Numerical methods and algebra

Assignment

1) Ans:-

We know that Area of the circular plate. $A(r) = \pi r^2$,

$A'(r) = 2\pi r$.

Change in Area $= A'(12.5)(0.15)$

$= 3.75\pi \ cm^2$

Exact calculation of the chage in Area $= A(12.65) - A(12.5)$

$= 160.0225\pi - 156.25\pi$

$= 3.7725\pi \ cm^2$

(i) Absolute error = Actual value - Approximate value

$= 3.7725\pi - 3.75\pi$

$= 0.0225\pi \ cm^2$

(ii) Relative error $= \dfrac{\text{Actual value} - \text{Approximate value}}{\text{Actual value}}$

$= \dfrac{3.7725\pi - 3.75\pi}{3.7725\pi}$

$= \dfrac{0.0225\pi}{3.7725\pi}$

$= 0.006 \ cm^2$

(iii) Percentage error = Relative error × 100

$= 0.006 \times 100$

$= 0.6\%$

2) Ans :-

(a) $y = y_1 + (x - x_1)\dfrac{(y_2 - y_1)}{(x_2 - x_1)}$

$= \dfrac{(5-4) \times (0.7781513 - 0.60206)}{(6-4)} + 0.60206$

$= \dfrac{0.1760913}{2} + 0.60206$

$= 0.088 + 0.60206$

$y = 0.69006$

(b) $y = y_1 + (x - x_1)\dfrac{(y_2 - y_1)}{(x_2 - x_1)}$

$= \dfrac{(5-4.5)(0.7403627 - 0.6532125)}{(5.5 - 4.5)} + 0.6532125$

$= \dfrac{0.0435751}{1} + 0.6532125$

$= \cancel{0.04}\ 0.0436 + 0.6532125$

$y = 0.6968125$

(3) $x^4 - x - 10 = 0$

$x_0 = 1.5$ $\qquad$ $y_1 = -6.4375$

$x_1 = 2$ $\qquad$ $y_2 = 4$

$\rightarrow$ $x_2 = \dfrac{x_0 + x_1}{2}$

$= 1.75$

$y_2 = -2.37109$

$\rightarrow$ $x_3 = \dfrac{x_1 + x_2}{2}$

$= 1.875$

$y_3 = 0.48462$

$\rightarrow$ $x_4 = \dfrac{x_2 + x_3}{2}$

$= 1.8125$

$y_4 = -1.02025$

$\rightarrow$ $x_5 = \dfrac{x_3 + x_4}{2}$

$= 1.84375$

$y_5 = -0.28773$

$\rightarrow$ $x_6 = \dfrac{x_4 + x_5}{2}$

$= 1.859375$

$y_6 = 0.93378$

4) Ans :-

Here $x^3 - x - 1 = 0$

let $f(x) = x^3 - x - 1$

$\therefore f'(x) = 3x^2 - 1$

Here

x	0	1	2
$f(x)$	-1	-1	5

$\therefore f(1) = -1 < 0$ and $f(2) = 5 > 0$

$\therefore$ Root lies between 1 and 2

$$x_0 = \frac{1+2}{2} = 1.5$$

1st iteration -

$f(x_0) = f(1.5) = 0.875$

$f'(x_0) = f'(1.5) = 5.75$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 1.5 - \frac{0.875}{5.75}$$

$x_1 = 1.34783$

2nd iteration -

$f(x_1) = f(1.34783) = 0.10068$

$f'(x_1) = f'(1.34783) = 4.44991$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1.34783 - \frac{0.10068}{4.44991}$$

$x_2 = 1.3252$

3^{rd} iteration -

$f(x_2) = f(1.3252) = 0.00206$

$f'(x_2) = f'(1.3252) = 4.26847$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 1.3252 - \frac{0.00206}{4.26847}$$

$x_3 = 1.32472$

4^{th} iteration

$f(x_3) = f(1.32472) = 0$

$f'(x_3) = f'(1.32472) = 4.26463$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 1.32472 - \frac{0}{4.26463}$$

$x_4 = 1.32472$

Approximate root of equation $x^3 - x - 1 = 0$ using Newton Raphson method is 1.32472

n	x_0	$f(x_0)$	$f'(x_0)$	x_1	update
1	1.5	0.875	5.75	1.34783	$x_0 = x_1$
2	1.34783	0.10068	4.44991	1.3252	$x_0 = x_1$
3	1.3252	0.00206	4.26847	1.32472	$x_0 = x_1$
4	1.32472	0	4.26463	1.32472	$x_0 = x_1$

5) Ans :-

$7A - (I + A)^3$

$= 7A - (I^3 + A^3 + 3I^2A + 3IA^2)$ .... $(A^2 = A)$

$= 7A - (I + A^3 + 3A + 3A^2)$

$= 7A - (I + A^2 \cdot A + 3A + 3A^2)$

$= 7A - (I + A \cdot A + 3A + 3A)$

$= 7A - (I + A \cdot A + 6A)$

$= 7A - (I + A^2 + 6A)$

$= 7A - (I + A + 6A)$

$= 7A - (I + 7A)$

$= -I$

$\therefore 7A - (I + A)^3 = -I$

6) Ans :-

$$\begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix} = A$$

$$A^{-1} = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 4 & 0 & 6 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$[R_2 = R_2 - 4R_1]$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 4 & -2 & -4 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$[R_2 = R_2/4]$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1/2 & -1 & 1/4 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$[R_1 = R_1 + R_2]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3/2 & 0 & 1/4 & 0 \\ 0 & 1 & -1/2 & -1 & 1/4 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

$[R_3 = R_3 - R_2]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3/2 & 0 & 1/4 & 0 \\ 0 & 1 & -1/2 & -1 & 1/4 & 0 \\ 0 & 0 & -1/2 & 1 & -1/4 & 1 \end{array}\right]$$

$[R_3 = -2R_3]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3/2 & 0 & 1/4 & 0 \\ 0 & 1 & -1/2 & -1 & 1/4 & 0 \\ 0 & 0 & 1 & -2 & 1/2 & -2 \end{array}\right]$$

$[R_1 = R_1 - 3R_3/2]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1/2 & 3 \\ 0 & 1 & -1/2 & -1 & 1/4 & 0 \\ 0 & 0 & 1 & -2 & 1/2 & -2 \end{array}\right]$$

$[R_2 = R_2 + R_3/2]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1/2 & 3 \\ 0 & 1 & 0 & -2 & 1/2 & -1 \\ 0 & 0 & 1 & -2 & 1/2 & -2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

7) Ans:-

The formula to calculate the angle between two vectors is given by:-

$$\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}\right]$$

Here, θ represents the angle between two vectors.

Substitute all the values.

$$\theta = \cos^{-1}\left[\frac{(8\hat{i}-9\hat{j})\cdot(7\hat{i}-9\hat{j})}{|(8\hat{i}-9\hat{j})||(7\hat{i}-9\hat{j})|}\right]$$

$$= \cos^{-1}\left[\frac{(8)(7)+(-9)(-9)}{\sqrt{(8)^2+(-9)^2}\sqrt{(7)^2+(-9)^2}}\right]$$

$$= 3.76^\circ$$

Thus, the angle between vector is 3.76°

8) Ans :-

Given - $A = 75\ m$

$B = 25\ m$

The angle $\theta = 30^\circ$.

Sol^n - $R^2 = A^2 + B^2 + 2AB \cos 30^\circ$

$$= (75)^2 + (25)^2 + 2(75)(25)\frac{\sqrt{3}}{2}$$

$$R = \sqrt{5625 + 625 + 3247.5}$$

$$R = 97.46\ m$$

9) Ans :-
The lowest 3 digit number is 101 and the highest number is 998 which when divided by '3' give remainder of '2'.

We know that the n^{th} term of AP is given by
$a_n = a + (n-1)d$
Now, substituting the values, we get:
$a_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$(n-1)3 = 998 - 101$

$(n-1)3 = 897$

$n-1 = \frac{897}{3}$

$n-1 = 299$

$n = 300$

We also know that the sum to n^{th} term in an A.P. is given by,
$S = \frac{n}{2}(1^{st}\text{ term} + \text{last term})$

$S = \frac{300}{2}(101 + 998)$

$= 150 \times 1099$

$= 164850$

Hence, the sum of all 3 digit numbers that leave a remainder of 2 when divided by 3 is 164,850.

10) Ans :-

Given - 6^{th} term of a 'gp' is 32 and its 8^{th} term is 128

To find - common ratio of the 'gp'.

Solution -

Let say GP is

$a, ar, ar^2, \ldots$

n^{th} term of a GP $= ar^{n-1}$

6^{th} term $= ar^5 = 32$

8^{th} term $= ar^7 = 128$

11) Ans :-

Using Binomial th^m

$$(4x+3x)^5 = \sum_{i=0}^{5} \binom{5}{i} (4x)^{(5-i)} (3x)^i$$

$$= \frac{5!}{0!\,(5-0)!} (4x)^5 (3x)^0 + \frac{5!}{1!\,(5-1)!} (4x)^4 (3x)^1 + \frac{5!}{2!\,(5-2)!} (4x)^3 (3x)^2$$

$$+ \frac{5!}{3!\,(5-3)!} (4x)^2 (3x)^3 + \frac{5!}{4!\,(5-4)!} (4x)^1 (3x)^4 + \frac{5!}{5!\,(5-5)!} (4x)^0 (3x)^5$$

$$= 1024x^5 + 3840x^5 + 5760x^5 + 4320x^5 + 1620x^5 + 243x^5$$

12) Ans :-

$$A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$$|A - \lambda I|$$

Enpanding using 3^{rd} raw

$$|A - \lambda I| = 0 - 0 + (3-\lambda)[(2-\lambda)(-5-\lambda) + 6]$$

$$= (3-\lambda)[-10-2\lambda+5\lambda+\lambda^2+6]$$

$$= (3-\lambda)[\lambda^2+3\lambda-4]$$

$$= 3\lambda^2 + 1\lambda - 12 - \lambda^3 - 3\lambda^2 + 4\lambda$$

$$= -\lambda^3 + 13\lambda - 12$$

$$-\lambda^3 + 13\lambda - 12 = 0$$

$$\lambda^3 - 13\lambda + 12 = 0$$

compairing coifficient

$$a+b+c+d = 1 - 13 + 12$$

$$= 0$$

only on root $= 1$

1	1	0	-13	12
	1	1	1	-12
	1	1	-12	0

$$\lambda^2 + \lambda - 12 = 0$$

$$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$$

$$\lambda(\lambda + 4) - 3(\lambda + 4) = 0$$

$$(\lambda - 3)(\lambda + 4) = 0$$

$$\lambda = 3 \quad \text{or} \quad -4$$

$\therefore$ Eigen value $= -4, 3$ & 1

14) Ans :-

$(1-x+x^2)^4$

$= (1-x+x^2)(1-x+x^2)(1-x+x^2)(1-x+x^2)$

$= (x^4-2x^3+3x^2-2x+1)(1-x+x^2)(1-x+x^2)$

$= (x^6-3x^5+6x^4-7x^3+6x^2-3x+1)(1-x+x^2)$

$= (x^6\cdot 1+x^6(-x)+x^6x^2-3x^5\cdot 1-3x^5(-x)-3x^5x^2+6x^4\cdot 1+6x^4(-x)$

$+6x^4x^2-7x^3\cdot 1-7x^3x^2+6x^2\cdot 1+6x^2(-x)+6^2(-x)+6x^2x^2-3x\cdot 1$

$-3x(-x)-3x x^2+1\cdot 1+1\cdot(-x)+1\cdot x^2$

$= x^8-4x^7+10x^6-16x^5+19x^4-16x^3+10x^2-4x+1$

15) $e^{-x} = 3.\log(x)$

$3\log x + e^{-x} = 0$

x	y
0.1	-2.09516
0.2	-1.27818
0.3	-0.82782
0.4	-0.5235
0.5	-0.29656
0.6	-0.11673
0.7	+0.03188

$x_0 = 0.6 \qquad y_0 = -0.11673$

$x_1 = 0.7 \qquad y_1 = 0.03188$

$\rightarrow x_2 = \frac{x_0 + x_1}{2} = 0.65 \quad , \quad y_2 = -0.00293$

$\rightarrow x_3 = \frac{x_2 + x_1}{2} = 0.675 \quad , \quad y_3 = -0.00293$

$\rightarrow x_4 = \frac{x_3 + x_2}{2} = 0.6875 \quad , \quad y_4 = 0.01465$

$\rightarrow x_5 = \frac{x_4 + x_3}{2} = 0.6825 \quad , \quad y_5 = 0.0059$