

Homework - 1 Lagrange's Interpolation:

Ques 1

x	7	8	9	10
$f(x)=y$	3	1	1	9

Find y ; $x_k = 9.5$

$$y = \frac{(x_k - x_2)(x_k - x_3)(x_k - x_4)}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)} x_1 + \frac{(x_k - x_1)(x_k - x_3)(x_k - x_4)}{(x_2 - x_1)(x_2 - x_3)(x_2 - x_4)} x_2$$

$$+ \frac{(x_k - x_1)(x_k - x_2)(x_k - x_4)}{(x_3 - x_1)(x_3 - x_2)(x_3 - x_4)} x_3 + \frac{(x_k - x_1)(x_k - x_2)(x_k - x_3)}{(x_4 - x_1)(x_4 - x_2)(x_4 - x_3)} x_4$$

$$= \frac{(9.5 - 8)(9.5 - 9)(9.5 - 10)}{(7 - 8)(7 - 9)(7 - 10)} \times 3 + \frac{(9.5 - 7)(9.5 - 9)(9.5 - 10)}{(8 - 7)(8 - 9)(8 - 10)} \times 1$$

$$+ \frac{(9.5 - 7)(9.5 - 8)(9.5 - 10)}{(9 - 7)(9 - 8)(9 - 10)} \times 1 + \frac{(9.5 - 7)(9.5 - 8)(9.5 - 9)}{(10 - 7)(10 - 8)(10 - 9)} \times 9$$

$$= 0.1875 + (-0.3125) + 0.9375 + 2.8125$$

$$\boxed{y = 3.625}$$

$$\text{Ques 2} \begin{array}{c|c|c|c|c|c} x & 3 & 7 & 9 & 10 \\ \hline y & 168 & 120 & 72 & 63 \end{array}$$

$$x = 6 \quad \text{find } y = ?$$

$$f(6) = \left[\frac{(6-3)(6-9)(6-10)}{(3-7)(3-9)(3-10)} \times 168 + \frac{(6-3)(6-9)(6-10)}{(7-3)(7-9)(7-10)} \times 120 \right. \\ \left. + \frac{(6-3)(6-7)(6-10)}{(9-3)(9-7)(9-10)} \times 72 + \frac{(6-3)(6-7)(6-9)}{(10-3)(10-7)(10-9)} \times 63 \right]$$

$$= \left[\frac{(+1)(-3)(-4)}{(-4)(-6)(-7)} \times \frac{84}{1} + \frac{(3)(-3)(-4)}{(4)(-2)(-3)} \times \frac{60}{1} \right. \\ \left. + \frac{(3)(-1)(-4)}{(6)(2)(-1)} \times 72 + \frac{(3)(-1)(-3)}{(7)(3)(1)} \times 6 \right]$$

$$= \left(\frac{84}{7} + 3 \times 60 - 72 + \frac{18}{7} \right)$$

$$y = \left(\frac{102}{7} + 180 - 72 \right)$$

$$= \frac{108 + 102}{7} = \frac{756 + 102}{7} =$$

$$\boxed{y = 122.5714}$$

Ques 3

x	1	2	7	8
y	4	5	5	4

$$f(6) = ?$$

$$n = 6$$

$$y = \frac{(6-2)(6-7)(6-8) \times 4}{(1-2)(1-7)(1-8)} + \frac{(6-1)(6-7)(6-8) \times 5}{(2-1)(2-7)(2-8)}$$

$$+ \frac{(6-1)(6-2)(6-8) \times 5}{(7-1)(7-2)(7-8)} + \frac{(6-1)(6-2)(6-7) \times 4}{(8-1)(8-2)(8-7)}$$

$$= \left(\frac{4 \times \cancel{-1} \times \cancel{-2} \times 4}{\cancel{-1} \times \cancel{-6} \times \cancel{-7}} + \frac{5 \times \cancel{-1} \times \cancel{-2} \times 5}{1 \times \cancel{-5} \times \cancel{-6}} \right)$$

$$+ \frac{\cancel{5} \times 4 \times \cancel{2} \times 5}{\cancel{6} \times \cancel{5} \times \cancel{1}} + \frac{\cancel{5} \times 4 \times \cancel{-1} \times 4}{7 \times 6 \times 1}$$

$$= \left(-\frac{32}{42} + \frac{16}{6} + \frac{20 - 80}{3 \times 42} \right)$$

$$= \left(\frac{\cancel{50}^{25}}{\cancel{63}} - \frac{\cancel{112}^{86}}{\cancel{42}^{21}} \right) = \frac{25 \times 7 - 56}{21}$$

$$= \frac{175 - 56}{21}$$

$$\boxed{y = 5.6666}$$

Ques 4:

x	1	2	3	4	7
y	2	4	8	16	128

$$f(5) = ? \quad x_k = 5$$

$$y = \left[\frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2 + \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4 \right. \\ \left. + \frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8 + \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16 \right. \\ \left. + \frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128 \right]$$

$$y = \left(\frac{3 \times 2 \times 1 \times -2}{-1 \times -2 \times -3 \times -6} \times 2 + \frac{4 \times 2 \times 1 \times -2}{1 \times -1 \times -2 \times -5} \times 4 \right. \\ \left. + \frac{4 \times 3 \times 1 \times -2}{2 \times 1 \times -1 \times -4} \times 8 + \frac{4 \times 3 \times 2 \times -2}{3 \times 2 \times 1 \times -3} \times 16 + \frac{4 \times 3 \times 2 \times 1 \times -2}{6 \times 5 \times 4 \times 3} \times 128 \right)$$

$$= \left(\frac{-24}{36} + \frac{64}{10} - \frac{192}{8} + \frac{8}{3} \times 16 + \frac{128}{3} \right)$$

$$= \frac{1006}{15}$$

$$y(5) = 67.0666$$

<u>Ques 5</u>	x	10	20	14	16
	$f(x)$	0.25	0.2	0.15	0.10

$$f(15) = ?$$

$$x = 15$$

$$y = \frac{(15-10)(15-20)(15-14)}{(16-10)(16-20)(16-14)} \times 0.10 + \frac{(15-10)(15-20)(15-16)}{(14-10)(14-20)(14-16)} \times 0.15$$

$$+ \frac{(15-10)(15-14)(15-16)}{(20-10)(20-14)(20-16)} \times 0.2 + \frac{(15-20)(15-14)(15-16)}{(10-20)(10-14)(10-16)} \times 0.25$$

$$= \left(\frac{5 \times -5 \times 1 \times 0.10}{6 \times -4 \times 2} + \frac{5 \times -5 \times -1 \times 0.15}{4 \times -6 \times -2} \right.$$

$$\left. + \frac{5 \times 1 \times -1 \times 0.2}{10 \times 6 \times 4} + \frac{-4 \times 1 \times -1 \times 0.25}{-10 \times -4 \times -6} \right)$$

$$= \left(\frac{-2.5}{-48} + \frac{3.75}{48} - \frac{1}{240} - \frac{1}{240} \right)$$

$$= (0.05208 + 0.0781 - 0.00833)$$

$$\boxed{y = 0.12185}$$

Homework -2 Regula Falsi Method.

Qus 1

$$y = x^3 + x - 1$$

x	y
0.5	-0.375
1	1

Using regula falsi method.

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= \frac{0.5 \times 1 - 1 \times (-0.375)}{1 - (-0.375)}$$

$$\bar{x} = \frac{0.5 + 0.375}{1.375} = \frac{0.875}{1.375}$$

$$\bar{x} = 0.63636$$

$$\bar{y} = -0.10594$$

Reapplying regula falsi method:

x_1	0.63636	1
y	-0.10594	1

$$\bar{x} = \frac{0.63636 \times 1 + 1 \times (-0.10594)}{1 + (-0.10594)}$$

$$\bar{y} = -0.02643$$

Spiral

Reapplying regula falsi method.

x	y
0.67119	-0.0264
1	1

$$\bar{x} = \frac{0.67119 + 0.0264}{1.0264}$$

$$\bar{x} = 0.6796$$

$$\bar{y} = -0.006409$$

Ans. Approximate root of eq. is 0.6796

Qn 2. $y = x^3 - 2x + 0.5$

x	y
0	0.5
1	-0.5

Using regula falsi method.

$$\bar{x} = \frac{x_1 y_2 - y_1 x_2}{y_2 - y_1} = \frac{0 \times -0.5 - 1 \times 0.5}{-0.5 - 0.5}$$

$$= \frac{-10.5}{-1} \quad \boxed{\bar{x} = 0.5}$$

$$\boxed{\bar{y} = -0.375}$$

Spiral

x	y
0	0.5
0.5	-0.375

Reapplying regula falsi method.

$$\bar{x} = \frac{0.5 \times -0.5 - 0 \times -0.375}{-0.5 + 0.375}$$

$$\bar{x} = \frac{+0.25}{+0.125} \quad \boxed{\bar{x} = 2}$$

$$\boxed{\bar{y} = 21.5}$$

Q3 $y = x^3 + 2x - 20 = 0$.

x	y
2	-8
2.5	0.625

Applying regula falsi method.

$$\bar{x} = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$$

$$= \frac{2 \times 0.625 + 8 \times 2.5}{0.625 + 8} = \frac{1.250 + 20}{8.625}$$

$$\boxed{\bar{x} = 2.4637}$$

$$\boxed{\bar{y} = -0.1183}$$

x	y
0.0	0.8
2.4637	-0.1183
2.5	0.625

Applying regula falsi method again -

$$\bar{x} = \frac{2.4637 \times 0.625 + 2.5 \times 0.1183}{0.625 + 0.1183}$$

$$\bar{x} = \frac{1.8355}{0.7433}$$

$$\boxed{\bar{x} = 2.4693}$$

$$\boxed{y = -0.004985}$$

Approximate of y is 2.4693

Ques 4: $y = 3x - \cos x - 1$

x	y
0	-2
1	1.0001

$$\bar{x} = \frac{0 \times 1.0001 + 2 \times 1}{1.0001 + 2}$$

$$1.0001 + 2$$

$$\boxed{y = 1.0000229}$$

$$\bar{x} = \frac{2}{2.0001} \quad \boxed{\bar{x} = 0.99995}$$

$\bar{x}$	$\bar{y}$
0	-2
0.99995	1.00002

$$\bar{x} = \frac{2 \times 0.99995}{2.00002}$$

$$\bar{x} = 0.99994$$

$$\bar{y} = 0.99997$$