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Sakshi Tripathi FYBSc - Section B Roll - 81

NMA Assignment

1) Observed radius = 12.65 cm
Actual radius = 12.5 cm

Observed area $= \pi r^2$

$A_O = \pi (12.65)^2$

$\therefore A_O = 502.725 \text{ cm}^2$

Actual Area $= \pi r^2$

$A_A = \pi \times (12.5)^2$

$A_A = 490.874 \text{ cm}^2$

i) Absolute error = Observed value − Actual value

$= 502.725 - 490.874$

$= 11.857 \text{ cm}^2$

ii) Relative error $= \dfrac{\text{Absolute error}}{\text{Actual val}}$

$\dfrac{11.857}{490.874} = 0.02414$

iii) Percentage error = Relative Error × 100.

2.414 %

2) i) $x_1 = 4$ $y_1 = 0.60206$

$x = 5$ $y = ?$

$x_2 = 6$ $y_2 = 0.7781513$

$\therefore y = \dfrac{y_2(x - x_1) + y_1(x_2 - x_1)}{(x_2 - x_1)}$

$= \dfrac{0.7781513(1) + 0.60206(1)}{2}$

$\therefore y = 0.6901$

3. $x^4 - x - 10 = 0$

$a = 1.5 \quad b = 2$

$f(a) = f(1.5) = (1.5)^4 - 1.5 - 10$
$= -6.4375$

$f(b) = f(2) = 2^4 - 2 - 10$
$= 4$

<u>1st iteration</u>

$x_1 = \frac{2+1.5}{2} = 1.75$

$f(x_1) = f(1.75) = 1.75^4 - 1.75 - 10 = -2.3711$

<u>2nd iteration</u>

$x_2 = \frac{1.75+2}{2} = 1.875$

$f(x_2) = 1.875^4 - 1.875 - 10$
$= 0.4846$

<u>3rd iteration</u>

$x_3 = \frac{1.75+1.875}{2} = 1.8125$

$f(x_3) = \cancel{\frac{1.75+1.875}{2} = 1.8125}$

$f(x_3) = f(1.8125) = 1.8125^4 - 1.8125 - 10$
$= -1.0202$

<u>4th iteration</u>

$x_4 = \frac{1.875+1.8125}{2} = 1.84375$

$f(x_4) = f(1.84375)$
$= 1.84375^4 - 1.84375 - 10$
$= -0.2877$

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5th iteration

$$x_5 = \frac{1.875 + 1.84375}{2} = 1.859375$$

$$f(x_5) = 1.859375^4 - 1.859375 - 10$$
$$= 0.093.$$

$\therefore$ Root of $x^4 - x - 10 = 10$ is 1.859375
which $\approx 1.86\%$

4) $f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

x	0	1	2
y	-1	-1	5

$$\therefore x_0 = \frac{1+2}{2} = 1.5$$

$f(x_0) = 0.875$ $\quad f'(x_0) = 5.75$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$1.5 - \frac{0.875}{5.75} = 1.34783.$$

$f(x_1) = 0.10068$ $\quad f'(x_1) = 4.44991$

$$x_2 = 1.34783 - \frac{0.10068}{4.44991}$$

$$= 1.3252.$$

$f(x_2) = 0.00205$

$f'(x_2) = 4.2684$

$$x_3 = 1.3252 - \frac{0.00205}{4.2684}$$

$$x_3 = 1.3247.$$

$f(x_3) = 0.0000007545$

$f'(x_3) = 4.2645$

$$x_4 = 1.3247 - \frac{0}{4.2645}$$

$x_4 = 1.3247$

Root of $x^3 - x - 1 = 0$ is 1.3247

5. $A^2 = A$

$$7A - (I+A)^3 = 7A - (I^3 + A^3 + 3A^2 I + 3AI^2)$$
$$= 7A - I^3 - A^2 A - 3A^2 - 3A$$
$$= 7A - I - A \cdot A - 3A - 3A$$
$$= 7A - I - A - 6A$$
$$= 7A - 7A - I$$
$$7A - (I+A)^3 = -I$$

6) Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$$\therefore |A| = 1 \begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} + 1 \begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} + 2 \begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix}$$

$$|A| = 1(-6) + 1(-4) + 2(4) = -2$$

$|A| \neq 0 \therefore$ inverse exists

$\therefore$ Co-factor matrix $= \begin{vmatrix} -6 & 4 & 4 \\ 1 & -1 & 1 \\ -6 & 1 & 4 \end{vmatrix}$

$\therefore$ Adjoint matrix $= \begin{vmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{vmatrix}$

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$$\therefore A^{-1} = \frac{1}{|A|} \times \text{Adjoint}$$

$$\frac{-1}{2}\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & -1/2 & 2 \end{bmatrix}$$

9) The smallest 3 digit no. that leaves remainder of 2 is 101 and the largest = 998

$d = 3$

$t_n = a + (n-1)d$

$998 = 101 + (n-1) \times 3$

$n-1 = \frac{998 - 101}{3}$

$n = 299 + 1$

$n = 300$

$S_n = \frac{n}{2}(a+l) = \frac{300}{2}(101 + 998)$

$S_n = 164850$

Sum of all no. that leave a remainder of 2 when divided by 2 is 164850.

10) $t_6 = 32$

$t_8 = 128$

$t_6 = ar^5$

$t_8 = ar^7$

$$\therefore \frac{ar^7}{ar^5} = \frac{128}{32}$$

$$r^2 = 4$$

$$\boxed{r = 2}$$

11) Expand $(4+3x)^5 = (3x+4)^5$

$a^{n-3}.b^3.$

$$(a+b)^n = a^n + na^{n-1}b + n(n-1)a^{(n-2)}b^2 + \frac{n(n-1)(n-2)}{3!}$$

$$\therefore (3x+4)^5 = (3x)^5 + 5(3x)^4\,4 + 5 \times \frac{4(3x)^3 4^2}{2}$$

$$+ \frac{5 \times 4 \times 3 (3x)^2 \times 4^3}{3 \times 2} + \frac{5 \times 4 \times 3 \times 2 \times 3x \times 4^4}{1 \times 3 \times 2}$$

$$= 243x^5 + 1625x^4 + 4320x^3 + 5160x^2 + 3840x + 1024$$

14) $(1-x+x^2)^4$

$\therefore (x^2 + (1-x))^4$

Let $1-x = y$.

$(x^2+y)^4 =$

using binomial theory

$$(a+b)^n = a^n + (n\,a)^{n-1}.b + \frac{n(n-1)a^{n-2}b^2}{2!}$$

$$+ \frac{n(n-1)(n-2)}{3!}$$

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$$[x^2+y]^4 = (x^2)^4 + 4(x^2)^3 y + \frac{4\times 3}{2}(x^2)^2 y^2 + \frac{4\times 3\times 2}{3\times 2}(x^2) + y^4$$

$$= x^8 + 4x^6 y + 6x^4 y^2 + 4x^2 y^3 + y^4$$

$$= [x^2 + (1-x)]^4 = x^8 + 4x^6(1-x) + 6x^4(1-x)^2 + 4x^2(1-x)^3 + (1-x)^4$$

15) $e^{-x} = 3\log(x)$

$\therefore f(x) = 3\log x - e^{-x}$

$a = 1 \quad b = 2$

$\therefore f(a) = -0.367879$

$f(b) = 1.944106$

$\frac{a+b}{2} = \frac{1+2}{2} = 1.5 = x_1$

$f(x_1) = 0.993265$

$x_2 = \frac{1+1.5}{2} = 1.25$

$f(x_2) = 3\log(1.25) - e^{-1.25}$

$f(x_2) = 0.382925$

$x_3 = \frac{1+1.25}{2} = 1.125$

$f(x_3) = 3\log(1.125) - e^{-1.125}$

0.0286964

$\therefore x_4 = \frac{1+1.125}{2} = 1.0625$

$f(x_4) = 3\log(1.0625) - e^{-1.0625}$

$= 0.16372$