INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Probability & Statistic 1

Chapter: Unit 1 & 2

Category: Assignment 1

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Q 1)

Calculate the mean, median, mode standard deviation and inter quartile range of the following data:

i) 4,7,9,9,11,15,18,18,18,25

ii)

Number of households	0	1	2	3	4
Number of people	5	11	26	15	3

Q 2)

It is assumed that claims arising on an industrial policy can be modelled as a Poisson Process at a rate of 0.5 per year.

i) Determine the probability that no claims arise in a single year.

ii) Determine the probability that, in three consecutive years, there is one or more claims in one of the years and no claims in each of the other two years.

iii) Suppose a claim has just occurred. Determine the probability that more than two years will elapse before the next claim occurs.

Q 3)

An analyst is interested in using gamma distribution with parameters $\alpha = 2$ and $\lambda = 1/2$. The range of x is defined as $0 < x < \infty$.

i) State the mean and standard deviation of this distribution.

ii) Hence comment briefly on its shape.

iii) Calculate the cumulative distribution function.

Q 4)

In an actuarial office, there are 10 male and 8 female staff. 6 of the males and 7 of the females are fully qualified actuaries. Calculate the probability that a team of two workers randomly selected consists of two qualified females given that it contains at least one female.

Q 5)

The supply chain service in a country has three levels of service: Level 1 (next day delivery), Level 2 (2-day delivery) and Level 3 (3-day delivery). 40% of packages are delivered via Level 1 and 50% are delivered via Level 2. The probability of a package arriving late using each level is 20%, 40% and 10% respectively. Calculate the probability that a late package was sent via Level 2 service.

Q 6)

If $X{\sim}U(1,2)$ and $Y = \frac{3}{6-X}$ determine the PDF of Y.

Q 7)

A discrete random variable X has the following probability distribution:

X	1	2	3	4	5
P(X=x)	0.05	0.15	0.35	0,4	0.05

Calculate:

i) F(3)

ii) E(X)

iii) V(X)

iv) Coefficient of skewness

Q 8)

Insurance claim amounts are independent and exceed 1000 with probability 0.2. In a sample of 15 claims, calculate the probability that fewer than 3 of them exceed 1000.

Q 9)

A man enters a raffle each week where the probability of winning a prize is 0.01. Calculate the probability that he takes seven weeks up to and including the week where he wins his third prize.

Q 10)

The number of claims submitted to an insurance office averages two per working week (of 5 days). Calculate the probability that more than two claims arrive in one day.

Q 11)

If the probability of having a male and female child is equal and a woman has 5 boys and two girls, calculate the probability that her next three children are boys.

Q 12)

Claims follow a Poisson process with rate 10 per day. Calculate the probability that the time between two claims is less than 0.1 days.

Q 13)

If X is following uniform distribution with parameters a = 75 and b = 120, calculate the probability that X lies between 80 and 100.

Q 14)

If $X \sim Gamma\ (5,0.4), calculate\ P(X < 22.89)$.

Q 15)

A continuous random variable X has the following probability density function:

$$\frac{k}{(x+5)^4} \qquad , x > 0$$

Calculate:

i) k

ii) F(10)

iii) E(X)

Answers

Q1) i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

Ascending order = 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

$n = 10$

$$\text{Mean} = \frac{4+7+9+9+11+15+18+18+18+25}{10}$$

$$= \frac{134}{10}$$

$$= 13.4$$

$$\text{Median} = \frac{n+1}{2}$$

$$= \frac{10+1}{2}$$

$$= 5.5$$

$$\therefore \text{Median} = \frac{\text{5th term} + \text{6th term}}{2}$$

$$= 11 + 15$$

$$\frac{\quad}{2}$$

$$= \frac{\cancel{26}^{13}}{\cancel{2}}$$

Median = 13

Mode = 18 [∵ highest frequency]

IQR = $q_2 = 13$

$q_1 = 8$

$q_3 = 18$

IQR $= q_3 - q_1$

$= 18 - 8$

$= 10$

x	$\bar{x}$	$(x-\bar{x})^2$
4	13.4	88.36
7	"	40.96
9	"	19.36
9	"	19.36
11	"	5.76
15	"	2.56
18	"	21.16
18	"	21.16
18	"	21.16
25	"	134.56

$$\sigma^2 = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$= \sqrt{41.648}$$

Stand deviation = 6.4535

(ii)

x	f	C.f.	$f \cdot x$	$f(x-\bar{x})^2$
0	5	5	0	20
1	11	16	11	11
2	26	42	52	0
3	15	57	45	15
4	3	60	12	12
			120	58

$$\text{Mean} = \frac{\sum f \cdot x}{\sum f}$$

$$= \frac{\cancel{120}^{2}}{\cancel{60}_{1}}$$

$$= 2$$

$$\text{Median} = \frac{N}{2}$$

$$= \frac{\cancel{60}^{30}}{\cancel{2}}$$

$$= 30$$

30 lies between 16 & 42

$\therefore$ Median = 2

Mode = 2 [$\because$ highest frequency]

IQR: $q_2 = 2$

$q_1 = 1$

$$q_3 = 3$$

$$IQR = q_3 - q_1$$

$$= 3 - 1$$

$$\boxed{IQR = 2}$$

$$\sigma^2 = \sqrt{\frac{\sum f(x-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{58}{59}}$$

$$\sigma^2 = 0.99149$$

Q2) $X \sim P(0.5)$ [Let claims be X]

(i) $P(X=0) \Rightarrow \frac{e^{-\lambda}\lambda^x}{x!}$

$$= \frac{e^{-0.5}\lambda^0}{0!}$$

$$= e^{-0.5}$$

$$= 0.6065$$

(ii) Y = no of years

$Y \sim$ binomial $(3, 0.3935)$

$P(Y=1) \Rightarrow {}^{3}C_{1} \times (0.3835)(0.6065)^{2}$

$= 0.434$

(iii) $X \sim \exp(0.5)$

$$P(X>2) = \int_{2}^{\infty} \lambda e^{-\lambda x} \, dx$$

$$= \int_{2}^{\infty} 0.5 e^{-0.5x}$$

$$= 0.5 \int_{2}^{\infty} e^{-0.5x}$$

$$= \frac{e^{-0.5x}}{-0.5} \times 0.5$$

$$= -\left[e^{-0.5x}\right]_{2}^{\infty}$$

$$= e^{-\infty} - e^{-1}$$

$$= -\left[0 - \frac{1}{e}\right]$$

$$= \frac{1}{e}$$

$= 0.368$

Q3) $\alpha = 2$ $\lambda = \frac{1}{2}$ $0 < x < \infty$

i) $x \sim \text{Gamma}(2, \frac{1}{2})$

$$\text{Mean} = \frac{\alpha}{\lambda} = \frac{2}{1/2} = 4$$

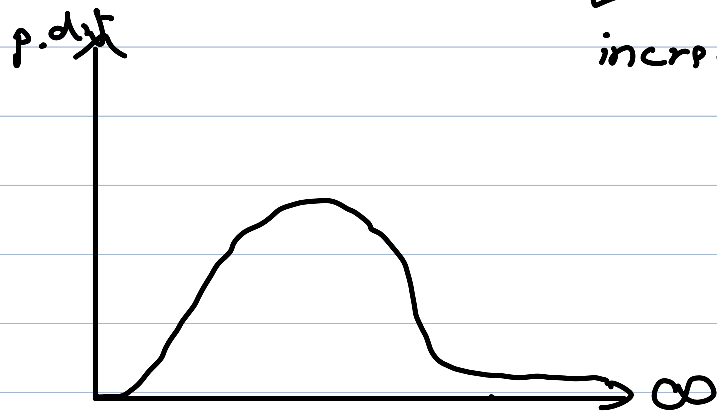
$$\text{varience} = \frac{\alpha}{\lambda^2} = \frac{2}{1/4} = 8$$

(ii) $\alpha = 2$, $\lambda = \frac{1}{2}$

$$\text{p.d.f} = \int_0^{?} \frac{x^{\alpha-1} e^{-\lambda x}}{\sqrt{\lambda}}$$

as $\alpha = 2$ $\underbrace{x^{\alpha-1} \lambda^{\alpha}}$ $e^{-\lambda x}$

increass then decrsases



(iii) c.d.f $\Rightarrow G\left(\alpha = 2, \lambda = \frac{1}{2}\right)$

$$= \begin{cases} 0, & x < 0 \\ \frac{\lambda^\alpha}{\sqrt{\alpha}} \int_0^x x^{\alpha - 1} e^{-\lambda x} . dx \end{cases}$$

Q4) 10 male 8 femals
6 mals - Q 7 femals - Q

$$P(B) = \frac{{}^6C_1}{{}^{18}C_2}$$

$$= \frac{6!}{5!\,1!} \div \frac{18!}{16!\,2!}$$

$$= 6 \div \frac{17 \times 18}{2}$$

$$= \frac{\cancel{6}^{2} \times \cancel{2}^{1}}{17 \times \cancel{18}\,\cancel{9}_{3}}$$

$$= \frac{2}{51}$$

$$= 0.0392$$

Q5) $P(L_1) = 0.4$

$P(L_2) = 0.05$

$P(L_3) = 0.1$

$P(A/L_1) = 0.2 \qquad P(A/L_2) = 0.4$

$P(A/L_3) = 0.1$

$$P(L_2/A) = \frac{P(A/L_2) \cdot P(L_2)}{P(A/L_2) \cdot P(L_2) + P(A/L_3) \cdot P(L_3) + P(A/L_1) \cdot P(L_1)}$$

$$= \frac{0.4 \times 0.5}{0.4 \times 0.5 + 0.2 \times 0.4 + 0.1 \times 0.1}$$

$$= \frac{0.2}{0.2 + 0.08 + 0.01}$$

$$= \frac{0.2}{0.29}$$

$$= \frac{20}{29}$$

$$= 0.6896$$

Q6) $X \sim U(1,2)$ $\quad y = \dfrac{3}{6-x}$

$F_y(y) = P(Y \leq y)$

$F_y(y) = P\left(\dfrac{3}{6-x} \leq y\right)$

$F_y(y) = P\left(\dfrac{6-x}{3} \geq y\right)$

$= P(6-x \geq 3y)$

$= P(x-6 \leq -3y)$

$F_y(y) = P(x \leq 6-3y$

$F_x(6-3y) = 6-3y$

$F_y(y) = 6-3y$

$$F_y(y) = \begin{cases} 0, & y > 2 \\ 6-3y, & 1 \leq y < 2 \end{cases}$$

$$f'y(y) = \begin{cases} -3, & 1 \leq y \leq 2 \end{cases}$$

Q7)

x	1	2	3	4	5
$P(X=x)$	0.05	0.15	0.35	0.4	0.05

i) $F(3) = P(X \leq 3) = P(1) + P(2) + P(3)$

$= 0.05 + 0.15 + 0.35$

$= 0.55$

ii) $E(X) = \sum_{0}^{5} x(Px)$

$= 5(0.05) + 4(0.04) + 3(0.35) + 2(0.15) + 0.05$

$= 0.25 + 1.6 + .05 + 0.3 + 0.05$

$= 1.3 + 0.35 + 0.16$

$= 1.65 + 1.6$

$= 3.25$

iii) $V(X) = E(X^2) - [E(X)]^2$

$= 0.05 + 0.6 + 31.5 + 6.4 + 1.25 - (3.25)^2$

$= 0.65 + 9.55 + 1.25 - 10.56$

$= 1.9 + 9.55 - 10.56$

$= 11.45 - 10.56$

$= 0.89$

Q8) $p = 0.2$ $\quad q = 0.8$

$x \sim \text{Binomial}(15, 0.2)$

$n = 15$

$P(x < 3) = P(0) + P(1) + P(2)$

$= {}^{15}C_0 \, p^0 q^{15} + {}^{15}C_1 \, p^1 q^{14}$

$+ {}^{15}C_2 \, p^2 q^{13}$

$= 1(0.8)^{15} + 15(0.2)(0.8)^{14}$

$+ 105(0.2)^2 (0.8)^{13}$

$= 0.39802$

Q9) $P(x = 7) = {}^6C_2 \, p^3 q^4$

$x \sim \text{binomial}(7, 0.01)$

$= \frac{6 \times 5}{2} (0.01)^3 (0.99)^4$

$= 15 (0.01)^3 (0.99)^4$

$= 0.0000014409$

Q10) $\lambda = \frac{2}{5} = 0.4$

$x \sim P(0.4)$

$P(x > 2) = 1 - P(x \leq 2)$

$= 1 - 0.99207$

$= 0.00793$

12) $X \sim \text{emp}(10)$

$$P(X<0.1) = \int_0^{0.1} 10(e^{-10x})\,dx$$

$$= \left[e^{-10x}\right]_0^{0.1}$$

$$= -\left[e^{-1} - e^{-0}\right]$$

$$= \left[1 - e^{-1}\right]$$

$$= 0.6321$$

13) $$P(80<x<100) = \int_{80}^{100} k \cdot dx$$

$$= \int_{80}^{100} \frac{1}{120-75} \cdot dx$$

$$= \frac{1}{45}\left[x\right]_{80}^{100}$$

$$= \frac{20}{45}$$

$$= \frac{4}{9}$$

Q14) $X \sim \text{Gamma}(5, 0.4)$

$\alpha = 5 \quad \lambda = 0.4$

$2\lambda x \sim \chi^2_{2\alpha = r}$

$0.8x \sim \chi^2_{10}$

$P(x < 22.89)$

$= P(2\lambda x < 2\lambda(22.89))$

$= P(\chi^2_{10} < 22.89 \times 0.8)$

$= P(\chi^2_{10} < 18.312)$

$18.0 \rightarrow 0.945$

$18.5 \rightarrow 0.9529$

$0.5 \rightarrow 0.0029$

$1 \rightarrow \frac{0.0079}{0.5}$

$0.31 \rightarrow \frac{0.0079}{0.5} \times 0.31$

$18.31 \rightarrow 0.945 + \frac{0.0079}{0.5} \times 0.31$

$18.31 = 0.949898$

$\therefore P(\chi^2_{10} < 18.312) = 0.949898$