Probability & Statistics
Assignment 2

Q1 X_i = home insurance
Y_i = Car insura..

So A to Q
$X \sim N(800, 100^2)$
$Y \sim N(400, 300^2)$

$P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + 800)$

$P(Y_1 + Y_2 + Y_3 - (X_1 + X_2 + X_3 + X_4)) > 800$

$(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) \sim N(3(1200) - 400,\ 300^2(3) + 100^2(4))$
$\sim N(400, 310000)$

$P\left(Z > \dfrac{800 - 400}{\sqrt{310000}}\right)$

$P(Z > 0.7184) \Rightarrow 1 - P(Z < 0.7184)$

$1 - P(Z < 0.72)$
$1 - 0.76424$

$= 0.23576$

Q2 $N \sim$ Neg. Binomial (K, P)

$$P(N=n) = {}^{n+2}C_{n} (0.9)^3 (0.1)$$

$$= {}^{(n+3)-1}C_{n-1} (0.9)^3 (0.1)^3$$

So, $K = 3,\ P = 0.9$

$MGF = M_P(\log M_X t)$

$X \sim Gamma(6, 2)$

$Y \rightarrow$ Total calcium amount

MGF of Y

$$M_Y(t) = E(E(e^{Yt}/N=n))$$

$$E(e^{Yt}/N=n) = E(e^{X_1 t + X_2 t + \dots + X_n t})$$

$$= E(e^{tX_1}) \times E(e^{tX_2}) \dots E(e^{tX_n})$$

$$= \left(1 - \frac{t}{2}\right)^{-6n}$$

So $E(E(e^{Yt}/N=n)) = E\left(\left(1 - \frac{t}{2}\right)^{-6n}\right)$

$$= E\left(e^{n\log\left(1-\frac{t}{2}\right)^{-6}}\right)$$

$$= M_n\left(\log\left(1-\frac{t}{2}\right)^{-6}\right)$$

$$M_n(t) = \left(\frac{pe^t}{1-qe^t}\right)^{k}$$

$$= \left(\frac{pe^{\log\left(1-\frac{t}{2}\right)^{-6}}}{1-qe^{\log\left(1-\frac{t}{2}\right)^{-6}}}\right)^{k}$$

$$= \frac{p\left(1-\frac{t}{2}\right)^{-6}}{q\left(1-\frac{t}{2}\right)^{-6}}$$

$$= \log \frac{0.9\left(1-\frac{t}{2}\right)^{-6}}{1-0.1\left(1-\frac{t}{2}\right)^{-6}}$$

$$V(Y) = E(N)\left(V(X) + [E(X)]^2 + V(N)\right)$$

$$= \left(\frac{3}{0.9} \times \frac{6}{4}\right) + \left(\frac{6}{2}\right)^2 \times \frac{3(0.1)}{(0.9)^2}$$

std = 2.887

Q3

Q4 $G_V(t) = \left(\frac{p}{1-qt}\right)^k$

$G_V t = \sum t^v \times P(V=v)$

$P(V=4) = \frac{\sqrt{k+4}}{\sqrt{5}\ \sqrt{k}} p^k q^k$

$= \frac{\sqrt{k+4}}{4!\sqrt{k}} p^k q^k$

$\frac{(k+3)!}{(k-1)\ 4!} p^k (1-p)^k$

Q5 $f(x,y) = ax(x+y)$

$P\left(Y \geq \frac{1}{2}X + 10\right)$

$a\int_{10}^{20}\int_{0}^{5}(x^2+xy)\,dx\,dy = 1$

$\int_{10}^{20}\left[\frac{x^3}{3} + \frac{x^2y}{2}\right]_0^5 dy = \frac{1}{a}$

$a\left(\frac{2500}{3} + 20500 = \frac{125}{3} - 625\right)$

$a\left(\frac{6875}{3}\right) = 1$

$a = \frac{3}{6875}$

$\int_{10}^{20}\left(\frac{125}{3} + \frac{25}{2}y\right)dy = \frac{1}{a}$

$= \left(\frac{125y}{3} \cancel{} + \frac{25y^2}{4}\right)^{20} = \frac{1}{\cancel{a3/6875}}$

$833.33 + 2500 - (416.667 + 625) = \frac{1}{a}$

$a = 0.0004363$

Q6 $X \sim Poi(\lambda)$ $\quad Y \sim Poi(\mu)$

$M_X(t) = e^{\lambda(e^t - 1)}$ $\quad M_Y(t) = e^{\mu(e^t - 1)}$

So, $Z = X - Y$

$M_Z(t) = E(e^{tZ}) = E(e^{t(X-Y)}) = E(e^{tX} \cdot e^{-tY})$

$= E(e^{tX}) \times E(e^{-tY})$

$= \frac{E(e^{tX})}{E(e^{tY})}$

$= \frac{e^{\lambda(e^t - 1)}}{e^{\mu(e^t - 1)}}$

$= e^{(\lambda - \mu)(e^t - 1)}$

So, by uniqueness MGF of Z

$= e^{(\lambda - \mu)(e^t - 1)} \sim Pois(\lambda - \mu)$

Now: let $Z = X + Y$

$E(e^{tZ}) = e^{\lambda(e^t - 1)} \times e^{\mu(e^t - 1)}$

$= e^{(\lambda+\mu)(e^t - 1)}$

So, by uniqueness $Z \sim Poi(\lambda + \mu)$

Q7 i) $V(X/Y=2)$
$= E(X^2/Y=2) - [E(X/Y=2)]^2$

$E(X^2/Y=2) = \frac{\sum x^2 P(x=x, Y=2)}{P(y=)}$

When $x = 1$

$\frac{(1)^2 P(x=1, y=2)}{P(y=2)}$

When $x = 2$

$\frac{(2)^2 P(x=2, y=2)}{P(y=2}$

when $x=3$

$$\frac{(3)^2 + P(x=3, y=2)}{P(y=2)}$$

when $x=4$

$$\frac{(4)^2 + P(x=4, y=2)}{P(y=4)}$$

$$= \frac{(1)(6)}{0.6} + \frac{(4)(0.3)}{0.62} + \frac{(9)(0.1)}{(0.6)} + \frac{16(0.2)}{0.6}$$

$$= 8.8333$$

$$E(X/Y=2) = \sum_{x} = \frac{x\,P(\cancel{x}=x, y=2)}{P(y=2)}$$

$$\frac{(1)\cdot(0)}{0.6} + \frac{(2)(0.3)}{0.6} + \frac{(3)(0.1)}{0.6} + \frac{4(0.2)}{0.6}$$

$$= \frac{0.6 + 0.3 + 0.8}{0.6}$$

$$= 0.807$$

ii) $E(U/V=v)$

$\frac{f(v,u)}{f(u)}$

$f(v) = \frac{48}{67}\int_0^1 (2uv - v^2)\,du = \frac{48}{67}\left(u^2 v - \frac{u^3}{3}\right)_0^1$

$\frac{48}{67}\left(v - \frac{1}{3}\right) \Rightarrow \frac{16}{67}(3v-1)$

$f(v) = \frac{16}{67}(3v-1)$

$\frac{f(u,v)}{f(v)} = \frac{\cancel{48}^{3}}{\cancel{67}}\left(\frac{2uv^2 - v^2 \times \cancel{67}}{\times \cancel{16}(3v-1)}\right)$

$= \frac{3}{3v-1}(2uv^2 - u^2)$

$= 2u^2v^2 - u^3$

$E(U/V=v) = \frac{3}{3v-1}\int_0^1 u \cdot (2uv^2 - u^2)\,du$

$$\frac{3}{3v-1}\left(\frac{2v^3v^2}{3}-\frac{v^4}{4}\right)^1$$

$$\frac{3}{3v-1}\left(\frac{2v^2}{3}-\frac{1}{4}\right)$$

$$=\frac{8v^2-3}{4(3v-1)}$$

Q8 $E(Y/X=x)=3x+1$ $\quad$ $Var(Y/X)=2x^2+5$

$E(E/Y/X=x)=E(3x+1)$

$=3E(x)+1$

$=3\left(\frac{3}{2}\right)+1$

$=11/2$

$V(y)=E(V/(x/y))+V(E/(x/y))$

$=E(2x^2+5)+V(3x+1)$

$=2E(x^2)+5+9V(x)$

$V(X)=E(x^2)-[E(x)]^2$

$=\frac{3}{4}+\frac{9}{4}=E(x^2)$

$$\frac{12}{4} = E(x^2)$$

$$E(x^2) = 3$$

$$V(y) = 2(3) + 5 + 9\left(\frac{3}{4}\right)$$

$$= 11 + \frac{27}{4}$$

$$= \frac{44+27}{4}$$

$$= \frac{71}{4}$$

Q9 i) $\text{cov}(x, x+y) = \text{cov}(x,x) + \text{cov}(x,y)$

$$= V(x) + \text{cov}(x,y)$$

$$= 4 + (-0.6)$$

$$= 3.4$$

ii) $\text{Var}(z) = \text{cov}(x+y, x+y)$

$$= \text{cov}(x,x) + \text{cov}(x,y) + \text{cov}(y, \ldots$$

$$= V(x) + 2\,\text{cov}(x,y) + V(y)$$

$$= 4 + 2(-0.6) + 1$$

$$= 3.8$$

Camlin Kokuyo