F-M Assignment

A1) $i^4 = 8\% = 0.08$

i) $1+i = e^{\delta}$

$\Rightarrow \cancel{1+} \left(1+\frac{i^{(p)}}{p}\right)^p \cancel{-1} \doteq e^{\delta}$

$\Rightarrow \left(1+\frac{0.08}{4}\right)^4 = e^{\delta}$

$\Rightarrow (1.02)^4 = e^{\delta}$

$\Rightarrow \ln[(1.02)^4] = \ln e^{\delta}$

$\Rightarrow 4 \ln(1.02) = \delta \ln(e)$

$\Rightarrow \boxed{\delta = 0.079210509}$

$\Rightarrow$ ~~$\delta = 7.9210509\%$~~

ii) $1+i = \left(1+\frac{i^{(4)}}{4}\right)^4$

$\Rightarrow i = \left(1+\frac{0.08}{4}\right)^4 - 1$

$\boxed{i = \cancel{0}\ 8.243216\%}$

iii) $d^{(12)} = 12\left[1-\left(\frac{1}{1+i}\right)^{1/12}\right]$

$\Rightarrow d^{(12)} = 12\left[1-\left(\frac{1}{1+0.08243216}\right)^{1/12}\right]$

$\Rightarrow \boxed{d^{(12)} = 7.8949654}$

A2) $\delta(t) = 0.004t + 0.0002t^2$

i) $AV_{10} = 1000$ ~~$t=10$~~

$$PV_0 = AV_{10} \times e^{-\int_0^{10}(0.004t + 0.0002t^2)\,dt}$$

$$= 1000 \times e^{-\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}}$$

$$= 1000 \times e^{(-0.2 - 0.06667)}$$

$$= 1000 \times e^{-26667}$$

$$= 1000 \times 0.765902808$$

$$\boxed{= 765.902808}$$

ii) $\frac{1}{(1+i)^{10}}$ ~~$(1+i)^{10}$~~ $= 0.765902808$

$\Rightarrow \frac{1}{0.765902808} = (1+i)^{10}$

$\Rightarrow (1+i)^{10} = 1.305648693$

$$\boxed{i = 2.7028827\%}$$

A3) i) Here, i is interest actually paid on a loan by the end of accumulation period and d is the interest payable at the beginning of the year.

$$d = iv.$$

Thus, d is equal to the present value at beginning of ~~payment~~ ~~period~~ period of i payment payable at the end.

ii). a) $\frac{d^{(4)}}{4} = 2.25\%$.

$$1 - d = \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$d = 1 - (1 - 0.0225)^4$$

$$d = 8.7007806\%$$

$$\cancel{d^6 = 6\left[1 - (1-d)^{1/6}\right]} \qquad d^2 = 2\left[1 - (1-d)^{1/2}\right]$$

$$\cancel{\boxed{d^6 = 9.0340920\%}} \qquad \boxed{d^2 = 8.89875}$$

b). $d^{(1/2)} = 5\%$.

$$D = 1 - \left[1 - (0.05 \times 2)\right]^{1/2}$$

$$D = 5.1316702\%$$

$$\cancel{d^6 = 6\left[1 - (1-D)^{1/6}\right]} \qquad d^{(2)} = 2\left[1 - (1-D)^{1/2}\right]$$

$$\cancel{\boxed{d^6 = 5.2449666}} \qquad \boxed{d^{(2)} = 5.1992507}$$

iii)
$$\left(1-\frac{91}{365}\times d\right)=(1+0.05)^{-91/365}$$
$$1-(1.05)^{-91/365}=\frac{91d}{365}$$
$$\boxed{d=4.8494619}$$

A4) ii) $i=0.08$

a)
$$d^{(12)}=12\left[1-\left(\frac{1}{1.08}\right)^{1/12}\right]$$
$$d^{(12)}=0.076714776$$
$$d^{(12)}=7.6714776\ \%$$

b)
$$i^{(365)}=365\times\left[(1.08)^{1/365}-1\right]$$
$$\underline{i^{(365)}=0.076969155}$$

c)
$$\delta=\ln(1+i)$$
$$=\ln(1.08)$$
$$\underline{\delta=0.076961041}$$

d)
$$i^{(0.5)}=0.5\times\left[1.08^{(2)}-1\right]$$
$$\underline{i^{(0.5)}=0.0832}$$

A5) i). $d^{(4)} = 0.06$

a) $\cancel{i^{6}} \quad d = 1 - \left(1 - \frac{d^{4}}{4}\right)^{4}$

$$d = 1 - \left(1 - \frac{0.06}{4}\right)^{4}$$

$$d = 0.058663449$$

$\cancel{i^{6} = 6\left[\frac{1}{(1-d)^{1/6}} - 1\right]}$ $\quad i^{(2)} = 2\left[\frac{1}{(1-d)^{1/2}} - 1\right]$

$\boxed{\cancel{i^{6} = 6.0760139\%}}$ $\Rightarrow$ $\boxed{i^{(2)} = 6.1377515\%}$

b) $d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$

$\boxed{d^{(12)} = 6.0302525\%}$

ii) a). A.V. after 1 year = 2,20,000.
Principal amount = 2,00,000

$$i = \frac{AV}{C} - 1$$

$$i = 10\%$$

Outstanding term = 3 months.

$\cancel{i^{4} = p[(1+i)}$ $\quad \frac{i^{4}}{4} = \cancel{4}[(1 + 0.10)^{1/4} - 1]$

i^{4}

$$\frac{i^{(4)}}{4} = 2.4113689\%$$

$$\text{A.V. after 3 months} = C \times \left(1 + \frac{i^{(4)}}{4}\right)^{1}$$

$$= 209000 \times \left(1 + \frac{i}{4}\right)$$

$$\boxed{= 209822.7378}$$

46) i) Let ~~A.V. = 30~~ the value of product be 100.

⇒ ~~real~~ retail price when ~~pay~~ paid in cash ~~is~~ = 70.
& retail price ~~when~~ after 6 months = 75.

$$70 = 75(1-d)^{6/12} \qquad \cancel{25 = 30(1-d)^{6/12}}$$

$$\boxed{d = 12.89\%} \qquad \boxed{\cancel{d = 30.555\%}}$$

ii) $$d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$$

$$\Rightarrow \boxed{d^{(12)} = 13.7208048\%}$$

$$\delta = -\ln(1-d) \Rightarrow \boxed{\delta = 0.13799 8498}$$

$$i^{(2)} = 2\left[\frac{1}{(1-d)^{1/2}} - 1\right] \Rightarrow \boxed{i^{(2)} = 14.2870809\%}$$

Ans

A2). i)

$$26{,}000 = 20{,}000\left(1+\frac{i^{(4)}}{4}\right)^{4\times 17/12}$$

$$\left(1+\frac{i^{(4)}}{4}\right)^{17/3} = \frac{26000}{20000}$$

$$\boxed{i^{(4)} = 18.9552546\ \%}$$

ii)

$$20{,}000 = 26{,}000\left(1-\frac{d^{(2)}}{2}\right)^{2\times 17/12}$$

$$\boxed{d^{(2)} = 17.6882353\ \%}$$

iii)

$$26{,}000 = 20{,}000\left[1+\left(i\times\frac{17}{12}\right)\right]$$

$$i = \left[\frac{26{,}000}{20{,}000}\cancel{\times\frac{12}{17}} - 1\right]\times\frac{12}{17}$$

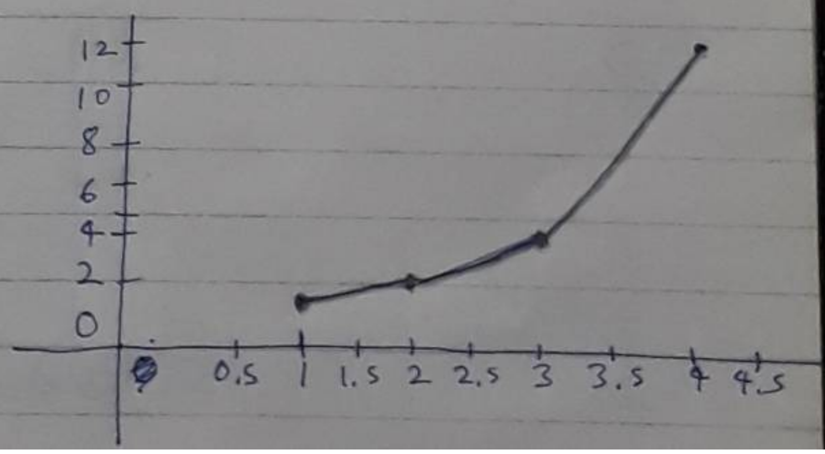
$$\boxed{i = 21.1764706\ \%}$$

iv) Simple interest needs to be higher than compound interest for same accumulated value.

A8). For 8%.

$i^{(1)}$	0.08
$i^{(2)}$	0.078461
$i^{(4)}$	0.077706
$i^{(12)}$	0.077208

Graph –



A9) (i). The nominal rate of interest per unit time convertible continuously (or momently) ~~this~~ is ~~also~~ referred to as the rate of continuously compounded. It is also called force of interest. (δ).
For each value of i -

$$\lim_{p\to\infty} i^{(p)} = \delta$$

ii) $i^{2} = 0.0775$

$$i = \left(1 + \frac{i^{(2)}}{2}\right)^{2} - 1$$

$\Rightarrow$ $\boxed{i = 0.079001563}$

$$\delta = \ln(1+i)$$

$\Rightarrow$ $\boxed{\delta = 0.076036134}$

$$d^{(12)} = 12\left[1 - \left(\frac{1}{1+i}\right)^{1/12}\right]$$

$\Rightarrow$ $\boxed{d^{(12)} = 0.075795747}$

$$i^{(1/2)} = \frac{1}{2}\left[(1+i)^{2} - 1\right]$$

$\Rightarrow$ $\boxed{i^{(1/2)} = 0.082122186}$

A10).

$$\delta(t) = \begin{cases} 0.08 & , 0 \le t \le 5 \\ 0.06 & , 5 \le t \le 10 \\ 0.04 & , t \ge 10 \end{cases}$$

$t \le 5$

$$v(t) = e^{-\delta t}$$

$$v(t) = e^{-0.08 \times 5}$$

$$v(t) = e^{-0.4}$$

~~$5 \le t \le 10$~~

$$v(t) = e^{-\left[0.4 + \int_5^{10}(0.06)\,dt + \int_{10}^{t}(0.04)\,dt\right]}$$

$$v(t) = e^{-[\cancel{0.4} + 0.3 + 0.04t - \cancel{0.4}]}$$

$$v(t) = e^{-[0.3 + 0.04t]}$$

$$v(t) = e^{-0.3 - 0.04t}$$ Ans

A11)(a). $C = 50{,}000$

$d = 18\%$

$$P.V. = 50{,}000\left[1 - \left(0.18 \times \frac{9}{12}\right)\right]$$

$$\boxed{P.V. = 43,250}$$

b). (i) $\delta = 6\%$.

$$d^{(12)} = 12\left[1 - (e^{-\delta t})^{1/12}\right]$$

$$\boxed{d^{(12)} = 5.9850250\%}$$

ii). $d^{4} = 6\% = 0.06$.

$$\frac{1}{1+i} = \left(1 - \frac{d^{p}}{p}\right)^{p}.$$

$$\Rightarrow \frac{1}{1+i} = \left(1 - \frac{d^{(4)}}{4}\right)^{4}$$

$$\Rightarrow \frac{1}{1+i} = \left(1 - \frac{0.06}{4}\right)^{4}$$

$$\Rightarrow \boxed{i = 6.2319315\ \%}$$

$$i^{(12)} = 12\left[(1+i)^{1/12} - 1\right]$$

$$\Rightarrow \boxed{i^{(12)} = 6.0607089}$$ Ans.

$$i^{(4)} = 4\left[(1+i)^{1/4} - 1\right]$$

$$\Rightarrow \boxed{i^{(4)} = 6.0913705}$$

c).

$$d < \delta < i^{(4)} < i$$

d is the interest paid in the beginning ~~of~~ and thus, is smaller amount, i is the ~~interest~~ interest paid in the end and δ is the payment throughout.

d). i is the interest paid by the end of the time period
~~A(2) d)~~ and d is the interest paid in the beginning.
Thus, i ~~must be~~ is equal to the present value at the beginning of the time period with a payment of i at the end.

A(2) $i^{12} = 6\% = 0.06$

$$\text{A.V. after 2 years} = 7049 \times \left(1 + \frac{\cancel{d}\, i^{(12)}}{12}\right)^{12\times 2}$$

$$= 7049 \times (1 + 0.005)^{24}$$

$$= 7945.349262$$

$i^{4} = 1.5\% = 0.015$

$$\text{A.V. after next 2.5 years} = (7945.349262) \times [1 + (2.5 \times 4 \times 0.015)]$$

$$\text{A.V. after 4.5 years} = 9137.151651$$

$d^{12} = 6\% = 0.06$

$$\frac{1}{1+i} = \left(1 - \frac{d^{(p)}}{p}\right)^{p}$$

$$\frac{1}{1+i} = \left(1 - \frac{0.06}{12}\right)^{12}$$

$$i = 6.1996367\%$$

$$\text{A.V. after 6 years} = (9137.151651) \times (1 + 0.061996367)^{1.5}$$

$$= 9999.893612$$

$$= 10{,}000 \text{ (approx)}$$ Ans

A13) Let the present value of investment be 1.

i) $i = 10\%$. $\quad AV. = P.V. (1+i)^t$

$$2 \times 1 = 1 (1+i)^t$$

$$2 = (1+i)^t$$

$$\ln(2) = t \ln(1+i)$$

$\cancel{\ln(2) = t \times i}$ $\quad \ln(2) = t \times \ln(1.10)$

$\cancel{\frac{\ln(2)}{0.10}} = t$ $\quad \underline{t = 7.2725 \text{ years}}$

or

~~t = 6.931 years~~

~~or~~

$\boxed{t = 7.2 \text{ years}}$ (approx)

ii) $i^{12} = 10\%$

$$2 = 1 \left(1 + \frac{i^{(12)}}{12}\right)^{t \times 12}$$

$$2 = \left(1 + \frac{0.10}{12}\right)^{t \times 12}$$

$$2 = (1 + 0.008333)^{12t}$$

$$\ln(2) = 12t \ln (1.008333)$$

$$\boxed{t = 6.96 \text{ years}}$$