Assignment 1

(i) Arbitrage opportunity exists when we can make profit with no risk

(ii) Law of one price:
This states that any 2 portfolio that behave in exactly the same way must have same price.

(iii) 3 month put
$S_t = 125$
$K = 120$
$r = 5\%$

(a) Put call parity

$$P_t = C_t + Ke^{-r(T-t)} - S_t e^{-\epsilon(T-t)}$$

$$= 30 + 120e^{-5(0.25)} - 125e^{-0.5(0.25)}$$

$$= 28.11$$

(c)

2. let $dX_t = A_t dt + B_t dZ_t$

$A_t = \frac{\alpha \mu (T-t)}{\sigma \sqrt{T-t}}$
$B_t =$

By Ito's lemma

$$dF = \frac{dy}{dx} B_t dZ_t + \left( \frac{df}{dt} + \frac{df}{dx} A_t + \frac{1}{2} \frac{d^2 y}{dx^2} B_t \right) dt$$

$$= -f B_t dZ_t + f \frac{dn}{dt} (T-t) dt - f A_t dt$$
$$\frac{1}{2} f B_t^2 dt$$

$$dF = f \left[ \frac{dn}{dt} (T-t) - A_t + \frac{1}{2} B_t^2 \right] dt - \int B_t dZ_t$$

y to be martingale
$\Rightarrow 0$

$$\frac{dn}{dt} (T-t) - A_t + \frac{1}{2} B_t^2 = 0$$

$$\frac{dn}{dt} (T-t) = A_t - \frac{1}{2} B_t^2$$

$$\frac{dn}{dt} = \alpha\mu + \frac{1}{2} \sigma^2$$

5. $S_t = S_0 e^{(\mu - \sigma^2/2)} + \sigma W_t$

$S_0$

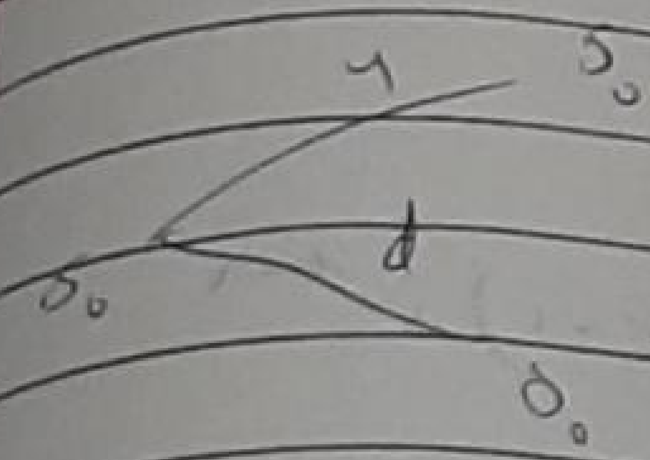
$\ln(S_t) \sim N\left(\ln(S_0) + \mu - \frac{\sigma^2}{2}\right)t, \sigma^2 t)$

$\ln(S_t) \sim \phi[\ln 254 + 0.06 - 0.35^2/2] \times 0.5$
$0.35 \times 0.5^{(1/2)}]$

$\ln(S_t) \sim \phi(5.57, 0.247)$

Hence put option is exercised when the stock price is less than 258 in 6 month

3.



$$C_u = S_0 u \phi + \psi e^{\psi}$$
$$C_d = S_0 d \phi + \psi e^{r}$$

$$C_u - C_d = \phi (S_0 u - S_0 d)$$

$$\phi = \frac{C_u - C_d}{S_0 (u - d)}$$

$$\psi = e^{-r}\left[\frac{u C_d - d C_u}{u - d}\right]$$

$$C_0 = \phi S_0 + \psi$$

$$C_0 = \frac{C_u - C_d}{S_0 (u-d)} \times S_0 + e^{-r}\left(\frac{u \cdot C_d - d C_u}{u - d}\right)$$

9. $13.534 + 90\,e^{(-0.\ldots)} = 0.120 + 5$

(i) $8.869 + 75\,e^{(-0.\ldots)} = 0.568 + 5$

$S = 82$

$r = 7.1$

forward price $= 82\,e^{(0.07 \times 0.25)}$

$= 83.45$

(ii) $e^{(r \times 0.5)} \cdot e^{(r_3 \times 0.25)} \times e^{(r_f \times 0.625)} = r_3 + \ldots$

$r_3 = 7.1$

$2.569 + 90\,e^{(0.5 \times r_6)} = 7.909 + 82$

$r_6 = 6.1$

$r_f = 5\%$

10. $q = \dfrac{(1-d)}{u-d}$

$q = \dfrac{1 - 1/u}{u - 1/u}$

$= \left(\dfrac{1}{u+1}\right)$

$u > 1 > d$

$u+1 > 2$

$q = \frac{1}{(u+1)} < 0.5$

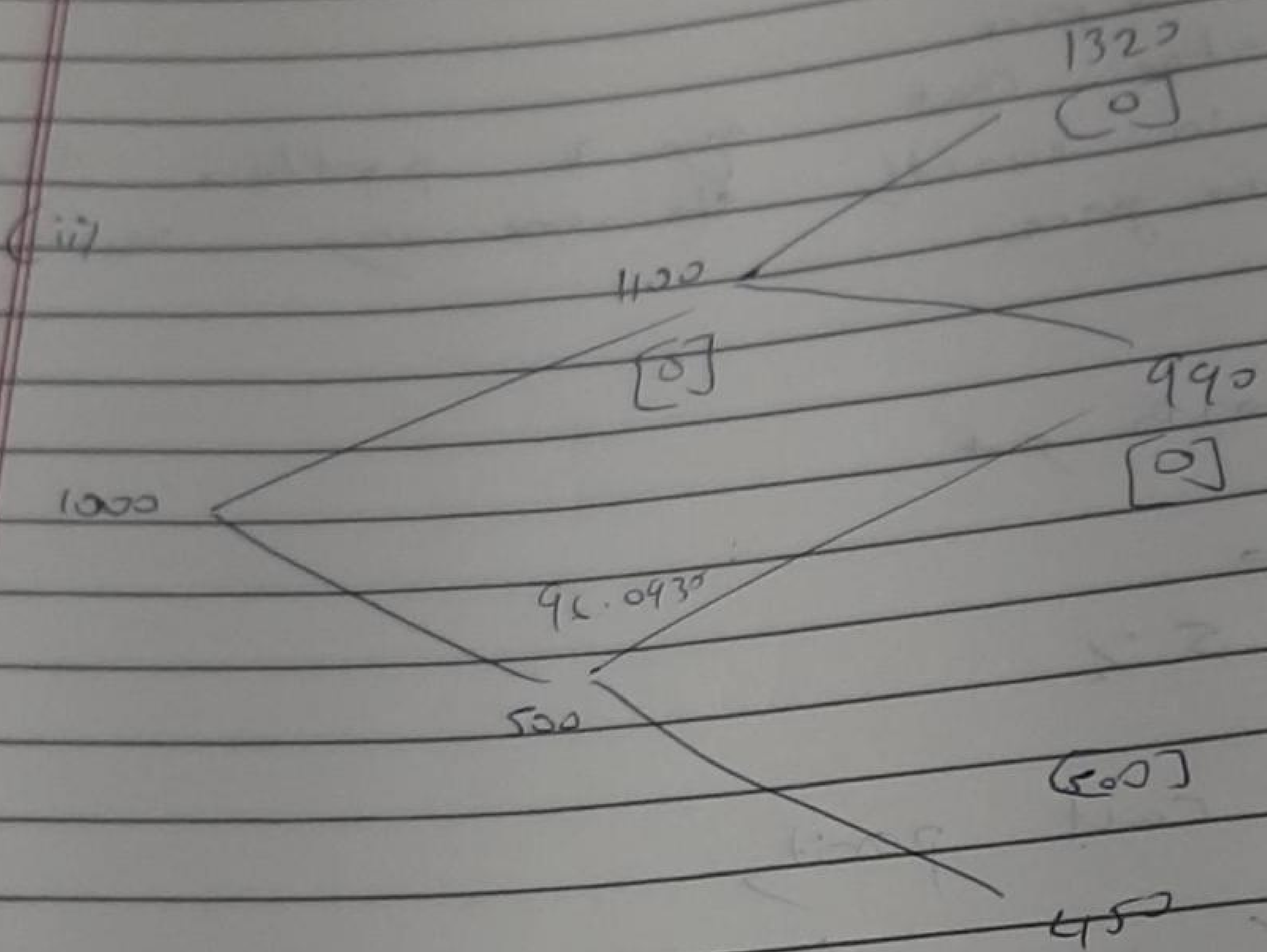
(ii) $q = \frac{1}{u+1}$

$q = \frac{1}{3}$

$u = 2$

11 (i)

(ii)




$K = \cancel{1000}\ 950$

(a) Risk-neutral of upward movement

$$q = \frac{e^{r} - d}{u - d}$$

$$= \frac{e^{0.0775} - 0.5}{1.1 - 0.5} = 0.80442$$

$$C_1[d] = e^{-r}\left[q(0) + (1-q)(500)\right]$$

$$= 96.0935$$

$$C_1 = e^{-0.0175}\left[0 + (1-q)(96.0935)\right]$$

$$= 18.467\cancel{5}$$

$B(1)12 = e^{-0.0120\times 2}\left[(1-g)^{2} \left(\right.\right.$

(b)

12

(a) $\partial/t$ ... 

$dv/t \quad = 2\,dz(t) = 0$

$0\,dt + 2dz/t)$

(3) $d(v/t) = d(z/t)^2 - dt$

$2\,z(t)\,d/z(t)\,\frac{+2}{2(dz/t)^2}$

$dv/t \quad = 2\,\cancel{z}(t) - d(z/t) + dt$

(c) $d(v/t) = d[t^2 z(t)) - 2t\,(z(t)\,dt$

$d(v/t) = t^2\,dz(t) + 2tz(t)\,dt$

$t^2\,d\,z/t)$

15. $f = f(S_t, t) = S^k$

$\frac{df}{ds} = kS^{k-1}$ $\frac{d^2y}{ds^2} = k(k-1)S^{k-2}$

$\frac{df}{dt} = 0$

$f_t = f_0\, e^{(\mu-1)2\sigma^2)t + \sigma^2 (w_t)}$

$\frac{f_t}{f_0} \sim \text{Log normal}\left( (\mu' - \frac{1}{2}\sigma'^2)t, \sigma'^2 t \right)$

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