NMA ASSIGNMENT

1. Observed Radius = 12.65 cm.
Actual Radius = ~~12 cm~~ 12.5 cm

Observed area = πr^2.
$A_0 = \pi \times (12.65)^2$
$\therefore A_0 = 502.725 \text{ cm}^2$

Actual area = πr^2.
$A_1 = \pi \times (12.5)^2$
$\therefore A_1 = 490.874 \text{ cm}^2$

(i) Absolute error = Observed value - actual value
= 502.725 − 490.874
= 11.851 cm^2

(ii) Relative error = $\dfrac{\text{Absolute error}}{\text{Actual value}}$
$= \dfrac{11.851}{490.874}$
= 0.02414

(iii) Percentage error = Relative error × 100
= 2.414 %

2.(i)

$x_1 = 4 \qquad y_1 = 0.60206$

$x = 5 \qquad y = ?$

$x_2 = 6 \qquad y_2 = 0.7781513$

$$\therefore \; y = \frac{y_2(x - x_1) + y_1(x_2 - x)}{(x_2 - x_1)}$$

$$= \frac{0.7781513(1) + 0.60206(1)}{2}$$

$$\therefore \; y = 0.6901$$

(ii)

$x_1 = 4.5 \qquad y_1 = 0.6532125$

$x = 5 \qquad y = ?$

$x_2 = 5.5 \qquad y_2 = 0.7403627$

$$\therefore \; y = \frac{y_2(x - x_1) + y_1(x_2 - x)}{x_2 - x_1}$$

$$= \frac{0.7403627(0.5) + 0.6532125(0.5)}{1}$$

$$\therefore \; y = 0.6901$$

3. $x^4 - x - 10 = 0.$

$a = 1.5 \qquad b = 2.$

$\therefore f(a) = f(1.5) = (1.5)^4 - 1.5 - 10$
$= -6.4375$

$f(b) = f(2) = 2^4 - 2 - 10$
$= 4$

$\therefore$ 1st iteration:

$$x_1 = \frac{2 + 1.5}{2} = 1.75$$

$f(x_1) = f(1.75) = 1.75^4 - 1.75 - 10 = -2.3711$

2nd iteration:

$$x_2 = \frac{1.75 + 2}{2} = 1.875$$

$f(x_2) = 1.875^4 - 1.875 - 10 = 0.4846$

3rd iteration:

$$x_3 = \frac{1.75 + 1.875}{2} = 1.8125$$

$f(x_3) = f(1.8125) = 1.8125^4 - 1.8125 - 10 = -1.0202$

4th iteration

$$x_4 = \frac{1.875 + 1.8125}{2} = 1.84375$$

$f(x_4) = 1.84375^2 - 1.84375 - 10 = -0.2877$

5th iteration

$$x_5 = \frac{1.875 + 1.84375}{2} = \cancel{1.859 \approx 1.86}\ 1.859375$$

$f(x_6) \cancel{= 1.86^4 - 1.86 - 10 =} = 1.859375^4 - 1.859375 - 10$
$= 0.093$

$\therefore$ root of $x^4 - x - 10 = 0$ is 1.859375
which is approximately 1.86.

4. $f(x) = x^3 - x - 1$

$$f'(x) = 3x^2 - 1$$

x	0	1	2
y	-1	-1	5

$$\therefore \; x_0 = \frac{1+2}{2} = 1.5$$

$f(x_0) = 0.875$ $\qquad$ $f'(x_0) = 5.75$

$$\therefore \; x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1.5 - \frac{0.875}{5.75} \qquad \cancel{= 1.65217}$$

$$\cancel{f(x_1) = 1.8577} \qquad = 1.34783$$

$f(x_1) = 0.10068$ $\qquad$ $f'(x_1) = 4.44991$

$$x_2 = 1.34783 - \frac{0.10068}{4.44991}$$

$$= 1.3252$$

$\therefore \; f(x_2) = 0.00205$ $\qquad$ $f'(x_2) = 4.2684$

$$x_3 = 1.3252 - \frac{0.00205}{4.2684}$$

$$\therefore \; x_3 = 1.3247$$

$f(x_3) = 0.000007545$ $\qquad$ $f'(x_3) = 4.2645$

$$\therefore \; x_4 = 1.3247 - \frac{0}{4.2645}$$

$$\therefore \; x_4 = 1.3247$$

$\therefore$ Root of $x^3 - x - 1 = 0$ is $\underline{\underline{1.3247}}$.

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5. $A^2 = A$

$7A - (I+A)^3$: To find

$\therefore 7A - (I+A)^3 = \cancel{7A - I + A}$

$= 7A - (I^3 + A^3 + 3A^2 I + 3AI^2)$

$= 7A - I^3 - A^3 - 3A^2 I - 3AI^2$

$= 7A - I^3 - A^2 . A - 3A^2 - 3A$

$= 7A - I - A . A - 3A - 3A$

$= 7A - I - A - 6A$

$= 7A - 7A - I$

$\therefore 7A (I + A)^3 = -I$

6. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$\therefore |A| = 1 \begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} + 1 \begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} + 2 \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$

$= 1(-6) + 1(-4) + 2(4) = -2$

$|A| \neq 0 \quad \therefore$ inverse exists.

$\therefore$ co-factor matrix $= \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & 1 \\ -6 & 2 & 4 \end{bmatrix}$

$\therefore$ Adjoint matrix $= \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} \times$ Adjoint

$= \frac{-1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & -1/2 & -2 \end{bmatrix}$$

9. The smallest 3 digit no. that leaves remainder ~~is~~ 1 of 2 is 101 and the largest is 998.

$\therefore$ $a = 101$, $l = 998$

$d = 3$.

$\therefore$ $t_n = a + (n-1)d$.

$998 = 101 + (n-1)3$

$\frac{998 - 101}{3} = n - 1$

$\therefore$ $\underline{n = 300}$

$S_n = \frac{n}{2}(a + l) = \frac{300}{2}(101 + 998)$

$= \underline{164850}$

$\therefore$ Sum of all numbers that leave a remainder of 2 when divided by 2 is $\underline{164850}$

10. $t_6 = 32$.

$t_8 = 128$

$\therefore$ $t_6 = ar^5$

$32 = ar^5$ — (1)

$t_8 = ar^7$

$128 = ar^7$ — (2)

Dividing (2) and (1)

$\frac{\cancel{ar}^{\cancel{7}2}}{\cancel{ar}^{\cancel{5}}} = \frac{128}{32}$

$\therefore$ ~~ar~~ $r^2 = 4$

$\therefore$ $\boxed{r = 2}$

11. expand $(4+3x)^5 = (3x+4)^5$

~~∴ a~~

$$\therefore (a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 \dots + b^n$$

$$\therefore (3x+4)^5 = (3x)^5 + 5(3x)^4 4 + \frac{5\times\cancel{4}^{2}}{\cancel{2}}(3x)^3 4^2 + \frac{5\times\cancel{4}^{2}\times\cancel{3}}{\cancel{3\times 2}}(3x)^2 4^3$$

$$+ \frac{5\times\cancel{4}\times\cancel{3}\times\cancel{2}}{\cancel{4}\times\cancel{3}\times\cancel{2}}(3x)4^4 + 4^5$$

$$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$$

12. $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$\therefore \lambda^3 - (0)\lambda^2 + (-13)\lambda - (-30 + 18 + 0) = 0.$$

$$\therefore \lambda^3 - 13\lambda + 12 = 0.$$

$\therefore \lambda = 1$ satisfies.

$$\therefore \begin{array}{c|cccc} 1 & 1 & 0 & -13 & 12 \\ & \downarrow & 1 & 1 & -12 \\ \hline & 1 & 1 & -12 & \boxed{0} \end{array}$$

$$\therefore \lambda^2 + \lambda - 12 = 0.$$

$$\lambda^2 + 4\lambda - 3\lambda - 12 = 0.$$

$$(\lambda + 4)(\lambda - 3) = 0.$$

$$\lambda = -4 \quad \text{or} \quad \lambda = 3.$$

12. $\cancel{2x_1 - 4x_2 = 0}$

$\cancel{(-) \quad x_1 \quad -}$

13. $f(x) = x^3 - 7x^2 + 8x - 3$.

$x_0 = 5$

$\therefore f(x) = x^3 - 7x^2 + 8x - 3$

$f(5) = -13$.

$f'(x) = 3x^2 - 14x + 8$.

$f'(5) = 13$.

$\therefore x_1 = 5 - \left(\frac{-\cancel{13}^{1}}{\cancel{13}}\right)$

$= \underline{6}$

$f(6) = 9$.

$f'(6) = 32$.

$x_2 = 6 - \left(\frac{9}{32}\right)$

$\therefore \boxed{x_2 = 5.71875}$

14. $(1 - x + x^2)^4$

$\therefore [x^2 + (1-x)]^4$

let $1-x = y$.

$\therefore [x^2 + y]^4$

using binomial theorem,

$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 \cdots + b^n$

$\therefore [x^2+y]^4 = (x^2)^4 + 4(x^2)^3 y + \frac{\cancel{4}^{2} \times 3}{\cancel{2}}(x^2)^2 y^2 + \frac{4 \times \cancel{3} \times \cancel{2}}{\cancel{3} \times \cancel{2}}(x^2) y^3 + y^4$

$= x^8 + 4x^6 y + 6x^4 y^2 + 4x^2 y^3 + y^4$

$\therefore [x^2 + (1-x)]^4 = x^8 + 4x^6(1-x) + 6x^4(1-x)^2 + 4x^2(1-x)^3 + (1-x)^4$

15. $e^{-x} = 3\log(x)$

$\therefore f(x) = 3\log x - e^{-x}$

$a = 1 \qquad b = 2.$

$\therefore f(a) = -0.367879.$

$f(b) = 1.944106$

$\therefore \frac{a+b}{2} = \frac{1+2}{2} = 1.5 = x_1$

$f(x_1) = 0.993265$

$x_2 = \frac{1+1.5}{2} = 1.25.$

$f(x_2) = 3\log(1.25) - e^{-1.25}$

$= 0.382925$

$$x_3 = \frac{1+1.25}{2} = 1.125$$

$$f(x_3) = 3\log(1.125) - e^{-1.125}$$
$$= 0.02869664$$

$$\therefore x_4 = \frac{1+1.125}{2} = 1.0625$$

$$f(x_4) = 3\log(1.0625) - e^{-1.0625}$$
$$= -0.16372$$