Page No.
Date

1) $d^{(4)} = 8\%$

$\frac{d^{(4)}}{4} = 2\%$

$1-d = \left(1 - \frac{d^{(4)}}{4}\right)^4 = 0.9224$

$d = 7.7632\%$

i) $\delta = -\ln(1-d)$

$\delta = 8.0776\%$

ii) $1+i = e^{\delta} = 1.084128$

$i = 8.4128\%$

iii) $1 - \frac{d^{(12)}}{12} = e^{-8/12}$

$= 0.9933$

$d^{(12)} = 8.0504\%$

2) $\delta(t) = 0.004t + 0.0002t^2 \in t$

i) $PV = C \cdot e^{-\int \delta(t)\,dt}$

$= 1000 \cdot e^{-\int_0^{10} 0.004t + 0.0002t^2\,dt}$

$= 1000 \cdot e^{-\left[0.002t^2 + \frac{0.0002t^3}{3}\right]_0^{10}}$

$= 10^3 \cdot e^{-\left[0.2 + \frac{0.2}{3}\right]} = 10^3 e^{-\frac{0.8}{3}}$

$= 765.92834$

ii) $e^{\int \delta(t)\,dt} = 1+i$

$e^{\frac{0.8}{3}} = 1+i$

$i = 30.56052\%$

2.7025

iii) $765.92834 \times (1+i)^{10} = 1000$

$i = 0.0270254$

3) $d = iv$

$\cancel{\text{or}}\; d = \dfrac{i}{1+i}$

$d = i\left(\dfrac{1}{1+i}\right) = i(1-d)$

$d = iv$

where $v = (1-d)$ is the discounting factor

$\therefore$ Interest earned at the end of the period is i and at beginning of the period is d on an investment of 1

$\therefore$ If we discount i from the end of the period to the beginning of the period with discount factor v, we get d. $i \cdot v = d$

ii) (a) $\frac{d^{(4)}}{4} = 2.25\%$

$d^{(4)} = 9\%$

$\left(1 - \frac{d^{(4)}}{4}\right)^4 = \left(1 - \frac{d^{(2)}}{2}\right)^2$

$(1 - 0.0225)^2 = \left(1 - \frac{d^{(2)}}{2}\right)$

$d^{(2)} = 8.8988\%$

(b) $\left(1 - \frac{d^{(2)}}{2}\right)^2 = \left(1 - \frac{d^{1/2}}{1/2}\right)^{1/2}$

$= \left(1 - (0.05 \times 2)\right)^{1/2}$

$= (1 - 0.1)^{1/2}$

$1 - \frac{d^{(2)}}{2} = (0.9)^{1/4}$

$d^{(2)} = 5.199251\%$

(iii) Let $C = 100$

$\therefore \ i = 5\%$

$(1+i)^{\frac{91}{365}} = A$

$A = (1.05)^{91/365} = 1.01224$

$A = \frac{1}{v} = \frac{1}{\left(1 - \frac{91}{365} d\right)}$

$1.01224 \left(1 - \frac{91}{365} d\right) = 1$

$d = 4.850086\%$

Page No.
Date

4) i) $i = 0.08$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = \frac{1}{1+i} = \frac{1}{1.08}$$

$$d^{(12)} = 7.671478\%$$

ii) $i^{(365)} = ?$

$$(1+i) = \left(1 + \frac{i^{(365)}}{365}\right)^{365}$$

$$\left((1.08)^{1/365} - 1\right)365 = i^{(365)}$$

$$i^{(365)} = 7.69692\%$$

iii) $\delta = \ln(1+i) = \ln(1.08)$

$$\delta = 7.696104\%$$

iv) $i^{(0.5)} = ?$

$$(1+i) = \left(1 + \frac{i^{(0.5)}}{0.5}\right)^{0.5}$$

$$\left[\frac{(1.08)^2 - 1}{2}\right] = i^{(0.5)}$$

$$i^{(0.5)} = 8.32\%$$

Page No.

Date

5) i)

$d^{(4)} = 6\%$

$\frac{d^{(4)}}{4} = 1.5\%$

a) $\left(\frac{i^{(2)}}{2} + 1\right)^2 = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^4}$

$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{(0.985)^4}$

$\frac{i^{(2)}}{2} = \left(\frac{1}{0.985}\right)^2 - 1$

$i^{(2)} = 6.137752\%$

b) $\left(1 - \frac{d^{(4)}}{4}\right)^4 = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$

$1 - \frac{d^{(4)}}{4} = \left(1 - \frac{d^{(12)}}{12}\right)^3$

$(0.985)^{1/3} = 1 - \frac{d^{(12)}}{12}$

$d^{(12)} = 12(1 - 0.994975)$

$d^{(12)} = 6.030253\%$

ii) $220000 = 200000(1+i)$

$1.1 = 1+i$ $\qquad n = 93$ days

$i = 10\%$

AV $\cancel{220000} = 200000(1+0.1)^{93/365}$

$= 204916.3564$

ii) $d^{(2)} = ?$

a) $\frac{d^{(4)}}{4} = 2.25\% = 0.0225$

$$\left(1-\frac{d^{(2)}}{2}\right)^2 = \left(1-\frac{d^{(4)}}{4}\right)^4$$

$$d^{(2)} = \left[1-\left(1-\frac{d^{(4)}}{4}\right)^{4/2}\right]\times 2$$

$$d^{(2)} = 0.0889875\%$$

(b) $d^{(1/2)} = 5\%$

$$d^{(2)} = \left[1-\left(1-\frac{d^{(1/2)}}{1/2}\right)^{1/4}\right]\times 2$$

$$= \left[1-\left(1-0.1\right)^{1/4}\right]2$$

$$= \cancel{0}5.1992507\%$$

iii) $i = 5\%$, $n = 91$ days

Let $C = 100$

$$AV = 100\,(1+i)^{n/365}$$

$$= 100\,(1+0.05)^{91/365}$$

$$= \cancel{100}\ 101.2238407$$

$\therefore$ ~~101.223841 = 100~~

$$100 = 101.223841\left(1-\frac{91}{365}d\right)$$

$$d = 4.8494631\%$$

Page No.
Date / /

6) Let 100 be the recommended retail price.

~~RRP~~

Amount paid by retailers giving cash payments $= 100(1-0.3)$
$= 70$

Amount paid by retailers giving 6 months credit $= 100(1-0.25)$
$= 75$

$$PV = C\cdot(1-d)^{1/2}$$
$$70 = 75\cdot(1-d)^{1/2}$$
$$d = \cancel{12\%}\ \cancel{p.a.}\ 12.88\%$$
$$(1-d) = \left(1-\frac{d^{(12)}}{12}\right)^{12}$$
$$12\left(1-(0.88)^{1/12}\right) = d^{(12)}$$
$$d^{(12)} = 12.7154\%$$
$$\left(1+\frac{i^{(2)}}{2}\right)^{2} = 1+i = \frac{1}{1-d}$$
$$i^{(2)} = 2\left[\left(\frac{1}{1-d}\right)^{1/2} - 1\right]$$
$$i^{(2)} = 13.200716\%$$
$$\delta = -\ln(1-d)$$
$$= -\ln(0.88)$$
$$= 12.7833\%$$

Page No.
Date

7) $C = 20{,}000$; $AV = 26000$

$n = 17$ months $= 1.41667$ yrs.

To find :- $i^{(4)}$, $d^{(2)}$, i

simple interest

→ $AV = A \cdot C$

$26000 = A \cdot 20000$

$A = 1.3$

$(1+i)^t = 1.3$

$(1+i)^{17 \times \frac{1}{12}} = 1.3$

1yr $i = 20.3457\%$

$\left(1 + \frac{17 i_s}{12}\right) = 1.3$

$\frac{17}{12} i_s = 0.3$

$i_s = 21.17647\%$

$\left(1 + \frac{i^{(4)}}{4}\right)^4 = 1 + i$

$= 1.203457$

$i^{(4)} = 18.9552\%$

$\left(1 - \frac{d^{(2)}}{2}\right)^2 = \frac{1}{1+i}$

$d^{(2)} = \left[1 - \left(\frac{1}{1+i}\right)^{1/2}\right] \times 2$

$d^{(2)} = 17.6882\%$

For the same ~~an~~ accumulated value and time for a given capital the simple interest must be greater than ~~effectiv~~ C.I. [effective rate of interest]
Nominal interest rate is always greater than nominal discount rate

9- i) Force of interest is the measure of interest at individual moments of time

ii) $i^{(2)} = 0.0775$

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = (1+i)$$

$$\left(1+\frac{0.0775}{2}\right)^2 = 1+i$$

$$i = 0.0790015$$

$$\delta = \ln(1+i)$$

$$= \ln(1.07900156)$$

$$= 0.07603613$$

$$1-\frac{d^{(12)}}{12} = e^{-\delta/12}$$

$$d^{(12)} = 12\left(1-e^{-\delta/12}\right)$$

$$d^{(12)} = 0.07579574$$

$$(1+2i^{(1/2)}) = e^{2\delta}$$

$$i^{(1/2)} = \frac{e^{2\delta}-1}{2}$$

$$= 0.08212218$$

Page No.

Date

10) $\delta(t) = \begin{cases} 0.08 & 0 \le t \le 5 \\ 0.06 & 5 \le t \le 10 \\ 0.04 & t \ge 10 \end{cases}$

Find $V(t)$

$V(0, t) = e^{-\int \delta(t)\, dt}$

${}_0^5\int \delta(t)\,dt = {}_0^5\int 0.08\, dt = [0.08 \cdot t]_0^5$

${}_0^5\int \delta(t)\,dt = 0.4$

${}_5^{10}\int \delta(t)\,dt = [0.06 t]_5^{10} = 0.3$

${}_{10}^{t}\int \delta(t)\,dt = [0.04]_{10}^{t} = 0.04(t-10)$

$V(0,t) = V(0,5) \times V(5,10) \times V(10,t)$

$= e^{-[{}_0^5\int \delta(t)dt + {}_5^{10}\int \delta(t)dt + {}_{10}^{t}\int \delta(t)dt]}$

$= e^{-[0.4 + 0.3 + 0.04t - 0.4]}$

$V(0,t) = e^{-[0.3 + 0.04t]}$

$V(0,t) = e^{-0.04t - 0.3}$

11) a) $PV = C(1-d)^n$

$= 50{,}000\,(1-0.18)^{9/12}$

$= 43085.42623$

b) i) $\delta = 6\%$ $d^{(12)} = ?$

$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = e^{-\delta}$

$d^{(12)} = \left(1 - e^{-\delta/12}\right)12$

$d^{(12)} = 5.98502\,\%$

ii) $d^{(4)} = 6\%$

$\left(1 - \frac{d^{(4)}}{4}\right)^4 = \frac{1}{\left(1 + \frac{i^{(12)}}{12}\right)^{12}}$

$0.9413365 = \left(1 + \frac{i^{(12)}}{12}\right)^{-12}$

$12\left(\frac{1}{0.994975} - 1\right) = i^{(12)}$

Page No.
Date

$$i^{(12)} = 6.060453\%.$$

$$\left(1+\frac{i^{(12)}}{12}\right)^{12/4} = 1+\frac{i^{(4)}}{4}$$

$$i^{(4)} = 4\left[\left(1+\frac{0.0\cancel{4}60460453}{12}\right)^{3} - 1\right]$$

$$i^{(4)} = 6.0911121\%$$

c) $i > i^{(4)} > \delta > d$

If δ is force of interest

$1+i = e^{\delta}$ ~~an~~ $\quad i = e^{\delta}-1$

$$1+\frac{i^{(4)}}{4} = e^{\delta/4}$$

$$i^{(4)} = 4(e^{\delta/4}-1)$$

$$1-d = e^{-\delta}$$

$$d = 1-e^{-\delta}$$

d) $v = 1-d$

= present value of 1 due at time ~~1~~ 1

i = effective annual rate of interest

d = effective annual rate of discount

$$1+i = \frac{1}{1-d}$$

$$d = \frac{i}{1+i} = i(1-d) = i\cdot v$$

$\therefore$ If we discount i by discounting factor v, we get ~~dis~~ d.

Page No

Date

7049

0 2 4.5 6

$i^{(12)} = 6\%$ $\frac{i}{4} = 1.5\%$ $d^{(12)} = 6\%$

$AV = C \cdot A(0,6)$

$A(0,2) = \left(1 + \frac{i^{(12)}}{12}\right)^{12 \times 2}$

$= \cancel{XA}\ 1.1271598$

$\cancel{A(2,4.5) = \left(1 + \frac{i^{(4)}}{4}\right)^{4 \times 4.5}}$

$A(2,4.5) = \left(1 + (4.5)(0.015)(4)\right)$

$A(2,4.5) = 1.27$

$A(4.5,6) = \left(1 - \frac{d^{(12)}}{12}\right)^{-12 \times 1.5}$

$A(4.5,6) = 1.094421$

$AV = 7049\ A(0,2) \cdot A(2,4.5) \cdot A(4.5,6)$

$= 7049\,(1.127159) \cdot (1.27) \cdot (1.094421)$

$AV = 11043.34989$

Page No.

Date

13) Let $\cancel{cc}\ C = x$ To find :- n yrs

$\therefore AV = 2x$

i) $AV = C(1+i)^n$

$2x = x(1+0.1)^n$

$2 = \cancel{x}(1.1)^n$

$\ln(2) = n \ln(1.1)$

~~0.69314~~ $n = 7.27254$ yrs

ii) ~~Assume~~ $d^{(10)} = 0.10$

$C = AV\left(1 - \frac{d^{(12)}}{12}\right)^{12 \times n}$

$x = 2x\left(1 - \frac{0.1}{12}\right)^{12n}$

$1 = 2(0.991667)^{12n}$

$\ln(0.5) = 12n \log(0.991667)$

$n = 6.902578$ yrs