Calculus Assignment - 2

1. $\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$

let $\frac{2}{w} \to t \Rightarrow w \to \frac{1}{50}, t = 100$

$w \to 2, t = 1$

$-\frac{2}{w^2}\, dw \to dt$

$\frac{dw}{w^2} = -\frac{1}{2}\, dt$

$= -\frac{1}{2}\int_{100}^{1} e^t\, dt$

$= -\frac{1}{2}[e^t]_{100}^{1} = -\frac{(e - e^{100})}{2}$

$= \frac{e^{100} - e}{2}$

2. $\int \frac{2t^3+1}{(t^4+2t)^3}\, dt$

let $t^4+2t \to a$

$(4t^3+2)\,dt = da$

$(2t^3+1)\,dt = \frac{da}{2}$

$= \frac{1}{2}\int \frac{da}{a^3}$

$= \frac{1}{2}\int a^{-3}\, da = \frac{1}{2}\left[\frac{a^{-2}}{-2}\right]$

$= -\frac{1}{4}\cdot\frac{1}{a^2}$

$= \frac{-1}{4a^2} = \frac{1}{4(t^4+2t)} + C$

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3. $f'(x) = x^4 + 3x - 9$

$f(x) = \int (x^4 + 3x - 9)\, dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$

4. $f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x \leq 1 \end{cases}$

$\int_{-2}^{1} (3x^2)\, dx + \int_{1}^{3} 6\, dx$

$= \frac{3[x^3]_{-2}^{1}}{3} + 6[x]_1^3$

$= 1 - (-2)^3 + 6[3-1]$

$= 1 + 8 + 12$

$= \underline{\underline{21}}$

5. $\int 3(8y-1)e^{4y^2-y}\, dy$

let $4y^2 - y = t$

$(8y-1)dy = dt$

$= 3\int e^t\, dt$

$= 3e^t + c$

$= 3e^{4y^2-y} + c$

6. ~~$f(x) = 15\sqrt{x}$~~ $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f'(x) = \int 15\sqrt{x} + 5x^3 + 6$

$= \frac{2 \times 15\, x^{3/2}}{3} + \frac{5x^4}{4} + 6x + c$

$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + c$

$f(x) = \int 10x^{3/2} + \frac{5}{4}x^4 + 6x + c$

$= \frac{2 \times 10 \times x^{5/2}}{5} + \frac{5x^5}{4 \cdot 5} + \frac{6x^2}{2} + cx + D$

$$f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + Cx + D$$

$$\cancel{f(x)}\ f(1) = -\frac{5}{4}$$

$$-\frac{5}{4} = 4 + \frac{1}{4} + 3 + C + D$$

$$-5 = 16 + 1 + 12 + 4C + 4D$$

$$-34 = 4C + 4D \quad \cancel{\text{(i)}}$$

$$\cancel{-34+8 = 4C+4D} \qquad 4C = 4D - 34 \quad \text{—— (i)}$$

$$\cancel{-26 = C+D}$$

$$f(4) = 404$$

$$404 = 4 \times 2^{2 \times 5/2} + \frac{4^5}{4} + 3(4)^2 + C(4) + D$$

$$404 = 128 + 256 + 48 + 4C + D$$

$$-28 = 4C + D$$

$$-28 = -34 + 4D + D \quad (\text{from (i)})$$

$$6 = -3D$$

$$D = -2$$

$$C = -\frac{13}{2}$$

$$\therefore f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - \frac{13}{2}x + 2$$

7. $z = f(x+iy) + g(x-iy)$

$p = \frac{\partial z}{\partial x} = f'(x+iy) + g'(x-iy)$

$q = \frac{\partial z}{\partial y} = if'(x+iy) - ig'(x-iy)$

$\frac{\partial z}{\partial y} = i\left(f'(x+iy) - g'(x-iy)\right)$

$r = \frac{\partial^2 z}{\partial x^2} =$

7. $z = f(x+iy) + g(x-iy)$

$p = \frac{dz}{dx} = f'(x+iy) + g'(x-iy)$

$q = \frac{dz}{dy} = if'(x+iy) - ig'(x-iy)$

$\frac{dz}{dy} = q = i(f'(x+iy) - g'(x-iy))$

$r = \frac{d^2z}{dx^2} = f''(x+iy) + g''(x-iy)$

$t = \frac{d^2z}{dy^2} = i^2(f''(x+iy) + g''(x-iy))$

So $r + t = 0$; required PDE.

8. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$; $r(1) = 2$

$\int r^{-2}\, dr = \int \theta^{-1}\, d\theta$

$\frac{r^{-2+1}}{-2+1} = \ln|\theta|$

$-\frac{1}{r} = \ln|\theta| + c$

$\frac{1}{r} = -\ln|\theta| - c$

$r = \frac{1}{-\ln|\theta| - c}$

$2 = \frac{1}{-\ln|1| - c}$

$c = \frac{-1}{2}$

$r = \frac{1}{-\ln|\theta| - c} = \frac{1}{-\ln|\theta| + 1/2}$

$r = \frac{2}{1 - 2\ln|\theta|}$

9. $\frac{dv}{dt} + 0.196v = 9.8$

$IF = e^{\int 0.196\,dt}$

$= e^{0.196t}$

$v(IF) = \int IF \times 9.8\,dt$

$v = \frac{1}{e^{0.196t}} \int 9.8e^{0.196t} + c$

$v = \frac{1}{e^{0.196t}} \left[\frac{9.8 \cdot e^{0.196t}}{0.196} + c\right]$

$v = 50 + ce^{-0.196t}$

10. $ty' + 2y = t^2 - t + 1 \qquad y(1) = \frac{1}{2}$

$\frac{dy}{dt} + \frac{2}{t}(y) = t - 1 + \frac{1}{t}$

$IF = e^{2\int 1/t\,dt}$

$= e^{2\ln|t|}$

$= \cancel{e^{\ln|t|^2}} \quad e^{2\ln|t|^2}$

$= t^2$

$y(t^2) = \int t^2\left(t - 1 + \frac{1}{t}\right) + c$

$yt^2 = \int t^3 - t^2 + t + c$

$yt^2 = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c$

$y = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + \frac{c}{t^2}$

$\frac{1}{2} = \frac{1}{4} - \frac{1}{3} + \frac{1}{2} + C$

$\cancel{72}\ \frac{1}{2} = \frac{3 - 4 + 6 + 12C}{12} \Rightarrow 6 = 5 + 12C$

$C = \cancel{\frac{1}{12}}\ \frac{1}{12} \qquad \therefore y = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{12t^2} + \frac{1}{2} \rightarrow$ required

11. $y' = \frac{xy^3}{\sqrt{1+x^2}}$ $\quad y(0) = -1$

$$\frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$$

$$y^{-3}\,dy = \frac{x}{\sqrt{1+x^2}}\,dx$$

$$\int y^{-3}\,dy = \int \frac{x}{\sqrt{1+x^2}}\,dx$$

Let $1+x^2 \to t$

$2x \to dt$

$$\int y^{-3}\,dy = \frac{1}{2}\int \cancel{2x}\,\frac{dt}{\sqrt{t}}$$

$$\frac{y^{-2}}{-2} = 2\cdot\frac{1}{2}\cdot\sqrt{t}$$

$$\frac{-1}{2y^2} = \sqrt{t} + c$$

$$\frac{-1}{2y^2} = \sqrt{1+x^2} + c$$

$$\frac{-1}{y^2} = 2\sqrt{1+x^2} + 2c$$

$$\frac{-1}{2} = 1 + c$$

$$c = \frac{-1}{2} - 1$$

$$c = \frac{-3}{2}$$

12.

$f(x) = x^3 - 10x^2 + 6$	$f(3) = 3^3 - 10(3)^2 + 6 = -57$
$f'(x) = 3x^2 - 20x$	$f'(3) = 3(9) - 20(3) = -33$
$f''(x) = 6x - 20$	$f''(3) = 6(3) - 20 = -2$
$f'''(x) = 6$	$f'''(3) = 6$
$f''''(x) = 0$	$f''''(3) = 0$

Now, all other high orderivatives are 0 (zero)

$$f(x) = f(3) + f'(3)(x-3) + \frac{f''(3)(x-3)^2}{2!} + \frac{f'''(x)(x-3)^3}{3!}$$

$$f(x) = -57 + -33(x-3) + \frac{(-2)(x-3)^2}{2!} + \frac{6(x-3)^3}{3!}$$

$$f(x) = -57 - 33(x-3) - (x-3)^2 - (x-3)^3$$

13.

$f(x) = \sqrt{1+x}$	$f(0) = \sqrt{1} = 1$
$= (1+x)^{1/2}$	
$f'(x) = \dfrac{1}{2(1+x)^{1/2}}$	$f'(0) = \dfrac{1}{2}$
$f''(x) = \dfrac{-1}{4}(1+x)^{-3/2}$	$f''(0) = \dfrac{-1}{4}$
$f'''(x) = \dfrac{3}{8}(1+x)^{-5/2}$	$f'''(0) = \dfrac{3}{8}$

$$f(x) = f(0) + f'(0)(x) + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$$

$$f(x) = 1 + \frac{1}{2}x + \frac{-1}{4}\frac{x^2}{2!} + \frac{3}{8}\frac{x^3}{3!} + \dots$$

$$f(x) = 1 + \frac{1}{2}x - \frac{x^2}{8} + \frac{3x^3}{16} + \dots$$

15. $f(x) = e^{-6x}$

$f'(x) = -6e^{-6x}$

$f''(x) = 36e^{-6x}$

$f'''(x) = 216e^{-6x}$

$f(-4) = e^{24}$

$f'(-4) = -6e^{24}$

$f''(-4) = 36e^{24}$

$f'''(-4) = -216e^{24}$

Taylor series → $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \cdots$

$f(x) = e^{-6x} = f(-4) + f'(-4)(x+4) + \frac{f''(-4)(x+4)^2}{2!} + \frac{f'''(-4)(x+4)^3}{3!} + \cdots$

$f(x) = e^{24} + -6e^{24}(x+4) + \frac{36e^{24}(x+4)^2}{2!} + \frac{216e^{24}(x+4)^3}{3!} + \cdots$

$f(x) = e^{24} - 6e^{24}(x+4) + 8e^{24}(x+4)^2 - 36e^{24}(x+4)^3 + \cdots$

16. $f(x) = \ln|3+4x|$

$f'(x) = \frac{1(4)}{3+4x}$

$f''(x) = -4(3+4x)^{-2}$

$= -16(3+4x)^{-2}$

$f'''(x) = 128(3+4x)^{-3}$

$f(0) = \ln|3|$

$f'(0) = \frac{4}{3}$

$f''(0) = \frac{-16}{9}$

$f'''(0) = \frac{128}{27}$

$f(x) = f(0) + f'(0)(x) + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \cdots$

$f(x) = \ln|3| + \frac{4}{3}x - \frac{16}{9}\frac{x^2}{2!} + \frac{128}{27}\frac{x^3}{3!} + \cdots$

$f(x) = \ln|3+4x| = \ln|3| + \frac{4}{3}x - \frac{8}{9}x^2 + \frac{128}{162}x^3 + \cdots$