ASSIGNMENT-1

1) Actual length = r_L = 12.5 cm

actual area = $\pi r_L^2 = \pi(12.5)^2 = 490.87385 \, cm^2$

approximate radius = 12.65

approximate area = $\pi r^2 = \pi(\text{approximate radius})^2 = 502.72551 \, cm^2$

(i) absolute error = approx area − actual area = $11.85166 \, cm^2$

(ii) relative error = $\frac{\text{absolute error}}{\text{actual error}} = \frac{11.85166}{490.87385} = 0.024144$.

(iii) Percentage error = (relative error) * 100 = 2.4144%.

2) (i) Using linear interpolation

$x_1 = 4, y_1 = 0.60206 \qquad x_2 = 6, y_2 = 0.7781513$

$x = 5, y = ?$

$$\frac{x_2 - x_1}{x - x_1} = \frac{y_2 - y_1}{y - y_1}$$

$$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y = 0.60206 + \frac{0.7781513 - 0.60206}{2} * (5-4)$$

$$y = 0.60206 + \left(\frac{0.1760913}{2}\right)$$

$$y = 0.60206 + 0.08804565$$

$$y = 0.69010565$$

(ii) $x_1 = 4.5, \; y_1 = 0.6532125$

$x_2 = 5.5, \; y_2 = 0.7403627$

$x = 5, \; y = ?$

$$y = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y = 0.6532125 + \left(\frac{0.7403627 - 0.6532125}{1}\right) * (5 - 4.5)$$

$$y = 0.6532125 + 0.0871502 \times \frac{1}{2}$$

$$y = 0.6967876.$$

3) $x^4 - x - 10 = 0$

x	$f(x)$
$x_0 = 1.5$	-6.4375
$x_1 = 2$	4

(i) $x_2 = \dfrac{x_0 + x_1}{2} = 1.75$

$f(x_2) = -2.37109$

(ii) $x_3 = \dfrac{x_1 + x_2}{2} = 1.875$, $f(x_3) = 0.48462.$

(iii) $x_4 = \dfrac{x_2 + x_3}{2} = 1.8125$, $f(x_4) = -1.0202.$

(iv) $x_5 = \dfrac{x_4 + x_3}{2} = 1.84375$, $f(x_5) = -0.28773$

(v) $x_6 = \dfrac{x_5 + x_4}{2} = 1.859375$, $f(x_6) = 0.0933.$

4) $x^3 - x - 1 = 0$

$f'(x) = 3x^2 - 1$

x	$f(x)$
0	-1
1	-1
2	5

Let $x_0 = 1$, $f(x_0) = -1$

$\cancel{x_1 =}$ $f'(x_0) = 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \left(\frac{-1}{2}\right) = 1.5$$

$f(x_1) = 0.875$

$f'(x_1) = 5.75$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.5 - \left(\frac{0.875}{5.75}\right) = 1.34783$$

$f(x_2) = 0.10069$

$f'(x_2) = 4.4499.$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.3252$$

$f(x_3) = 0.00205.$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.32472.$$

$f(x_4) = 0.000008712.$

5) $A^2 = A$

$$7A - (I+A)^3 = 7A - (I^3 + 3IA(I+A) + A^3)$$
$$= 7A - (I^3 + 3A + 3IA^2 + A^3) \quad \text{....} \quad (AI = IA = A)$$
$$= 7A - (I^3 + 3A + 3A^2 + A^3)$$
$$= 7A - (I^3 + 3A + 3A + A^2(A)) \quad \text{.....} \quad (A^2 = A \text{ given})$$
$$= 7A - (I^3 + 6A + A) \quad \text{...} \quad (A^2(A) = A(A) = A^2 = A)$$
$$= -I^3$$
$$= -I$$

6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$, $|A| = 1(-6) - (-1)(-4-0) + 2(4-0)$

$|A| = -6 - 4 + 8 = -2$

$\therefore |A| = -2$

as $|A| \neq 0$, then A^{-1} is pssble.

$$adj(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$\boxed{A^{-1} = \frac{1}{|A|} * adj(A) = -\frac{1}{2}\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}}$$

Q.7) $\vec{a} = 8\hat{i} - 9\hat{j}$

$\vec{b} = 7\hat{i} - 9\hat{j}$

$\vec{a}\cdot\vec{b} = ab\cos\theta = (8\hat{i} - 9\hat{j})(7\hat{i} - 9\hat{j}) \quad \text{....} \quad (\hat{i}\cdot\hat{i} = 1 = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k})$

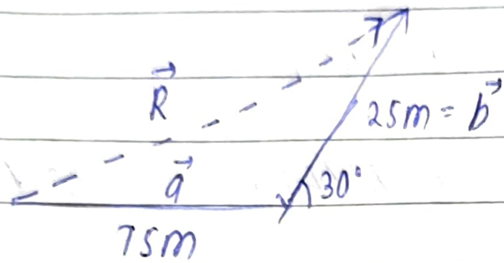
$= 56 + 81 = 137$

$\vec{a}\cdot\vec{b} = 137$, $|a| = \sqrt{64+81} = \sqrt{145}$, $|b| = \sqrt{49+81} = \sqrt{130}$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|a||b|} = \frac{137}{\sqrt{130}\times\sqrt{145}} = 0.99784914$$

$$\theta = 3.758°$$

8)



Let $\vec{R}$ be the displacement vector; let $\vec{a}$ be 75m towards east and $\vec{b}$ be ~~th~~ 25m and 30° to the left.

$\vec{R} = \vec{a} + \vec{b}$ ... (triangle law)

$R^2 = a^2 + b^2 + \cancel{2AB} 2ab\cos\theta$ ... $(\theta = 30°)$

$= 75^2 + 25^2 + 2\times75\times25\times\cos30°$

$R^2 = 9497.595264$

$|\vec{R}| = 97.4556066\,3\,m.$

~~dist~~ displacement is 97.45560663m.

9) common diff = 3

$a = 101$

~~an = a~~ $t_n = a + (n-1)d$

$998 = 101 + (n-1)3 = 101 + 3n - 3$

$n = 300$

$S_n = \frac{n}{2}(a + t_n) = \frac{300}{2}(101 + 998) = 150 * 1099 = 164850.$

10) $t_6 = 32$, $t_8 = 128$

$\frac{t_8}{t_6} = \frac{128}{32} = 4$

$\frac{\cancel{a}r^7}{\cancel{a}r^5} = r^2 = 4$

$r = 2$ or -2

$\therefore$ r = common ratio is 2 or -2.

11) $(3x+4)^5$

Using Pascal Δ.

$(3x+4)^5 = (3x)^5 + 5(3x)^4 * 4 + 10(3x)^3(4)^2$
$+ 10(3x)^2(4)^3 + 5(3x)(4)^4 + 4^3$

$(3x+4)^5 = 243x^5 + 1620x^4 + 4320x^3 + 5760x^2$
$+ 3840x + 1024.$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$|A - \lambda I| = 0 - 0 + (3-\lambda)\left[(2-\lambda)(-5-\lambda) - (-3)(2)\right]$ .... (using 3rd row)

$= 3-\lambda\left[-10 - 2\lambda + 5\lambda + \lambda^2 + 6\right]$

$= \cancel{-\lambda^3}\ 3-\lambda\left[\lambda^2 + 3\lambda - 4\right]$

$= -\lambda^3 - \cancel{3\lambda^2} + 4\lambda + \cancel{3\lambda^2} + 9\lambda - 12$

$= -\lambda^3 + 13\lambda - 12$

$|A - \lambda I| = 0$

$\therefore \lambda^3 - 13\lambda + 12 = 0$

$(\lambda - 1)(\lambda^2 + \lambda - 12) = 0$

$(\lambda - 1)(\lambda + 4)(\lambda - 3) = 0$

$\lambda = 1, -4, 3$

Eigen values $= 1, -4, 3$.

13) $F(x) = x^3 - 7x^2 + 8x - 3$

differentiating w.r.t x

$F'(x) = 3x^2 - 14x + 8$

$x_0 = 5$, $f(5) = -13$, $F'(5) = 13$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 6$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_0)} = 6 - \frac{9}{32} = 5.71875$$

$x_2 = 5.71875$, $f(x_2) = 0.84787$

14) $(x^2-x+1)^4 = \{(x-1)^2+x\}^4$

Using Pascal Triangle :

1	
1 1	$(x+y)$
1 2 1	$(x+y)^2$
1 3 3 1	$(x+y)^3$
1 4 6 4 1	$(x+y)^4$

$$\left[(x-1)^2+x\right]^4 = \left((x-1)^2\right)^4 + 4(x-1)^6 x + 6(x-1)^4 x^2 + 4(x-1)^2 x^3 + x^4$$

$$\left[(x-1)^2+x\right]^4 = (x-1)^8 + 4(x-1)^6 x + 6(x-1)^4 x^2 + 4(x-1)^2 x^3 + x^4$$

15) $e^{-x} = 3\log(x)$

$e^{-x} - 3\log(x) = 0$

x	$f(x)$
1.1	0.046940544
1.2	-0.245770458

$x_0 = 1.1$, $x_1 = 1.2$

i) $x_2 = \frac{x_0+x_1}{2} = \frac{1.1+1.2}{2} = 1.15$

$f(x_2) = -0.102649058$.

ii) $x_3 = \frac{x_0+x_2}{2} = 1.125$ $f(x_3) = -0.0286966$

iii) $x_4 = \frac{x_0+x_3}{2} = 1.1125$ $f(x_4) = 0.00890688$

iv) $x_5 = \frac{x_3+x_4}{2} = 1.11875$ $f(x_5) = -0.009948072$.