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FINANCIAL MATHEMATICS - ASSIGNMENT 2

1) (i) $20000 = 427.90\, a_{\overline{60}|} @ i^{(12)}/12$

$$\frac{20000}{427.90} = \left[\frac{1-(1+i)^{-60}}{i}\right]$$

$$46.73989 = \frac{1-(1+i)^{-60}}{i}$$

$i = 10.79\%$

$$1+i = \left[1+\frac{i^{(12)}}{12}\right]^{12}$$

$$\frac{i^{(12)}}{12} = 0.008583$$

$= 0.8583\%$

$i^{(12)} = 10.2996\%$

(ii)

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(2)(i)(a) Discounted Payback Period:
In many practical problems, the net cash flow changes sign only once, this change being from negative to positive. In these circumstances the balance in the investor's account will change from negative to positive at a unique time t, or it will always be negative, in which case the project is not viable.
If this time t, exist, it is referred to as the discounted payback period.

(b) Payback Period:
In many practical problems, the net cash flow changes sign only once, this change being from negative to positive. In these circumstances the balance in the investor's account will change from negative to positive at a unique time t, or it will always be negative, in which case the project is not viable.
It is the smallest value of t such that $A(t) \geq 0$, where
$A(t) = \sum_{s \leq t} C_s (1+j_1)^{(t-s)} + \int_0^t \rho(s)(1+j_1)^{(t-s)} ds$
where, j_1 = rate of interest applicable to investors borrowings
The payback period of a project is found by counting the number of years it takes before the cumulative forecasted cash flow equals the initial investment.

(ii)(a) Project A:
$170000 = 20000(1+i)^{-1} + 20000(1+i)^{-2} + 20000(1+i)^{-3}$
$- 20000(1+i)^{-1} - 10000(1+i)^{-2}$
$170000 = 10000(1+i)^{-2} + 20000(1+i)^{-3}$
$i = 0.07427$
$\boxed{i = 7.424\%}$

Project B:

$200000 = 14000 a_{\overline{6}|} @ 9\%$

$200000 = 14000 \left[\frac{1 - (1+i)^{-6}}{i} \right]$

$\frac{100}{7} = \left[\frac{1 - (1+i)^{-6}}{i} \right]$

$7 - 7(1+i)^{-6} - 100i = 0$

$200000 = 14000 \, a_{\overline{6}|} @ 9\% + 200000 (1+i)^{-6}$

$200 = 14 \left[\frac{1 - (1+i)^{-6}}{i} \right] + 200 (1+i)^{-6}$

$200 i = 14 - 14(1+i)^{-6} + 200i [1+i]^{-6}$

$\boxed{i = 7\%}$

(b) Project A = $-170000 - 20000 (1.06)^{-1} - 10000 (1.06)^{-2}$
$+ 20000 (1.06)^{-1} + 20000 (1.06)^{-2} + 20000 (1.06)^{-3}$
$= -170000 + 10000 (1.06)^{-2} + 20000 (1.06)^{-3}$
$= 6823.82.$

Project B = $-200000 + 14000 \, a_{\overline{6}|} @ 6\% + 200000 (1.06)^{-6}$
$= 9834.65$

(3) Annuity 1 = $200 (v)^{1/4} \left(\ddot{a}_{\overline{23}|} @ 8\% \right) = 2161.363$
Annuity 2 = $320 (1+i)^{1/12} \left[a^{(4)}_{\overline{6.25}|} @ 8\% \right] = 2997.872$
Annuity 3 = $180 \left(a^{(12)}_{\overline{18.75}|} @ 8\% \right) = 1780.665$
Total = 6899.90
$6899.90 = A (1+p)^{1/4} \left(a^{(2)}_{\overline{21.5}|} \right)$
$A = 656.56$

(5) i) TWRR $= \frac{16.4}{12.4} - 1 = 0.3226 = 32.26\%$

TWRR of equity fund $= \frac{15.5}{12.1} - 1 = 28.10\%$

(ii) (a) $1240(1+i) + 1310(1+i)^{3/4} + 1480(1+i)^{1/2} + 1580(1+i)^{1/4} = 4(164\ldots)$

$i = 29.32\%$

(b) $400(\ddot{s}^{(4)}_{\overline{1}|}) = 100(16.4)\left[\frac{1}{12.4} + \frac{1}{13.1} + \frac{1}{14.8} + \frac{1}{15.8}\right]$

$\boxed{i = 27.8\%}$

(iii) (a) $121(1+i) + 920(1+i)^{3/4} + 1030(1+i)^{1/2} + 1310(1+i)^{1/4} = 4(1550)$

$i = 67.5\%$

(b) $400[\ddot{s}^{(4)}_{\overline{1}|}] = 100(15.5)\left[\frac{1}{12.1} + \frac{1}{9.2} + \frac{1}{10.3} + \frac{1}{13.1}\right]$

$400(1.4135) = 100(15.5)\left[\frac{1}{12.1} + \frac{1}{9.2} + \frac{1}{10.3} + \frac{1}{13.1}\right]$

$i = 7.688\%$

(4) The notation $(I\ddot{a})_{\overline{n}|}$ is used to denote the present value of an increasing annuity due payable for n time units the nth payment (of amount n) being made at time n-1.

Present value

$(I\ddot{a})_{\overline{n}|} = 1 + 2v + 3v^2 + \ldots + nv^{n-1} \to 1$

Multiplying both sides by v.

$v(I\ddot{a})_{\overline{n}|} = v + 2v^2 + 3v^3 + \ldots (n-1)v^{n-1} + nv^n \to 2$

Subtracting (1) from (2)

$(I\ddot{a})_{\overline{n}|}(1-v) = 1 + v + v^2 + v^3 + \ldots + v^{n-1} - nv^n$

$(I\ddot{a})_{\overline{n}|}(1-v) = \ddot{a}_{\overline{n}|} - nv^n$

$$(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$$

(6) (i) $160000 = Xa_{\overline{10}|}$ @ 8%

$X = 23844.65$

(ii) Loan outstanding after 4th payment

$23844.65\,[a_{\overline{n}|} @ 8\%] = 110231.44$

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$\therefore Y a_{\overline{7}|} = 110231.44$

$\therefore Y = 25309.72$

(iii) Loan outstanding after 7th payment

$25309.72 \, [a_{\overline{3}|} @ 10\%] = 62942.74$

$Z a_{\overline{3}|} @ 9\% = 62942.74$

$\therefore Z = 24869.77$

$\therefore$ Yield $= 16000 + 1465(a_{\overline{4}|}) - 443.95(a_{\overline{7}|})$
$- 24865.72(a_{\overline{10}|}) = 0$

$\therefore i = 8.4\%$

(8) (i) $PV = 2.7 + 0.2 a_{\overline{3}|} @ 9\% \rightarrow (1)$

$0.1v^3 + \bar{a}_{\overline{10}|}(0.025v + 0.2v^2 + 0.1v^3) @ 9\% \rightarrow (2)$

$(1) = (2) \quad \therefore i = 9.28\%$

(ii) $0.1v^3 + 0.85 \bar{a}_{\overline{10}|} v^3$

$= 0.1v^3 + 0.85 \bar{a}_{\overline{10}|} v^3 - 2.7 - 0.2 a_{\overline{3}|} @ 10\%$

$= 1.0089$

as NPV is positive, its good to invest

(11) (i) TWRR $\Rightarrow (1+i)^2 = \frac{500}{460}\left(\frac{5x+50}{500+40}\right)\left(\frac{x}{550}\right) = 1.44$

$\therefore x = 655.78$

(ii) MWRR $= 460(1+i)^2 + 40(1+i)^{7/4} - 50(1+i)^3 = 655.78$

MWRR $= 19.85\%$

(iii) $460(1+i) + 40(1+i)^{3/4} = 500$

$i = 20.445\%$

$(1+i) = \frac{655.78}{56} = 1.1923$

LIRR $= (1+i)^n = 1.9923 / (1.2044) = 19.83\%$

(12) (ii) (a) DPP $= -800(1+i)^t - 50(1+i)^{t-1} + 100(\bar{s}_{\overline{t-2}|} @ 7\%) = 0$

$t = 17.8$ yr

(b) $AV > 100(\bar{s}_{\overline{4}|} @ 6\%) = 480$

$AV = 480000$

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(iii) $i = 6\%$

$100\, s_{\overline{7}|} = 102.971$

$OPP \Rightarrow -800(1+i)^t - 50(1+i)^{t-1} + 102.97\, s_{\overline{t-7}|}$ @ 7% $= 0$

$t = 18$ yrs

$-800(1+i)^{10} - 850(1+i)^{17} + 102.97 + s_{\overline{17}|}$ @ 7% $= 9.7714$

$AV = 9.7714(1+i)^9 + 100\, s_{\overline{9}|}$ @ 6% $= 462.79$

$AV = 462.790$.

13 (i) $X a^{(12)}_{\overline{25}|}$ @ 7% $= 9880$

$X = 821.76$.

(ii) Loan outstanding after 10/03/29

$X a^{(12)}_{\overline{39/2}|}$ @ 7% $= 648600$

(iii) Loan outstanding after 10/9/26 $\Rightarrow X a^{(12)}_{\overline{83/6}|}$ @ 7%

Loan outstanding after 10/10/26 $\Rightarrow X a^{(12)}_{\overline{13.75}|}$ @ 7%

Capital repaid $= X \left[\dfrac{v^{13.75} - v^{83/6}}{i^{2}} \right]$ @ 7% $= 26.86$

(iv) Capital repayment continue in 12 installment is

$X \left[a^{(12)}_{\overline{22/3}|} - a^{(12)}_{\overline{19/3}|} \right]$ @ 7% $= 516.2$

Total interest $= X - 516.20$

$= 305.56$