Homework - 1

1] Q1)

x	7	8	9	10
$f(x)$	3	1	1	9

Find $f(9.5)$?

$$f(9.5) = \frac{(9.5-8)(9.5-9)(9.5-10)(3)}{(7-8)(7-9)(7-10)} + \frac{(9.5-7)(9.5-9)(9.5-10)(1)}{(8-7)(8-9)(8-10)}$$
$$+ \frac{(9.5-7)(9.5-8)(9.5-10)(1)}{(9-7)(9-8)(9-10)} + \frac{(9.5-7)(9.5-8)(9.5-9)(9)}{(10-7)(10-8)(10-9)}$$
$$= \frac{3}{16} - \frac{5}{16} + \frac{5}{16} + \frac{45}{16}$$

$$= 3.625$$

Q2]

x	3	7	9	10
$f(n)$	108	120	72	63

$f(6) = ?$

$$f(6) = \frac{(6-7)(6-9)(6-10)(108)}{(3-7)(3-9)(3-10)} + \frac{(6-3)(6-9)(6-10)(120)}{(7-3)(7-9)(7-10)}$$
$$+ \frac{(6-3)(6-7)(6-10)(72)}{(9-3)(9-7)(9-10)} + \frac{(6-3)(6-7)(6-9)(63)}{(10-3)(10-7)(10-9)}$$

$$= 12 + 180 + 27 - 72$$
$$= 147$$

Q3]

x	1	2	7	8
$f(x)$	4	5	5	4

$f(6) = ?$

$$f(6) = \frac{(6-2)(6-7)(6-8)(4)}{(1-2)(1-7)(1-8)} + \frac{(6-1)(6-7)(6-8)(5)}{(2-1)(2-7)(2-8)}$$
$$+ \frac{(6-1)(6-2)(6-8)(5)}{(7-1)(7-2)(7-8)} + \frac{(6-1)(6-2)(6-7)(4)}{(8-1)(8-2)(8-7)}$$

$$= -\frac{16}{21} + \frac{5}{3} + \frac{20}{3} - \frac{40}{21}$$

$$= 5.6667$$

Q4]

x	1	2	3	4	7
$f(x)$	2	4	8	16	128

$f(5) = ?$

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)(2)}{(1-2)(1-3)(1-4)(1-7)} + \frac{(5-1)(5-3)(5-4)(5-7)(4)}{(2-1)(2-3)(2-4)(2-7)}$$
$$+ \frac{(5-1)(5-2)(5-4)(5-7)(8)}{(3-1)(3-2)(3-4)(3-7)} + \frac{(5-1)(5-2)(5-3)(5-7)(16)}{(4-1)(4-2)(4-3)(4-7)}$$
$$+ \frac{(5-1)(5-2)(5-3)(5.4)(16)}{(7-1)(7-2)(7-3)(7-4)}$$

$$= -\frac{2}{3} + \frac{32}{5} - 24 + \frac{128}{3} + \frac{128}{15}$$

$$= 32.9334$$

Q5]

x	10	20	14	16
$f(x)$	0.25	0.20	0.15	0.10

$f(15) = ?$

$$f(15) = \frac{(15-20)(15-14)(15-16)(0.25)}{(10-20)(10-14)(10-16)} + \frac{(15-10)(15-14)(15-16)(0.20)}{(20-10)(20-14)(20-16)} + \frac{(15-10)(15-20)(15-16)(0.15)}{(14-10)(14-20)(14-16)} + \frac{(15-10)(15-25)(15-14)(0.10)}{(16-10)(16-20)(16-14)}$$

$$= \frac{-1}{112} - \frac{1}{240} + \frac{5}{64} + \frac{5}{46}$$

$$= 0.1208$$

Q2]

Q1. $x^3 + x - 1 = 0$ b/w 0.5 and 1

$\therefore y = x^3 + x - 1$

x	y
0.5	-0.315
	-0.184
	0.043

$x_n : \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1}$

$x_1 = 0.6$ $\quad x_2 = 0.7$

$y_1 = 0.184$ $\quad y_2 = 0.043$

$$x_n = \frac{(0.043)(0.6)+(0.7)(0.184)}{0.043+0.184} = 0.6810$$

$$h = -0.003042$$

$x_1 = 0.6810$ $\quad x_2 = 0.7$

$y_1 = -0.003042$ $\quad y_2 = 0.043$

$$x_n = \frac{(0.6810)(0.043)+(0.7)(0.003042)}{0.043+0.003042}$$

$$= 0.6823$$

$$y_n = -0.000045$$

∴ Root $y'x^3 + x - 1 = 6$ is $= 0.6823$

Q2] $x^3 + 2x + 0.5 = 0$, b/w 0 and 1

$\therefore y = x^3 + 2x + 0.5$

x	y
0	0.5
0.1	0.301
0.2	0.108
0.3	-0.073

$x_1 = 0.2$ $\quad x_2 = 0.3$

$y_1 = 0.108$ $\quad y_2 = -0.073$

$$x_n = \frac{(-0.073)(0.2)-(0.3)(0.10)}{-0.073-0.108}$$

$$= 0.259669$$

$$y_n = -0.001829$$

$x_1 = 0.2$ $\quad x_2 = 0.259669$

$y_1 = 0.108$ $\quad y_2 = -0.001829$

$$x_n = \frac{(-0.001829) - (0.259669)(0.10)}{-0.001829 - 0.103}$$

$$= 0.255675$$

$y_n = -0.000041$

∴ Root of "$x^3 - 2x + 0.5$" is $= 0.258675$

Q3) $x^3 + 2x - 20 = 0$ b/w 2 and 2.5

$\therefore y = x^3 + 2x - 10$

x	y
2.0	-8
2.1	-6.534
2.2	-4.953
2.3	-3.233
2.4	-0.1625
2.5	0.625

$x_1 = 2.4$ $\quad x_2 = 2.5$

$y_1 = -0.1371$ $\quad y_2 = 0.625$

$$x_n = \frac{(0.625)(2.4) + (1.376)(2.5)}{0.025 + 1.376}$$

$$= 2.46878$$

$y_n = -0.01582$

$x_1 = 2.426878$ $x_1 = 2.5$

$y_2 = -0.01582$, $y_2 = 6.625$

$$x_x = \frac{(0.625)(2.46176) + (0.01582)(2.5)}{0.625 + 20.01882}$$

$$= 2.46954$$

$y_x = -0.000176$

$\therefore$ Root of " $x^3 + 2x - 20$ is 2.46954

Q4 $3x - 105x - 1 = 0$ b/w 0 and 1

$\therefore y = 3x - 105x - 1$

x	y
0	$-y_2$
0.1	-1.69998
0.2	-1.39994
0.3	-1.09998
0.4	-0.79994
0.5	-0.49999
0.6	-0.1999
0.7	0.10001

$x_1 = 0.6$ $x_2 = 0.1$

$y_1 = 0.1999$ $y_2 = 0.10001$

$$x_x = \frac{(0.10007)(0.6) + (0.7)(0.1999)}{0.10001 + 0.1999} = 0.6666$$

$y_x = 0.0000003$

$\therefore$ Root of $3x - 105x - 1$ is -0.6666

Q5. $x^3 - 3x - 5 = 0$ b/w 0 and 2.5

$y = x^3 - 3x - 5$

x	y
2.0	-3
2.1	-2039
2.2	-0.952
2.3	0.267
2.4	1.629

$x_1 = 2.2$ $\quad x_2 = 0.3$

$y_1 = -0.952$ $\quad y_2 = 0.267$

$$x_x = \frac{(0.267)(2.2) + (-0.952)(2.3)}{0.267 + 0.952} = 2.27809$$

$y_x = -0.01159$

$x_1 = 2.27809$ $\quad x_2 = 2.3$

$y_1 = -0.01159$ $\quad y_2 = 0.267$

$$x'_x = \frac{(0.267)(2.27809) + (2.3)(0.01159)}{0.267 + 0.01159} = 2.27901$$

$y'_x = 0.0001$

∴ Root of $x^3 - 3x = 5 = 0$ is 2.27901.

Q6. $x \log_{10} x - 1.2 = 0$ b/w 2 and 3

$\therefore y = x \log_{10} x - 1.2$

x	y
2.0	-0.59794
2.1	-0.52339
2.2	-0.44667
2.3	-0.36806
2.4	-0.28749
2.5	-0.20514
2.6	-0.12106
2.7	-0.03523
2.8	0.05205

$x_1 = 2.7$ $\quad x_2 = 2.8$

$y_1 = 0.03523$ $\quad y_2 = 0.05205$

$$x_3 = \frac{(0.05205)(2.7) + (2.8)(0.03523)}{(0.05205) + (0.03523)} = 2.7404$$

$y_3 = -0.00019$

$\therefore$ Root of $x \log_{10} x = 1.2$ is 2.7407

Q7. $x^3 - 4x + 1 = 0$

$\therefore y = x^3 - 4x + 1$

x	y
0	1
0.1	0.60
0.2	0.208
0.3	-0.173

$x_1 = 0.2$ $\quad x_2 = 0.3$

$y_1 = 0.208$ $\quad y_2 = -0.173$

$$x_x = \frac{(-0.173)(0.3) - (0.3)(0.208)}{-0.173 - 0.208} = 0.2546$$

$$y_x = -0.00187$$

$x_1 = 0.2$ $\quad x_2 = 0.2546$

$y_1 = 0.208$ $\quad y_2 = 0.00187$

$$x_{in} = \frac{(-0.00187)(0.2) - (0.208)(0.2546)}{-0.00187 - 0.208}$$

$$= 0.2541$$

$y''x = -0.000002$

∴ Root of '$x^3 - 4x + 1 = 0$' is 0.254

Q8. $x^3 + 2x^2 - 8 = 0$

∵ $y = x^3 + 2x^2 - 8$

x	y
0	-8
1	-5
2	8

$x_1 = 1$ $\quad x_2 = 2$

$y_1 = -5$ $\quad y_2 = 8$

$$x_n = \frac{(8)(1)+(5)(2)}{8+5} = 1.3846$$

$$y_n = -1.5112$$

$x_1 = 1.3846 \quad x_2 = 2$

$y_1 = -1.5112 \quad y_2 = 8$

$$x'_n = \frac{(8)(1.3846)+(2)(1.3112)}{8+1.5112} = 1.4824$$

$$y'_n = -0.3475$$

$x_1 = 1.4824 \quad x_2 = 2$

$y_1 = -0.3475 \quad y_2 = 8$

$$x''_n = \frac{(8)(1.4824)+(0.3475)(2)}{8+0.3475}$$

$$= 1.5039$$

$$y''_n = 0.0747$$

$x_1 = 1.5033 \quad x_2 = 2$

$y_1 = -0.0747 \quad y_2 = 8$

$$x'''_n = \frac{(8)(1.5033)+(0.0747)(2)}{8+0.0747} = 1.5085$$

$$y'''_n = -0.0158$$

$x_1 = 1.5085 \quad x_2 = 2$

$y_1 = 0.1158 \quad y_2 = 5$

$x''_n = \frac{(8)(1.5085)+(0.0958)(2)}{8+0.0158} = 1.5095$

$y_n = -0.0033$

$x_1 = 1.8095 \qquad x_2 = 2$

$y_1 = -0.0033 \qquad y_2 = 8$

$x_n = \frac{(8)(1.8095)+(0.0033)(2)}{8+0.0033} = 1.5097$

$y_n = -0.0007$

∴ Root of '$x^3 + 20x^2 - 8$' is 1.5097

Q9 $x^3 - 9x + 1 = 0$

∴ $y = x^3 - 9x + 1$

x	y
0	1
0.1	0.101
0.2	0.792

$x_1 = 0.1 \qquad x_2 = 0.2$

$y_1 = 0.101 \qquad y_2 = -0.792$

$x_n = \frac{(-0.192)(0.1)-(0.2)(0.101)}{-0.792-0.101} = 0.1113$

$y_n = 0.00041$

∴ Root of '$x^3 - 9x + 1.15$' ≈ 0.1113

10) $x^3 - x - 4 = 0$

$\therefore y = x^3 - x - 4$

x	y
0	-4
1	-4
2	2

$x_1 = 1$ $\quad$ $x_2 = 2$

$y_1 = -4$ $\quad$ $y_2 = 2$

$$x_n = \frac{(2)(1) + (4)(2)}{2+4} = 1.3334$$

$y_n = -2.9629$

$x_1 = 1.3334$ $\quad$ $x_2 = 2$

$y_1 = -2.9629$ $\quad$ $y_2 = 2$

$$x_n = \frac{(2)(1.334) + (2)(2.9624)}{2 + 2.9624} = 1.7313$$

$y_n = -0.5415$

$x_1 = 1.7313$ $\quad$ $x_2 = 2$

$y_1 = -0.5415$ $\quad$ $y_2 = 2$

$$x_n = \frac{(2)(1.7313) + (2)(0.5415)}{2 + 0.5415} = 1.2885$$

$y_n = 0.6680$

$x_1 = 1.7885$ $\quad x_2 = 2$

$y_1 = 0.6680$ $\quad y_2 = 2$

$$x_n = \frac{(2)(1.7885) + (2)(0.6680)}{2 + 0.6680} \Rightarrow 1.7954$$

$y_n = -0.0078$

$x_1 = 1.7954$ $\quad x_2 = 2$

$y_1 = 0.0078$ $\quad y_2 = 2$

$$x_n = \frac{(2)(1.7984) + (2)(0.0078)}{(2) + 0.0078} = 1.7912$$

$\therefore y_n = 0.00091$

$\therefore$ Root of '$x^3 - 3 - 4$' is 1.7962