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Assignment Unit 1 & 2

1.(i) let T be time of default

$P(T_1 < 1, T_2 < 1, T_3 < 1) = C(u_1, u_2, u_3)$

$\exp\left(-\left((-\ln u_1)^2 + (-\ln u_2)^2 + (-\ln u_3)^2\right)^{\frac{1}{2}}\right)$

$\exp\left(-\left(3(-\ln 0.1)^2\right)^{\frac{1}{2}}\right)$

0.0185

(ii) The Gumbel copula exhibits upper-tail dependence and no lower tail dependence.

Gumbel is appropriate if three investment are likely to behave similarly

If one bond defaults early on, the other investments may also be likely to default

2. (i) The three parameters are:
- a location ~~param~~ parameter, α
- a scale parameter β
- a shape parameter γ

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(ii) The parameter α and β just rescale the distribution.
The parameter γ determine the overall shape of the distribution.

(iii) The parameter α and β are analogous to the mean and standard deviation.

The parameter γ are analogous to the skewness.

3. (i) The existence of moment
limiting density ratio
the hazard rate
the mean residual life.

(ii) $\alpha = 0.2$, $\alpha = 0.3$

$$\lim_{x \to \infty} \frac{f_{\alpha=0.2}(x)}{f_{\alpha=0.3}(x)}$$

$$= \lim_{x \to \infty} \frac{0.2\lambda^{0.2}}{(\lambda+x)^{1.2}} \Big/ \frac{0.3\lambda^{0.3}}{(\lambda+x)^{1.3}}$$

$$= \frac{0.2}{0.3\lambda^{0.1}} \lim_{x \to \infty} (\lambda+x)^{0.1}$$

$$\infty$$

$\alpha = 0.2$ has thicker tail

4. P(male survive) = 0.6
P(female survive) = 0.65

Death prob = 0.4 , 0.35

(i) Gumbel Copula , $\alpha = 3.5$

$u = 0.4$, $v = 0.35$

$P(\cancel{X \le 25}, \cancel{Y \le 2})$
$P(= C(u,v)$

$= \exp\left(-\left((-\ln 0.4)^{3.5} + (-\ln 0.35)^{3.5}\right)\right)^{1/3.5}$

$= \exp\left(-\left((0.73640) + (1.185509)\right)\right)^{1/3.5}$

$= 0.2996$

(ii) Clayton copula $\alpha = 3.5$

$P = C(u,v)$
$\left(u^{-\alpha} + v^{-\alpha} - 1\right)^{-1/\alpha}$

$\left(0.4^{-3.5} + 0.35^{-3.5} - 1\right)^{-1/3.5}$

$= 0.30594$

(iii) Frank copula $\alpha = 3.5$

$C(u,v)$

$$= \frac{-1}{\alpha} \ln\left(1 + \frac{(e^{-\alpha u} - 1)(e^{-\alpha v} - 1)}{e^{-\alpha} - 1}\right)$$

$$\frac{-1}{3.5} \ln\left(1 + \frac{(e^{-0.4 \times 3.5} - 1)(e^{-0.35 \times 3.5} - 1)}{(e^{-3.5} - 1)}\right)$$

$$\frac{-1}{35} \ln\left(1 + \frac{(\overset{-0.75340}{\cancel{3.055199}})(-0.70624)}{-0.96980}\right)$$

$\cancel{= 0.45}$

0.22728

(iv) Clayton gives the highest probability because it exhibits lower tail dependence.

The ~~Gumbel~~ Frank copula copula give the lowest probability.

5.

(i) $e(x) = \int_x^\infty \frac{(y-x)f(y)\,dy}{\int_0^\infty f(y)\,dy}$ $\lambda = 0.125$

$= \int_x^\infty \frac{1-F(y)}{1-F(x)}\,dy$

$\frac{\int_x^\infty e^{-\lambda y}\,dy}{e^{-\lambda x}}$

$= \frac{\left[\frac{-1}{\lambda}e^{-\lambda y}\right]_x^\infty}{e^{-\lambda x}} = \frac{\frac{1}{\lambda}e^{-\lambda x}}{e^{-\lambda x}}$

$= 1/\lambda = 1/0.125$

$= 8$

(ii) The exponential mean residual life is constant.