%.

PV of contribution:

$2{,}00{,}000\ \ddot{a}_{\overline{10}|} + x(Ia)_{\overline{9}|}$ } @ i = 10%.

$= 2{,}00{,}000 \times 6.759026 + x \times 25.18046$

$= 1351805.20 + 25.18046x$

PV of higher edu. expense:

$3000000 \times (1.06)^{10} \times v^{10}$ @ 10%

$= 2071350$

$\therefore \boxed{x = 28{,}575}$

Q2.

$$\begin{tikzcd} h \arrow[r, "\sigma_x"] \arrow[dr, "\mu_x"'] & i \arrow[d, "\nu_x"] \\ & d \end{tikzcd}$$

$i = 5\%$

$\mu_x = 0.004$

$\nu_x = 0.005$

$\sigma_x = 0.002$

$x = 40$

$n = 20$

$S = 100{,}000$ (imm)

$$EPV = 100{,}000 \int_0^{20} v^t \cdot {}_tp_{40}^{h} \left(\sigma_{40+t} + \mu_{40+t}\right) dt$$

$$= 100{,}000 \int_0^{20} e^{-\ln(1.05)t} \cdot e^{-0.006t} \cdot 0.006 \, dt$$

$$= 600 \cdot \int_0^{20} e^{-\ln(1.05)t - 0.006t}$$

$$= \frac{600}{0.5497} \left[-e^{-0.5479t}\right]_0^{20}$$

$$\boxed{EPV = 7290.3}$$

Q3. $x = 40$

$n = 20$

$i = 4\%$

AM92 select - disability

AM92 ult - illness.

i. EPV of prem $= P\ddot{a}^{(12)}_{40:\overline{20}|}$

$$= P\left[\ddot{a}_{40:\overline{20}|} - \frac{11}{24}\left(1 - \frac{D_{60}}{D_{40}}\right)\right]$$

$$= \left[13.927 - \frac{11}{24} \times \left(1 - \frac{882.85}{2052.96}\right)\right] P$$

$$= 13.66576724\,P \quad \text{— ①}$$

EPV of claims:

$$75\,00\,000\, \underbrace{A^{1}_{40:\overline{20}|}}_{A_{40:\overline{20}|} - \frac{D_{60}}{D_{40}}} + 50{,}000\, \underbrace{A^{1}_{[40]:\overline{20}|}}_{A_{[40]:\overline{20}|} - \frac{D_{60}}{D_{[40]}}}$$

$$= 4{,}27{,}714.965824 \quad \text{— ②}$$

EPV of expenses:

$$0.03 \times 42{,}714.965824 + 0.3P.\ddot{a}^{(12)}_{40:\overline{1}|} + 0.05P\left[\ddot{a}^{(12)}_{40:\overline{20}|} - 1\right]$$

$$+ 0.1P \times \ddot{a}^{(12)}_{40:\overline{20}|} \times 1.04^{0.5} + 500 \quad \text{— ③}$$

① = ② + ③

solving for $P \Rightarrow \boxed{39009.7}$

$\therefore$ monthly $P = \frac{39009.7}{12} = \boxed{3250.8}$

ii. Policies with only accidental claims

$$7500,000 \, A^{1}_{45:\overline{15}|} - 39009.7 \, \ddot{a}^{(12)}_{45:\overline{15}|}$$

$$= 269387.1975 - 434750.6352$$

$${}_5V_1 = 165,363.4377$$

policies with only illness claims:

$$50,00,000 \, A^{1}_{45:\overline{15}|} - 390097 \, \ddot{a}^{(12)}_{45:\overline{15}|}$$

$$= 50,00,000 \times 0.035918293 - 434750.6352$$

$${}_5V_2 = 255,159.1702$$

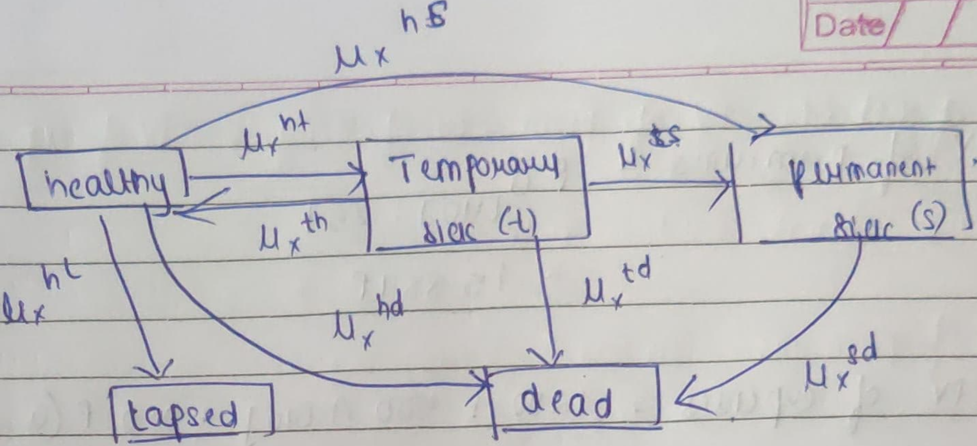
policies with no claims

$$= [7500,000 + 50,00,000] \times A^{1}_{45:\overline{15}|} - 390097 \, \ddot{a}^{(12)}_{45:\overline{15}|}$$

$${}_5V_3 = 14,228.0273$$

Q4

i.



ii. $n=15$

$x=50$

EPV (prem) = EPV (benefit)

$$\text{EPV of benefit} = 10{,}000\int_0^{15} v^u \cdot {}_uP_{50}^{ht}\,dt$$

$$+\ 20{,}000\int_0^{15} v^u\ {}_uP_{50}^{hs}\,du$$

$$+\ 100{,}000\int_0^{15} v^u\left({}_uP_{50}^{hh}\ \mu_{50+u}^{hd} + {}_uP_{50}^{ht}\cdot\mu_{50+u}^{td} + {}_uP_{50}^{hs}\cdot\mu_{50+u}^{sd}\right)du$$

$$\text{EPV of Premium} = P\times\int_0^{15} v^u\cdot{}_uP_{50}^{hh}\,du$$

$$\therefore P = \frac{\left[100{,}000\int_0^{15} v^u\left[{}_uP_{50}^{hh}\ \mu_{50+u}^{hd} + {}_uP_{50}^{ht}\cdot\mu_{50+u}^{td} + {}_uP_{50}^{hs}\cdot\mu_{50+u}^{sd}\right]du + 10{,}000\int_0^{15} v^u\ {}_uP_{50}^{ht}\,du + 20{,}000\int_0^{15} v^u\ {}_uP_{50}^{hs}\,du\right]}{\int_0^{15} v^u\ {}_uP_{50}^{hh}\,du}$$

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Q5

i. EPV of premiums $= P\ddot{a}_{[40]:\overline{25}|}$

$= 15.887P$

EPV of expenses $= 1200 + 500\bar{A}_{[40]} + 0.02P(\ddot{a}_{[40]:\overline{25}|} - 1)$

$= 1317.487 + 0.29774P$

EPV benefits $= 2,00,000\,(v^{0.5}\,q_{[40]} + v^{1.5}\,(1.04)\;{}_{1|}q_{[40]} + \dots\dots)$

$= \dfrac{2,00,000}{1.04^{0.5}}\left[q_{[40]} + 1.04\;{}_{1|}q_{[40]} + \dots\right]$

$= \dfrac{2,00,000}{1.04^{0.5}} = 1,96,116.14$

$15.887P = 1,96116.14 + 1317.487 + 0.29774P$

$\boxed{P = 12,664}$

ii. Reveid bonus (simple)

EPV of benefits $= 2,00,000 \times x \times (I\bar{A})_{[40]} + 2,00,000(1-x)\bar{A}_{[40]}$

$= (1.04)^{0.5}\left[200000x\,(I\bar{A})_{[40]} + 2,00,000(1-x)A_{[40]}\right]$

$\therefore$ ~~to~~ equate this to EPV of benefits (compound)

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$\therefore 1,96,116.14 = (1.04)^{0.5}[2,00,000\, x\,(\bar{IA})_{[40]} + 200000(1-x)\,\bar{A}_{[40]})$

$\therefore \boxed{x = 9\%}$

iii. $P\ddot{a}_{40:25} = 2,00,000\,\bar{A}_{40}$

$P = 2,960.54$

$_{10}V = 3,00,000\,\bar{A}_{50} - P\ddot{a}_{50:15}$

$= 300000 \times 1.04^{0.5} \times 0.32907 - 2960.54 \times 11.253$

$= \boxed{67,361.10}$

Q6 force of decrement: death $= \mu_x$

cancer $= \sigma_x$

$$\text{Benefit} = 100,000 \int_0^{20} v^t\,{}_tP_{45}^{hh}\,(\mu_{45+t} + \sigma_{45+t})\,dt$$

$$= 100,000 \int_0^{20} e^{-\ln(1.05)t} \cdot e^{-0.008t}\,dt$$

$$= 800 \int_0^{20} e^{-0.05679t}$$

$$= 9562.8$$

$\therefore$ return of prem $= P \times e^{-\int_{45}^{65}(0.006+0.002)\,dx} \times v^{20}$

$= P \times e^{-0.16} \times 0.37689$

$\therefore P = 9562.8 + 0.321164 \times P + 0.02P + 0.02P + 0.1P$

$\therefore$ Premium $= 17,747.15$

Q7. PV of net CF = PV of prem − PV of benefits − PV of expenses − PV of comm

max initial commission be I

PV of premium $= 30{,}000 \times \ddot{a}_{[45]:\overline{20}|}$

$= 356{,}640$

PV of benefits $= 1000\,000 \times A_{[45]:\overline{20}|}$

$= 327{,}110$

PV of expenses $= 5000 + 0.025\left[30{,}000 \times (\ddot{a}_{[45]:\overline{20}|} - 1)\right]$

$+ 500 \times p_{[45]} \times 1.06^{-1}\left[1 + 1.019231 \times v \times {}_1p_{[46]} + 1.019231^2\, v^2\, {}_2p_{[46]} + \dots + 1.019231^{18} \times v^{18}\, {}_{18}p_{[46]}\right]$

$= 19{,}438$

PV of renewal commission $= 0.02 \times 30{,}000 \times (\ddot{a}_{[45]:\overline{20}|} - 1)$

$= 6533$

$\therefore$ PV of net CF $= 356\,640 - 327110 - 19438 - 6533 - I$

$= 3559 - I$

$\therefore$ margin = PV of net CF / Annual premium

$= \dfrac{(3559 - I)}{30{,}000}$

$= 10\%$

$\boxed{I = ₹559}$

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Q8 i. Assumption of independence of decrements is used to derive single decrement table from multiple decrement table

$(a\mu)_x^j = \mu_x^j$ for all j & all x

ii. In case of term assurance, people with lower than average mortality are more likely to lapse

iii.

a. $1000 \int_0^5 e^{-\delta t} \left({}_tP_{45}^{PI} \mu_{45+t} + {}_tP_{45}^{PP} \nu_{45+t} \right) dt$

b. $250 \int_0^5 e^{-\delta t} \cdot {}_tP_{45}^{IP} \nu_{45+t} \, dt$

c. $150 \int_0^5 e^{-\delta t} \cdot {}_tP_{45}^{II} \, dt$

d. $250 \int_0^5 e^{-\delta t} \cdot {}_tP_{45}^{PP} \nu_{45+t} \, dt$

iv. $(aq)_{45}^c = \dfrac{0.05}{0.18} \times 1 - e^{-0.18} = 0.045758$

$(aq)_{45}^s = \dfrac{0.13}{0.18} (1 - e^{-0.18}) = 0.118972.$

$(ap)_{45} = 1 - 0.045758 - 0.118972 = 0.835270$

Similarly,

$(aq)_{46}^c = 0.055446$

$(aq)_{46}^s = 0.092410$

$(ap)_{46} = 0.852144$

$\therefore$ prob of being in state inforce at 47 (start)

$= 0.835270 \times 0.852144$

$\boxed{= 0.711770}$

Q9.

i. under with-profit polices, the contract is such that whatever surplus arises on the underlying fund, would be shared between policyholders and shareholders in a pre-determined manner.

$\therefore$ As surplus arises over future years, the same is shared with policyholders in the form of bonuses & hence the need of bonuses under with profit policies

ii. Investment surplus, expense surplus, mortality surplus, Lapse/surrender surplus, reinsurance surplus.

iii. Terminal bonus = build down guarantees over the lifetime of with profit policies.

Q10 EPV of premiums $= 12P\,\ddot{a}^{(12)}_{50:\overline{10}|}$ @6%.

$= 89.7648P$

i. EPV of benefits $= \dfrac{10,00,000}{1.019231} \cdot A_{50:\overline{15}|} + 10,00,000\, v^{15}\, {}_{15}p_{50} \times \left[1.019231^{15} - 1.019231^{14}\right]$ @4%

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$= 566,004.1$

EPV of expense & commission =

$\frac{(20+35)}{100} \times 12P + \frac{2+1.5}{100}, P_{50} \times 1.06^{-1} \times 12P \times \ddot{a}^{(12)}_{51:\overline{9}|}$

@ 6%

$+ 2100 + 400 \times {}_1P_{50} \times 1.06^{-1} \times \ddot{a}_{51:\overline{9}|}$

$= 9.3333P + 6115.06$

$\therefore 80.4315P = 572119.16$

$\boxed{P = 7113.12}$

ii SA @ end of 10^{th} year + Accrued bonus

$x = 10,00,000 \times 1.02^{10}$
$= 12,18,994$

PV of future benefits @ end of year 10 =

$\frac{x}{1.05} \times q_{60} + \frac{x \times 1.01 \times {}_1|q_{60}}{1.05^2} + \dots + \frac{x}{1.05^5} \times 1.04^5 \times {}_5P_{60}$

$= 0.952948x$
$= 11,60,774$ — ①

PV of expenses $= 400\ \ddot{a}_{60:\overline{5}|}$ @ 5%

$= 1787.39$ — ②

$\therefore$ reserves required $= \boxed{11,62,652}$ [① + ②]

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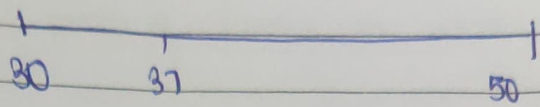
Q4:



$i = 6\%$

$SA = 400{,}000$

bonus = 40 per 1000 SA (simple)

EPV of prem = $P\ddot{a}_{30:\overline{20}|}$

EPV of benefit = $400000\ A_{30:\overline{20}|}$

$\therefore P = \dfrac{400{,}000\ (0.315817)}{12.08705}$

$\boxed{P = 10457.42}$

Total bonus at time 7 $= 7 \times 0.04 \times 400{,}000$
$= 112000$

SA at time 7 $= 400{,}000 + 112000$
$= 512000$

$\therefore {}_7V = 512000\ A_{37:\overline{13}|} - 10457.42\ \ddot{a}_{37:\overline{13}|}$
$= 512000\ \cancel{A_{37}}\ (0.471638)$
$- 10457.42\ (4.33835)$
$= \boxed{143926.86}$