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NMA assignment

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1. Observed radius = 12.65 cm
actual radius = 12.5 cm
Observed area = πr^2
$A_0 = \pi \times (12.65)^2$
$A_0 = 502.725\ cm^2$
Actual area = πr^2
$A_1 = 490.874\ cm^2$

(i.) Absolute error = observed value − actual value
= 502.725 − 490.874
= 11.851 cm^2

(ii.) Relative error = $\frac{\text{Absolute error}}{\text{Actual error}}$
$= \frac{11.851}{490.874} = 0.02414$

(iii.) Percentage error = Relative error × 100
= 2.414 %

2(i) $x_1 = 4$ $\quad y_1 = 0$
$x = 5$ $\quad y = ?$
$x_2 = 6$ $\quad y_2 = 0.778151$

$\therefore y = \frac{y(x - x_1) + y_1(x_2 - x_1)}{(x_2 - x_1)}$

$= \frac{0.778151(1) + 0.60206(1)}{2}$

@ $y = 0.6901$

(4.) $x_1 = 4.5$ $y_1 = 0.6532125$

$x = 5$ $y = ?$

$x_2 = 5.5$ $y_2 = 0.7403627$

$$\therefore y = \frac{y_2(x - x_1) + y_1(x_2 - x_1)}{x_2 - x_1}$$

$$= y = \underline{0.6901}$$

3. $x^4 - x - 10 = 0$

$a = 1.5$, $b = 2$

$\therefore f(a) = f(1.5) = (1.5)^4 - 1.5 - 10$

$= -6.4375$

$f(b) - f(2) = 24 - 2 - 10 = 4$

$\therefore$ 1st iteration :-

$$x_1 = \frac{2 + 1.5}{2} = 1.75$$

$f(x_1) = f(1.75) = 1.75^4 - 1.75 - 10 = \underline{-2.3711}$

2nd iteration :-

$$x_2 = \frac{1.75 + 2}{2} = 1.875$$

$f(x_2) = 1.875^4 - 1.875 - 10 = \underline{0.4846}$

3rd iteration :

$$x_3 = \frac{1.75 + 1.875}{2} = 1.8125$$

$f(x_3) = f(1.8125) = 1.8125^4 - 1.8125 - 10 = \underline{-1.0202}$

4th iteration

$$x_4 = \frac{1.875 + 1.8125}{2} = 1.84375$$

$f(x_4) = 1.84375^2 - 1.84375 - 10 = -0.2871$

5^{th} iteration:-

$$x_5 = \frac{1.875 + 1.84375}{2} = 1.859375$$

$$f(x_6) = 1.859375^4 - 1.859375 - 10$$
$$= 0.093$$

$\therefore$ root of $x^4 - x - 10 = 0$ is ~~1.85993~~ 1.859375 which is approximately 1.86.

4. $f(x) = x^3 - x - 1$

$$f'(x) = 3x^2 - 1$$

x	0	1	2
y	-1	-1	5

$$\therefore x_0 = \frac{1+2}{2} = 1.5$$

$f(x_0) = 0.875$ $\qquad$ $f'(x_0) = 5.75$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= \cancel{0}\ 1.5 - \frac{0.875}{5.75}$$

$$= 1.34783$$

$f(x_1) = 0.10068$ $\qquad$ $f'(x_1) = 4.44991$

$$x_2 = 1.34783 - \frac{0.10068}{4.44991}$$

$$= 1.3252$$

$\therefore f(x_2) = 0.00205$ $\qquad$ $f'(x_2) = 4.2684$

$$x_3 = 1.3252 - \frac{0.00205}{4.2684}$$

$$\therefore x_3 = 1.3247$$

$f(x_3) = 0.0000007545$ $f'(x_3) = 4.2645$

$\therefore x_4 = 1.3247 - \dfrac{0}{4.2645}$

$x_4 = 1.3247$

$\therefore$ Root of $x^3 - x - 1 = 0$ is 1.3247

c) $A^2 = A$

$7A - (I + A)^3 =$ To Find

$\therefore 7A - (I+A)^3 = 7A - (I^3 + A^3 + 3A^2 I + 3AI^2)$

$= 7A - I^3 - A^3 - 3A^2 I - 3AI^2$

$7A - I^3 - A^2 \cdot A - 3A^2 - 3A$

$= 7A - I - A \cdot A - 3A - 3A$

$= 7A - I - A - 6A$

$= 7A - 7A - I$

$\therefore 7A (I + A)^3 = -I$

6) Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$\therefore |A| = 1\begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} + 2\begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix}$

$= -1(-6) + 1(-4) + 2(4)$

$= -2 \neq 0$ $\therefore$ inverse exists

$\therefore$ Co-factor matrix $= \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & 1 \\ -6 & 2 & 4 \end{bmatrix}$

$\therefore$ Adjoint matrix $= \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$

$\therefore A^{-1} = \dfrac{1}{|A|} \times \text{Adjoint}$

$= \dfrac{1}{-2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & 1 & 4 \end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & -1/2 & -2 \end{bmatrix}$

9. The smallest 3 digit no that leaves remainder of 2 is 101 and the largest is 998.

$\therefore a = 101 \quad , \quad l = 998$

$d = 3$

$\therefore t_n = a + (n-1)d$

$998 = 101 + (n-1)3$

$\dfrac{998 - 101}{3} = n - 1$

$n = 300$

$S_n = \frac{n}{2}(a+l) = \frac{300}{2}(100+998)$

$= 164850$

$\therefore$ Sum of all numbers that leave a remainder of 2 when divided by 2 is 164850.

10) $t_6 = ar^5 = 32$ — (i)

$t_8 = ar^7 = 128$ — (ii)

Dividing (ii) & (i)

$\frac{ar^7}{ar^5} = \frac{128}{32}$

$r^2 = 4$

$r = 2$

11) expand $(4+3x)^5 = (3x+4)^5$

$\because (a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{(n)(n-1)(n-2)}{3!}a^{n-3}b^3$

$\therefore (3x+4)^5 = (3x)^5 + 5(3x)^4 4 + \frac{5\times4}{2}(3x)^3 4^2 + \frac{5\times4\times3}{3\times2}(3x)^2 4^3$

$+ \frac{5\times4\times3\times2}{4\times3\times2}(3x)4^4 + 4^5$

$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2 + 3840x + 1024$

12) $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\therefore \lambda^3 - (0)\lambda^2 + (-13)\lambda - (-30+18+0) = 0$

$\therefore \lambda^3 - 13\lambda + 12 = 0$

$\lambda = 1$ satisfies.

$$\therefore 1 \left| \begin{array}{cccc} 1 & 0 & -13 & 12 \\ \downarrow & 1 & 1 & -12 \\ \hline 1 & 1 & -12 & \boxed{0} \end{array} \right.$$

$\therefore \lambda^2 + \lambda - 12 = 0$

$\lambda^2 + 4\lambda - 3\lambda - 12 = 0$

$\lambda(\lambda+4) - 3(\lambda+4) = 0$

$\lambda = -4, \lambda = 3$

13) $f(x) = x^3 - 7x^2 + 8x - 3$

$x_0 = 5$

$\therefore f(x) = x^3 - 7x^2 + 8x - 3$

$f(5) = -13$

$f'(x) = 3x^2 - 14x + 8$

$f'(5) = 13$

$\therefore x_1 = 5 - \left(\frac{-13}{13}\right)$

$= 6$

$f(6) = 9$

$f'(6) = 32$

$$x_2 = 6 - \left(\frac{9}{32}\right)$$

$$x_2 = 5.71875$$

14) $(1-x+x^2)^4$

$\therefore [x^2 + (1-x)]^4$

let $1-x = y$

$\therefore [x^2+y]^4$

using binomial theorem

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}b^2}{2} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + b^n$$

$$\therefore [x^2+y]^4 = (x^2)^4 + 4(x^2)^3y + 4\times3(x^2)^2 + \frac{4\times\cancel{3}\times2(x^2)y^3}{\cancel{3}\times\cancel{2}} + y^4$$

$$= x^8 + 4x^6y + 6x^4y^2 + 4x^2y^3 + y^4$$

$$\therefore [x^2+(1-x)]^4 = x^8 + 4x^6(1-x) + 6x^4(1-x)^2 + 4x^2(1-x)^3 + (1-x)^4$$

15) $e^{-x} = 3\log(x)$

$\therefore f(x) = 3\log x - e^{-x}$

$a = 1 \quad b = 2$

$\therefore f(a) = -0.367879$

$f(b) = 1.944106$

$$\therefore \frac{a+b}{2} = \frac{1+2}{2} = 1.5 = x_1$$

$f(x_1) = 0.0993265$

$x_2 = \frac{1+1.5}{2} = 1.25$

$f(x_2) = 3\log(1.25) - e^{-1.25}$

$= 0.382925$

$x_3 = \frac{1+1.25}{2} \cdot 1.125$

$\therefore x_4 = \frac{1+1.125}{2} = 1.0625$

$f(x_4) = 3\log(1.0625) - e^{-1.0625}$

$= -0.16372.$