Date : / /
Page No :

FM HW Assignment

Q1] $d^{(4)} = 8\%$

i) $\delta = -\ln(1-d)$

$= -\ln(1-0.077632)$

$= 8.081\%$

$d = 1 - \left[1 - \frac{d^{(4)}}{4}\right]^4$

$= 7.7632\%$

ii) & $1 + i = \dfrac{1}{\left[1 - \frac{d^{(4)}}{4}\right]^4}$

$= i = 8.417\%$

iii) $d^{(12)} = \left[1 - [1-d]^{1/12}\right] \times 12$

$= 7.913\%$

Q2] $\delta(t) = 0.004t + 0.0002t^2$

i) $PV_0 = 1000 \times v(0,10)$

$= 1000 \times e^{-\int_0^{10} 0.004t + 0.0002t^2 dt}$

$= 1000 \times \exp\left[-\left[0.002t^2 + \frac{0.0002t^3}{3}\right]_0^{10}\right]$

$= 1000 \times \exp\left[-4/15\right]$

$= 765.928$

ii) ~~$i = \frac{A(10) - A(9)}{A(10)}$~~

$\cancel{765.928}$ $AV_{10} = C \times (1+i)^{10}$

$1000 = 765.928 \times (1+i)^{10}$

$i = 2.703\%$ p.a.

Q3] i) $d = iv$ or $1 - d = \frac{1}{1+i}$

$\therefore V = \frac{1}{1+i}$

$\therefore d = i \times \frac{1}{1+i}$

$= i / 1+i$

$d = 1 - \frac{1}{1+i}$

$= \frac{\cancel{1} + i - \cancel{1}}{1+i}$

$= i / 1+i$

Hence, they are equal.

ii)

a) $\frac{d^{(4)}}{4} = 2.25\%$ per quarter

$$\left[1 - \frac{d^{(2)}}{2}\right]^2 = \left[1 - \frac{d^{(4)}}{4}\right]^4$$

$$\left[1 - \frac{d^{(2)}}{2}\right]^2 = 0.91299$$

$$d^{(2)} = 8.899\%$$

b) $d^{(1/2)} = 5\%$

$$\left[1 - \frac{d^{(2)}}{2}\right]^2 = \left[1 - \frac{d^{(1/2)}}{1/2}\right]^{1/2}$$

$$d^{(2)} = 5.199\%$$

iii) ~~AV~~ Let the bill be 100

$\therefore$ $AV = 100 \times 1.05 = 105$

$\therefore$ $PV = AC.(1-d)$

$100 = 105(1-d)$

$d = 4.762\%$

Q4] i)

ii) $i = 0.08$

a) $\left[1 - \frac{d^{(12)}}{12}\right]^{12} = \left[\frac{1}{1+0.08}\right]$

$d^{(12)} = 7.671\%$

b) $(1+i) = \left(1 + \frac{i^{(365)}}{365}\right)^{365}$

$(1.08)^{1/365} = \left(1 + \frac{i^{(365)}}{365}\right)$

$i^{(365)} = 7.696\%$

c) $(1+i) = e^{\delta}$

$\ln(1.08) = \delta$

$\delta = 7.6961\%$

d) $(1+i) = \left(1 + \frac{i^{(0.5)}}{0.5}\right)^{0.5}$

$(1.08)^{1/0.5} = \left(1 + \frac{i^{(0.5)}}{0.5}\right)$ $\quad i^{(0.5)} = 8.32\%$

Date: / /
Page No.:

Q5] i) $d^{(4)} = 6\%$

a] $\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^4}$

$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \cancel{\frac{1}{0.94}} \left(\frac{1}{0.9413}\right)$

$i^{(2)} = \cancel{6.284}\% \quad 6.1378\%$

b] $\left(1 - \frac{d^{(12)}}{12}\right)^{12} = \left(1 - \frac{d^{(4)}}{4}\right)^4$

$d^{(12)} = 6.03\%$

ii)

200000 — 220000

15/4/05 — 17/7/05 — 15/4/06

a) $\therefore 220000 = 200000(1+i)^1 \quad i^{(365)} = 9.532\%$

$i = 0.1 = 10\%$

$\therefore AV_{17/7/05} = 200000 \times \left(1 + \frac{i^{(365)}}{365}\right)^{93}$

$= 204916.356$

Q6 i)

a) ~~∴ Let Retail price be 100~~

~~$\cancel{C.D}$ = C.P = 70~~

~~∴ Effective discount = $\frac{100-70}{}$~~

Let Retail price be 100

∴ Cash payment = 70

Hence annual effective discount to retailers that pay cash

$$70 = 100(1-d)$$

$$d = 0.3 = 30\%$$

ii) $$(1-d) = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

$$d^{(12)} = 35.142\%$$

$$\cancel{(1-d)} = \frac{1}{(1-d)} = \left[1 + \frac{i^{(2)}}{2}\right]^2$$

$$\left[\frac{1}{0.7}\right]^{0.5} = \left[1 + \frac{i^{(2)}}{2}\right]$$

$$i^{(2)} = 39.04\%$$

$$(1-d) = e^{-\delta}$$

$$\ln(1-d) = -\delta$$

$$\delta = -\ln(1-d)$$

$$= 35.667\%$$

$n = 17$ months

7] i) $26000 = 20000\left(1+\frac{i^{(4)}}{4}\right)^{17/3}$

$i^{(4)} = 18.955\%$

ii) ~~26000 = 20000~~

$20{,}000 = 26000\left(1-\frac{d^{(2)}}{2}\right)^{17/6}$

$d^{(2)} = 17.688\%$

iii) $\cancel{20{,}000 = 26000\left(1-\frac{17}{12}\times d\right)}$ $\quad 26000 = 20000\left(1+\frac{17}{12}i\right)$

$\cancel{d = 16.289\%}$ $\quad i = 21.1764\%$

→ iv) Hence the rate with simple discount is the smallest followed by nominal discount and then nominal interest rate.

Q8]

∴ As the p-th ly convertible rate ~~increases~~ decreases due to an increase the amount of time ~~compounding happens~~ interest is paid in a period (p times)

Hence we ~~see~~ a decrease in the $i^{(p)}$ value with increase in p as it approaches infinity compounding the $i^{(p)}$ is limited to d interest rate.

Q9] i) Force of interest is called the rate continously compounded. ∴ δ is the nominal rate of interest convertible continously. i.e interest paid in every moment of time.

ii) $i^{(2)} = 7.75\%$

& $(1+i) = \left(1+\frac{i^{(2)}}{2}\right)^2$

$(1+i) = \left(1+\frac{0.0775}{2}\right)^2$

$\boxed{i = 7.90\%}$

$\delta = \ln(1+i)$

$\boxed{= 7.603\%}$

$d^{(12)} =$

$\left(1-\frac{d^{(12)}}{12}\right)^{12} = \frac{1}{\left(1+\frac{i^{(2)}}{2}\right)^2}$

$d^{(12)} = 7.57\%$

$i^{(1/2)} =$

$\left(1+\frac{i^{(2)}}{2}\right)^2 = \left(1+\frac{i^{(1/2)}}{0.5}\right)^{0.5}$

$i^{(1/2)} = 8.21\%$

Q10] ∴ $PV = C \times v(t)$

$= 1 \times \exp\left[-\left[\int_0^5 0.08\,dt + \int_5^{10} 0.06\,dt + \int_{10}^t 0.04\,dt\right]\right]$

$= \exp\left[-\left[(0.08t)_0^5 + (0.06t)_5^{10} + (0.04t)_{10}^t\right]\right]$

$= \exp\left[-\left[\cancel{0.4} + 0.3 + 0.04t - \cancel{0.4}\right]\right]$

$= \exp - \left[0.04t + 0.3\right]$

$= v(t) = e^{-0.04t - 0.3}$

Q11] a)

50,000

9 months

Dis = 18% p.a

$$\therefore PV_0 = C \cdot v(0, 9/12)$$
$$= 50{,}000 \times (1 - 0.18)^{9/12}$$
$$= 43{,}085.426$$

b) i] $\delta = 6\%$

$$\left(1 - \frac{d^{(12)}}{12}\right)^{12} = e^{-\delta}$$

$$d^{(12)} = 5.985\%$$

ii] $d^{(4)} = 6\%$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{-4} = \left(1 + \frac{i^{(12)}}{12}\right)^{12}$$

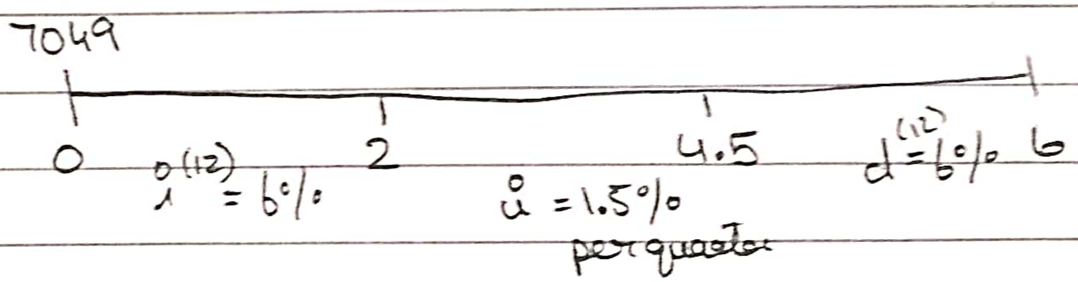
$$i^{(12)} = 6.0607\%$$

c) $d < \delta < i^{(4)} < i$

As ~~discounting~~ d is the lowest value and i is the largest value. force of interest is limit that comes from both d and i when compounded continously. ~~Hence~~ and $i^{(4)}$ is ~~compo~~ paid per quarter hence has lower than i. ~~as~~ ~~compounded~~

d) As d is the ~~disco~~ equal to discounting the interest earned from 0 to 1 for same ~~effective~~ ~~rate of~~ amount of accumalation and principle.

Q12



$$\therefore AV_6 = C \times A(0,6)$$

$$= C \times \left(1 + \frac{i^{(12)}}{12}\right)^{24} \times (1 + 10 \times i) \times \left(\frac{1}{\left(1 - \frac{d^{(12)}}{12}\right)^{18}}\right)$$

$$= 7049 \times (1.1271598) \times (1.15) \times (1.094421)$$

$$= 9999.8938$$

Q13

i) $AV = C(1+i)^n$

$2 = 1(1+0.10)^n$

$\ln 2 = n \ln(1.1)$

$n = 7.27$ years

ii) $AV = C \cdot \frac{1}{(1 - d^{(12)}/12)^{12 \times n}}$

$2 = \cancel{\frac{1}{}} \quad \frac{1}{2} = \left(1 - \frac{d^{(12)}}{12}\right.$

$\ln(0.5) = 12n \ln(0.991667)$

$n = 6.9$ years.