Assignment - 2

Prob and stats

Ans (1) let $x_i \rightarrow$ claim size on home insurance

$y_i \rightarrow$ —— " —— car —— " ——

$$\therefore \quad x \sim N(800, 100^2)$$

$$y \sim N(1200, 300^2)$$

$$P(y_1 + y_2 + y_3 > x_1 + x_2 + x_3 + x_4 + 800)$$

$$\Rightarrow P(y_1 + y_2 + y_3 - (x_1 + x_2 + x_3 + x_4) > 800)$$

$$y_1 + y_2 + y_3 - (x_1 + x_2 + x_3 + x_4) \sim N(3 \times 1200 - 4 \times 800,\ 300^2(3) + 100^2(4))$$

$$\sim N(400, 310000)$$

$$P(z > 0.71842) \Rightarrow 1 - P(z < 0.71842)$$

$$= 1 - P(z < 0.72)$$

$$= 1 - 0.76424$$

$$= \underline{0.23576}$$

Ans (2) $N \sim$ Negative Binomial (k, p)

$$P(N = n) = {}^{n+2-1}C_{n-1}\ (0.9)^3 (0.1)^n$$

$k = 3,\ p = 0.9$

$\Rightarrow X \sim$ Gamma $(6, 2)$ $\qquad M_Y t = M_N(\log M_X t)$

$y \rightarrow$ Total claim Amount

$\therefore$ For y, MGF =

$$M_y(t) = E(e^{ty}/N=n)$$

$$E(e^{ty}/N=n) = E(e^{x_1t+x_2t+\dots x_n})$$

$$= \left(1-\frac{t}{2}\right)^{-6n}$$

Putting this value,

$$= E(E(e^{yt}/N=n)) = E\left(\left(1-\frac{t}{2}\right)^{-6n}\right)$$

$$= E\left(e^{N\log\left(1-\frac{t}{2}\right)^{-6}}\right)$$

$$= M_N\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$$

$$M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^k$$

$$= \left(\frac{pe^{\log(1-t/2)^{-6}}}{1-qe^{\log(1-t/2)^{-6}}}\right)^k$$

$$= \left[\frac{(0.9)(1-t/2)^{-6}}{1-0.1\left(1-\frac{t}{2}\right)^{-6}}\right]^3$$

$$V(y) = E(N)\cdot V(x) + (E(x))^2\, V(N)$$

Page No.

Date / /

$= \frac{\cancel{5 \times 6} \cancel{\times 0.3}}{\cancel{0.9 \times 4}} + \left(\frac{6}{2}\right)^2 \frac{\times 3 (0.1)}{0.9^2}$

0.3

$= 5 + \frac{\cancel{9 \times 0.3}}{\cancel{0.9 \times 0.9}}$ 10 / 3

$= 5 + \frac{10}{3}$

$= \frac{25}{3}$

$\therefore \; s.d = \sqrt{25/3} = \frac{5}{1.732} = 2.887$

Q3. i) $f(x) = \lambda e^{-\lambda(x-\alpha)}$

$= MGF = E(e^{tx}) = \int^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)} dx$

$= M_x t = \lambda \int_{\alpha}^{\infty} e^{tx} e^{-\lambda x} e^{\lambda \alpha} dx$

$= e^{\lambda\alpha} \lambda \int_{\alpha}^{\infty} e^{-(\lambda - t) x} dx$

$= \frac{e^{\lambda\alpha}}{t - \lambda} \lambda \left(- e^{-(\lambda - t)\alpha} \right)$

$= e^{t\alpha} \left(\lambda / \lambda - t \right)$

Shrikrupa

$$M_x(t) = \frac{\lambda e^{t\alpha}}{\lambda - t} = \lambda e^{t\alpha}(\lambda - t)^{-1}$$

$$M_x'(t) = \lambda\left[e^{t\alpha}(\lambda-t)^{-2}(t-1) + (\lambda-t)^{-1} e^{t\alpha}(\alpha)\right]$$

$$= \lambda\left[(\lambda-t)^{-1} e^{t\alpha}(\alpha) + e^{t\alpha}(\lambda-t)^{-2}\right]$$

$$= \lambda(\lambda)^{-2}\left[\alpha\lambda+1\right]$$

$$= \frac{\lambda}{\lambda^2}(\alpha\lambda+1) = \frac{\lambda\alpha+1}{\lambda}$$

(iii) $$M_x''(t) = \lambda\left[\alpha e^{t\alpha}(\lambda-t)^{-2} + \alpha^2(\lambda-t)^{-1}e^{t\alpha} + e^{t\alpha}(2)(\lambda-t)^{-3} + (\lambda-t)^{-2}e^{t\alpha}\right]$$

Putting $t=0$

$$= \lambda\left[\alpha\lambda^{-2} + \alpha^2\lambda^{-1} + 2\lambda^{-3} + \alpha\lambda^{-2}\right]$$

$$= \lambda\left[\frac{2\alpha}{\lambda^2} + \frac{\alpha^2}{\lambda} + \frac{2}{\lambda^3}\right]$$

$$= E(x^2) = \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= \frac{2\alpha}{\lambda} + \alpha^2 + \frac{2}{\lambda^2} - \left(\alpha^2 + \frac{1}{\lambda^2} + \frac{2\alpha}{\lambda}\right)$$

$$= \frac{1}{\lambda^2}$$

(4) $G_V(t) = \left[\dfrac{p}{1-qt}\right]^k$

$\rightarrow$ $= \sum t^v \, P(V=v)$

$P(V=4) = \dfrac{\sqrt{k+4}}{\sqrt{5} \cdot \sqrt{k}} \, p^k q^k$

$= \dfrac{(k+3)!}{(k-1)!\,4!} \, p^k (1-p)^k$

$0<x<5, \; 10<y<20$

(5) $f(x,y) = ax(x+y)$

$\rightarrow$ (i) $P\left(y > \dfrac{x}{3} + 10\right)$

$= a\int_{10}^{20}\int_{0}^{5} (x^2 + xy)\,dx\,dy = 1$

$= \int_{10}^{20} \left.\dfrac{x^3}{3} + \dfrac{x^2 y}{2}\right|_0^5 dy = \dfrac{1}{a}$

$= \int_{10}^{20} \left(\dfrac{125}{3} + \dfrac{25y}{2}\right) dy = \dfrac{1}{a}$

$$= a\left(\frac{2500}{3} + 2500 - \frac{1250}{3} - 625\right) = 1$$

$$= a\left(\frac{1250}{3} + 1875\right) = 1$$

$$a\left(\frac{6875}{3}\right) = 1$$

$$a = 3/6875$$

$$= \therefore a = 0.0004363$$

(iii) $E(Y/x)$

$$f_{Y|X} = \frac{f(x,y)}{f(x)} =$$

$$f(x) = \frac{3}{6875}\int_{10}^{20} x^2 + xy \, dy$$

$$= \frac{3}{6875}\left[10x^2 + 150x\right]$$

$$= \frac{30x}{6875}\left[x + 15\right]$$

$$\therefore f_{Y/X} = \frac{3x(x+y)}{6875(30)x} \cdot (6875)(x+15)$$

$$= \frac{(x+y)}{10(x+15)}$$

(6) $X \sim Poi(\lambda)$ $\quad Y \sim Poi(\mu)$

$$M_X(t) = e^{\lambda(e^t - 1)}$$

$$M_Y(t) = e^{\mu(e^t-1)}$$

$$= Z = X - Y$$

$$= M_Z(t) = E(e^{tZ}) = E(e^{t(X-Y)})$$

$$= e^{(\lambda-\mu)(e^t-1)}$$

So, by uniqueness MGF of $Z =$

$\sim Poi(\lambda - \mu)$

let $Z = X + Y$

$$\therefore E(e^{tZ}) = e^{(\lambda+\mu)(e^t-1)}$$

$||^{rly}$ Now, $Z \sim Poi(\lambda+\mu)$

(7)

			x		
		1	2	3	4
y	1	0.2	0	0.05	0.15
	2	0	0.3	0.1	0.2

→ (i) $V(X/y=2) = E(X^2/y=2) - [E(X/y=2)]^2$

$$E(X^2/Y=2) = \sum_x x^2 \frac{P(X=x, Y=2)}{P(Y=2)}$$

When $X=1$, when $X=2$, when $X=3$ & when $X=4$

$$= (1)^2 \frac{P(X=1, Y=2)}{P(Y=2)} + (2)^2 \frac{P(X=2, Y=2)}{P(Y=2)} + (3)^2 \frac{P(X=3, Y=2)}{P(Y=2)}$$

$$= + (4)^2 \frac{(P(X=4, Y=2))}{P(Y=2)}$$

$= 8.83333$

$$E(X/Y=2) = \sum_x x \cdot \frac{P(X=x, y=2)}{P(Y=2)}$$

$$= \frac{0.6 + 0.3 + 0.8}{0.6} = 2.8333$$

$= 0.807$

Page No.______
Date __/__/__

(ii) $f(u,v) = \frac{48}{67}(2uv - u^2)$

$E(u \mid v = v) = ?$

$\frac{f(u,v)}{f(v)} = f_{u \mid v = v}$

$f(v) = \frac{48}{67}\int_0^1 2uv - u^2 \, du$

$f(v) = \frac{48}{67}\left[v - \frac{1}{3}\right] = \frac{16}{67}[3v - 1]$

$\frac{f(u,v)}{f(v)} = \cancel{\frac{48}{67}}\left(\frac{2uv^2 - u^2}{\cancel{16}(3v-1)}\right) \times \cancel{67}$

$= \frac{3}{3v-1}(2uv^2 - u^2)$

$E(u \mid v = v) = \frac{3}{3v-1}\int_0^1 u(2uv^2 - u^2)\, du$

$= \frac{3}{3v-1}\left[\frac{2v^2}{3} - \frac{1}{4}\right]$

$= \frac{\cancel{3}}{3v-1}\left[\frac{8v^2 - 3}{\cancel{12}\,4}\right]$

Shrikrupa

Page No.

Date / /

$= \dfrac{8v^2-3}{4(3v-1)}$

(8) $X \sim \text{Gamma}(3, 2)$

$= E(X) = \dfrac{3}{2}, \quad V(X) = \dfrac{3}{4}.$

$= E(Y / X=x) = 3x+1,$

$V(Y/X=x) = 2x^2+5$

$= E(Y) = E(E(Y/X=x))$

$= 3E(X)+1$

$= 3\dfrac{3}{2}+1 = \dfrac{9}{2}+1 = \dfrac{11}{2}$

$V(Y) = E(V(X/Y)) + V(E(X/Y))$

$= E(2X^2+5) + V(3X+1)$

$= 2E(X^2)+1 + 9V(X)$

$\Rightarrow V(X) = E(X^2) - (E(X))^2$

$= E(X^2) = \dfrac{3}{4} + \dfrac{9}{4}$

$= E(X^2) = 3$

$\therefore V(Y) = 2(3) + 5 + 9 \times 3$

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$= \frac{71}{4}$

(9) $E(x) = 3$, $V(x) = 4$
$E(y) = 4$, $V(y) = 1$

$=$ correlation coeff $= -0.3 \times 2 = cov(x, y)$
$= -0.6$

$= V(x) + cov(x, y)$
$= 4 + -0.6$
$= 3.4$

$= var(z) = cov(x+y, x+y)$
$= cov(x, x) + cov(y, y) + 2cov(x, y)$
$= 4 + 2 \times -0.6 + 1$
$= 5 - 1.2$
$= 3.8$

(10) $f(x, y) = kx^{-\alpha} e^{-y/\beta}$

$= k \int_1^{\infty} \int_1^{\infty} x^{-\alpha} e^{-y/\beta} \, dx \, dy = 1$

$\alpha \geq 1, \beta > 0$
$1 < x < \infty,$
$1 < y < \infty$

$$= K \int e^{-y/\beta} \left[\frac{x^{-(\alpha-1)}}{-\alpha+1} \right]_1^{\infty} = 1$$

$$= \cancel{\frac{K}{\beta - \alpha} \int e^{-y/\beta} [-1]}$$

$$= \frac{K}{\alpha-1} \int e^{-y/\beta} dy = 1$$

$$= \frac{K}{\alpha-1} \left[\frac{e^{-y/\beta}}{-1/\beta} \right]_1^{\infty} = 1$$

$$= \frac{-K}{\alpha-1} \beta (e^{-1/\beta}) = 1$$

$$= \left[K = \frac{\alpha-1}{\beta(e^{-1/\beta})} \right]$$