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Page No. | Date

Numerical methods and Algebra.

Assignment

Q.1.] Radius of a circular plate (measured) = r_m = 12.65 cm.

Actual radius = r_a = 12.5 cm.

i) Absolute error $= |r_m - r_a|$

$= 12.65 - 12.5$

$= 0.15$.

ii) Relative error $= \left|\frac{12.65 - 12.5}{12.5}\right|$

$= 0.001$

iii) Percentage error $= \left|\frac{r_m - r_a}{r_a}\right| \times 100$

$= \left|\frac{12.65-12.5}{12.5}\right| \times 100$

$= 0.1\%$.

Q.2.]

x	4.0	4.5	5.5	~~6.0~~	6.0
f(x) = log(x)	0.60206	0.6532125	0.7403627		0.7781513

a) Interpolate log(5) using the points $x=4$ and $x=6$.

∴ By Lagrange's Interpolation

$$f(5) = \frac{(5-6)}{(4-6)} \times f(4) + \frac{(5-4)}{(6-4)} \times f(6)$$

$$= \frac{-1}{-2} \times 0.60206 + \frac{1}{2} \times 0.7781513$$

$$= \left(\frac{1}{2} \times 0.60206\right) + \left(\frac{1}{2} \times 0.7781513\right)$$

$= 0.690105$

b) Interpolate log(5) using the points $x = 4.5$ and $x = 5.5$

$$\therefore f(5) = \frac{5-5.5}{4.5-5.5} \times f(4.5) + \frac{5-4.5}{5.5-4.5} \times f(5.5)$$

$$= \left(\frac{-0.5}{-1} \times 0.6532125\right) + \left(\frac{0.5}{1} \times 0.7403627\right)$$

$$= 0.3266 + 0.37018$$

$$= 0.69678.$$

Q.3] To find:- root of $x^4 - x - 10 = 0$

$a = 1.5$

$b = 2$

	$\therefore x$	$f(x) = y$
x_1	1.5	-6.4375
x_2	2	4

$\therefore$ By bisection method

$$x_3 = \frac{x_1 + x_2}{2} = \frac{1.5+2}{2} = 1.75.$$

$\therefore y_3 = -2.371$

$\therefore x_3 = 1.75 \qquad y_3 = -2.371$

$x_2 = 2 \qquad y_2 = 4$

$$\therefore x_4 = \frac{1.75+2}{2} = 1.875$$

$y_4 = 0.4846.$

$\therefore x_3 = 1.75 \qquad y_3 = -2.371$

$x_4 = 1.875 \qquad y_4 = 0.4846.$

Page No.
Date

$\therefore x_5 = \frac{1.75 + 1.875}{2} = 1.8125$

$y_5 = -1.0202$

$\therefore x_5 = 1.8125 \quad y_5 = -1.0202$

$x_4 = 1.875 \quad y_4 = 0.4846$

$\therefore x_6 = \frac{1.8125 + 1.875}{2} = 1.8438$

$\therefore y_6 = -0.2865$

$\therefore x_6 = 1.8438 \quad y_6 = -0.2865$

$x_4 = 1.875 \quad y_4 = 0.4848$

$\therefore x_7 = \frac{1.8438 + 1.875}{2} = 1.8594$

$\therefore y_7 = 0.0939$

$\therefore$ The root of $x^4 - x - 10$ is 1.8594 [By bisection method]

Q.4.] Find the root of an equation $f(x) = x^3 - x - 1$ using Newton Raphson Method.

~~Let~~

$\therefore$ ~~x~~ $\quad f(x) = x^3 - x - 1$

$f'(x) = 3x^2 - 1$

x	$f(x)$	
0	-1	
1	-1	- -ve
2	5	- +ve.

$\therefore$ Answer lies between 1 and 2

$x_1 = 1$

$f(x_1) = -1$

$f'(x_1) = 2$

X.

Page No.
Date

Q.4] Find the root of the equation $f(x)=x^3-x-1$ using Newton Raphson Method.

We make the table as follows:-

x	$f(x)=x^3-x-1$	
0	-1	
0.5	-1.375	
1	-1	(-ve)
1.5	0.875.	(+ve)

$f(x)=x^3-x-1$

$f'(x)=3x^2-1$

Let $x_0=1$

$\therefore f(x_0)=-1 \qquad f'(x_0)=2$

$\therefore$ By Newton Raphson Method.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1-\left(\frac{-1}{2}\right)$$

$$x_1 = 1.5$$

$\therefore f(x_1)=0.875$

$f'(x_1)=5.75$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.5-\left(\frac{0.875}{5.75}\right) = 1.35$$

$f(x_2)=0.1104$

$f'(x_2)=4.4675$

$$\therefore x_3 = 1.35-\left(\frac{0.1104}{4.4675}\right) = 1.325$$

$f(x_3)=0.0012$

$f'(x_3)=4.2669$

$\therefore x_4 = 1.325 - \left(\frac{0.0012}{4.2669}\right) = 1.3247$

$f(x_4) = -0.00007$

$f'(x_5) = 4.26$

$\therefore$ We can say that root of the equation $f(x) = x^3 - x - 1$ is

$\underline{1.3247}$

Q.5.] If A is a square matrix such that $A^2 = A$, then find the value of $7A - (I+A)^3$. I - is an identity matrix.

$$\begin{aligned}
\therefore 7A - (I+A)^3 &= 7A - (I^3 + 3I^2 A + 3IA^2 + A^3) \\
&= 7A - (I^3 + 3A + 3A^2 + A \cdot A^2) \\
&= 7A - (I^3 + 3A + 3A + A \cdot A) \quad [\because A^2 = A] \\
&= 7A - (I^3 + A^2 + 6A) \\
&= 7A - (I + A + 6A) \\
&= 7A - (I + 7A) \\
&= \cancel{7A} - I - \cancel{7A} \\
&= -I.
\end{aligned}$$

$$\boxed{\therefore 7A - (I+A)^3 = -I}$$

Q.6] Find out the inverse of :-

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\begin{aligned}
\therefore |A| &= 1(0-6) - (-1)[-4-0] + 2[4-0] \\
&= -6 - 4 + 8 \\
&= -10 + 8 \\
&= -2
\end{aligned}$$

$|A| \neq 0 \therefore$ It is an invertible matrix.

$\therefore$

$$M_{11} = \begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} = (0-6) = -6$$

$$M_{12} = \begin{vmatrix} 4 & 6 \\ 0 & -1 \end{vmatrix} = (-4-0) = 4$$

$$M_{13} = \begin{vmatrix} 4 & 0 \\ 0 & 1 \end{vmatrix} = (4-0) = 4$$

$$M_{21} = \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} = (1-2) = -1$$

$$M_{22} = \begin{vmatrix} 1 & 2 \\ 0 & -1 \end{vmatrix} = (-1-0) = -1$$

$$M_{23} = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = (1-0) = 1$$

$$M_{31} = \begin{vmatrix} -1 & 2 \\ 0 & 6 \end{vmatrix} = (-6-0) = -6$$

$$M_{32} = \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix} = (6-8) = -2$$

$$M_{33} = \begin{vmatrix} 1 & -1 \\ 4 & 0 \end{vmatrix} = (0-(-4)) = 4.$$

$\therefore$ Now, the new matrix

$$A = \begin{bmatrix} -6 & 4 & 4 \\ -1 & -1 & 1 \\ -6 & -2 & 4 \end{bmatrix}$$

Page No.

Date

We multiply the matrix by alternating +, - signs to obtain the cofactor matrix.

$$A = \begin{bmatrix} -6 & 4 & 4 \\ -1 & -1 & 1 \\ -6 & -2 & 4 \end{bmatrix} \times \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} -6 & -4 & 4 \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}$$

We will take transpose of A (cofactor matrix) to obtain Adj(A)

$$\therefore Adj(A) = \begin{bmatrix} -6 & 1 & -6 \\ -4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot Adj(A)$$

$$= \frac{1}{-2} \cdot \begin{bmatrix} -6 & 1 & -6 \\ -4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ 2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

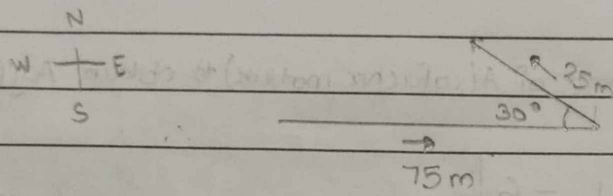
Q7] Using the definition of the Scalar product, find the angle between the following vectors.

a) $A = 8i - 9j$ and $B = 7i - 9j$

∴ By Scalar product

$$\bar{A}\cdot\bar{B} = A_1B_1 + A_2B_2$$
$$= 8(7) + (-9)(-9)$$
$$= 56 - 81$$
$$= -25.$$

Q.8.] Sabrina walked 75m to the east.
She turned 30 degrees to the left and walked 25m.
– To find Magnitude of Sabrina's displacement vector.



∴ The vector can be defined

as $75\hat{i} + 25\hat{j}$

∴ Let $R = 75\hat{i} + 25\hat{j}$

∴ Magnitude of $R = \sqrt{75^2 + 25^2} = \sqrt{6250}$

$= 79.05$

Q.9.] What is the sum of all three digit numbers that leave a remainder of '2' when divided by 3?

Smallest three digit number divisible by 3 is '101'
Largest three digit number divisible by 3 is '998'

We also know that the first smallest number is 101 and the next number divisible by 3 that gives remainder 4 is 104 and after than 107. and so on

Thus we can form an A.P

Page No.

Date

101, 104, 107, ... 998

$\therefore t_1 = 101$ is a $\quad t_2 = 104$

$t_n = 998$

$\therefore$ common difference $= 104 - 101 = 3.$

$t_n = a + (n-1)d$

$\therefore 998 = 101 + (n-1)3$

$\therefore 998 - 101 = 3n - 3$

$\therefore 897 + 3 = 3n$

$900 = 3n$

$\therefore n = \frac{900}{3} = 300$

$\therefore$ The total no. of terms are 300.

$\therefore S_n = \left(\frac{a + t_n}{2}\right) \times n = \left(\frac{101 + 998}{2}\right) \times 300$

$= \left(\frac{1099}{2}\right) \times 300$

$=$ ~~[illegible]~~ $164{,}850.$

Q.10 $t_6 = 32 \qquad t_8 = 128$

the terms are in G.P

Let 'r' be the common ratio.

$\therefore \frac{t_7}{t_6} = \frac{t_8}{t_7} = r \qquad \cancel{t_9} =$

$\therefore \frac{t_7}{t_6} = \frac{t_8}{t_7}$

$\frac{t_7}{t_6} \times \frac{t_7}{t_8} - \frac{t_8}{t_7} = 0$

$\therefore t$ -

We Know that

$t_{\cancel{6}} = t_n = ar^{(n-1)}$

$\therefore\ t_6 = ar^{(6-1)} = ar^5$ | $t_6 = 32$

$t_8 = ar^{(8-1)} = ar^7$ | $t_8 = 128$

$\therefore\ ar^5 = 32 \quad - (1)$

$ar^7 = 128 \quad - (2)$

Dividing eq. (2) by (1)

$\therefore\ \frac{ar^7}{ar^5} = \frac{128}{32}$

$\therefore\ \cancel{a}r^2 = 4$

$\therefore$ Taking +ve sq. root

$\therefore\ \boxed{r = 2}$

Q.11.] Use Binomial Theorem to expand $(4+3x)^5$

According to Binomial Theorem

$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}.b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}.b^3 \ldots$

$\therefore (4+3x)^5 = 4^5 + 5 \times 4^4\ 3x + \left(\frac{5\times4}{2!}\right).\ 4^3.(3x)^2 + \frac{5\times4\times3}{3!}.\ \cancel{4}^2.(3x)^3$

$+ \frac{5\times4\times3\times2}{4!}.\ 4.(3x)^4 + \frac{5\times4\times3\times2\times1}{5!}\ 4^0.(3x)^5$

$= 1024 + (5)(256)(3x) + (10)(64)\,9x^2 + (10)(16)(27x^3)$
$+ (5)(4)(81x^4) + (243x^5)$

$\therefore (4+3x)^5 = 1024 + 3840x + 5760x^2 + 4320x^3 + 1620x^4 + 243x^5$

Q.12] Find all eigenvalues of matrix A.

$$A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$[A - \lambda I] = \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

Eigenvalues can be found as follows:-

$$\lambda^3 - \underbrace{(a_{11}+a_{22}+a_{33})}_{\text{main diagonal addition}}\lambda^2 + \underbrace{(m_{11}+m_{22}+m_{33})}_{\text{minors of main diagonal}}\lambda - \underbrace{|A|}_{\text{determinant}} = 0$$

$\therefore \lambda^3 - (2-5+3)\lambda^2 + [(-15)+6+(-4)]\lambda - [2(-15) - (-3)(6) + 0(6)] = 0$

$\therefore \lambda^3 - 0\lambda^2 + (-13)\lambda - (-30+18+0) = 0$

$\therefore \lambda^3 - 13\lambda + 12 = 0$ — (1)

put $\lambda = 1$ in eq. (1)

$\therefore 1 - 13 + 12 = -12 + 12 = 0$

$\therefore \lambda = 1$ satisfies.

1	1	0	-13	12
		1	1	-12
	1	1	-12	0

$\therefore \lambda^2 + \lambda - 12 = 0$

$\therefore \lambda^2 + 4\lambda - 3\lambda - 12 = 0$

$\therefore \lambda(\lambda+4) - 3(\lambda+4) = 0$

$\therefore \lambda + 4 = 0$ or $\lambda - 3 = 0$

$\therefore \lambda = -4$ or $\lambda = 3$

$\boxed{\therefore \lambda = 1, -4, 3}$

Eigenvector corresponding to the value $\lambda = 1$

$$\therefore \begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Setting $x = 1$

$\therefore$ $1 - 3y = 0$ $\quad \therefore 1 = 3y \quad \therefore \boxed{y = 1/3}$

$2 - 6y = 0$

$\boxed{2 = z}$

$\therefore$ when $\lambda = 1$

Eigenvector $(1, 1/3, 2)$. or we can also write $(3, 1, 6)$

$\lambda = -4$

$$\begin{bmatrix} 6 & -3 & 0 \\ 0 & -9 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\therefore$ Let $x = 1$

$\therefore$ $6 - 3y = 0 \quad 6 = 3y \quad \therefore y = 6/3 = 2$

$\boxed{z = 7}$

$\therefore$ Eigenvector $(1, 2, 7)$

$\lambda = 3$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Let $x = 1$

$\therefore$ $-1 - 3y = 0 \quad \therefore -1 = 3y \quad \therefore \boxed{y = -1/3}$

$\boxed{z = 0}$

$\therefore$ Eigenvector $(1, -1/3, 0)$ or $(3, -1, 0)$.

Page No.
Date

Q.13] Use Newton's method to determine x_2.

$$f(x) = x^3 - 7x^2 + 8x - 3$$

$$x_0 = 5.$$

$\therefore x_0 = 5 \quad , \quad f(x_0) = -13$

$$f(x) = x^3 - 7x^2 + 8x - 3$$

$$\therefore f'(x) = 3x^2 - 14x + 8$$

$\therefore f'(x_0) = 74$

$\therefore$ By Newton Raphson Method.

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \left(\frac{-13}{74}\right) = 5.176$$

$$f(x_1) = -138.82$$

$$f'(x_1) = 120.95$$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 5.176 - \left(\frac{-138.82}{120.95}\right) = 6.326$$

$$\boxed{\therefore x_2 = 6.326}$$

Q.14] Expand the following $(1 - x + x^2)^4$.

$(1 - x + x^2)^4$

Let $(1 - x) = a$

$\therefore \cancel{(a + x^2)^4 = a^4 +}$

According to Binomial Theorem

$(a+b)^n = a^n + na^{n-1} \cdot b + \frac{n(n-1)}{2!} a^{n-2} \cdot b^2 \ldots$

$$(a + x^2)^4 = a^4 + 4a^3 x^2 + \frac{4(3)}{2!} \cdot a^2 (x^2)^2 + \frac{4(3)(2)}{3!} \cdot a (x^2)^3 + \frac{4(3)(2)(1)}{4!} \cdot (x^2)^4$$

Page No.
Date

$\therefore (a+x^2)^4 = a^4 + 4a^3x^2 + 6a^2x^4 + 4ax^6 + x^8$

Resubstituting $a = (1-x)$.

$\therefore (1-x+x^2)^4 = (1-x)^4 + 4(1-x)^3 x^2 + 6(1-x)^2 x^4 + 4(1-x)x^6 + x^8$

$= \left[1^4 - 4(1)^3 x + \frac{4(3)}{2!}(1)^2 x^2 + \frac{4(3)(2)(1)}{3!} \cdot x^3 + x^4\right] + 4$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$+ 4\left[1^3 - 3(1)^2 x + \frac{3(2)(1)}{2!} x^2\right] x^2$

$+ 6\left[1^2 - 2(1)(x) + x^2\right] x^4 + (4x^6 - x^7) + x^8$

$= 1 - 4x + 6x^2 + 4x^3 + x^4 + 4x^2 - 12x^3 + 12x^4 + 6x^4 - 12x^5 + 6x^6$
$+ 4x^6 - x^7 + x^8$

$\therefore (1-x-x^2)^4 = 1 - 4x + 10x^2 - 8x^3 + 19x^4 - 12x^5 + 10x^6 - x^7 + x^8$

Q.15] Find the approximated value of x till 4 iterations for $e^{-x} = 3\log(x)$ using bisection method

$\therefore e^{-x} = 3 \cdot \log(x)$

$e^{-x} - 3 \cdot \log x = 0$

$\therefore f(x) = e^{-x} - 3 \cdot \log x$.

x	$f(x)$	
0.5	2.685	
1	0.3678	+ve
1.5	−0.9932	−ve

$\therefore x_1 = 1 \quad f(x_1) = 0.3678$

$x_2 = 1.5 \quad f(x_2) = -0.9932$

$\therefore$ By Bisection method,

① $x_3 = \frac{x_1 + x_2}{2} = \frac{1 + 1.5}{2} = 1.25$

$f(x_3) = -0.3829$

② $x_4 = \frac{x_1 + x_3}{2} = \frac{1 + 1.25}{2} = 1.125$

$f(x_4) = -0.0286$

③ $x_5 = \frac{x_1 + x_4}{2} = \frac{1 + 1.125}{2} = 1.0625$

$f(x_5) = 0.1637$

④ $x_6 = \frac{x_5 + x_4}{2} = \frac{1.0625 + 1.125}{2} = 1.0937$

$f(x_6) = 0.0662$

$\therefore \boxed{x \approx 1.0937}$