Financial Mathematics.

Q.1.A) i) $d^{(4)} = 8\%$

$\delta = \ln\left(\frac{1}{1-a}\right)$

$\delta = \ln\left(\frac{1}{1-\frac{d(P)}{P}}\right)^{P}$

$\delta = P\ln\left(\frac{1}{1-\frac{d(P)}{P}}\right)$

$\delta = 4\ln\left(\frac{1}{1-\frac{d^{(4)}}{4}}\right)$

$\delta = 4\ln\left(\frac{1}{1-\frac{0.08}{4}}\right)$

$\delta = 4\ln\left(\frac{1}{1-0.02}\right)$

$\delta = 4\ln\left(\frac{1}{0.98}\right)$

$\cancel{\delta = 4\ln\left(\frac{1}{\ }\right)}$

$\delta = 0.08081082927.$

$\delta = 8.081082927\%$

ii) $d^{(4)} = 8\%$

$1+i = \left(\frac{1}{1-\frac{d^{(4)}}{4}}\right)^4$

$1+i = \left(\frac{1}{1-0.02}\right)^4$

$1+i = 1.084165785$

$i = 0.084165785$

$i = 8.4165785\%$

iii) $\frac{d^{(12)}}{12} = ?$

$\left(1-\frac{d^{4}}{4}\right)^4 = \left(1-\frac{d^{(12)}}{12}\right)^{12}$

Raising both sides by 1/12.

$\left(1-\frac{d^{4}}{4}\right)^{1/3} = 1-\frac{d^{12}}{12.}$

$(1-0.02)^{1/3} = \frac{1-d^{12}}{12}$

$\frac{d^{12}}{12} = 0.006711611621$

$d^{(12)} = 8.0539339\%$

Q.2. A) $PV = ?$ $AV = 1000$

$\delta(t) = 0.004t + 0.0002t^2$

$AV = PV\, e^{\int_0^{10} \delta(t)}$

$PV = AV\, e^{-\int 0.004t + 0.0002t^2.}$

$PV = 1000\, e^{-\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]^3}$

$PV = 1000\, e^{-\left[0.002(10)^2 + \frac{0.0002(10)^3}{3}\right]}$

$PV = 1000\, e^{-0.266667}$

$PV = 1000 \times e$

$= 1000 \times 0.7659283384.$

$= 765.283384$

ii) $AV = PV\,(1+i)^{10}$

$1000 = 765.9283384\,(1+i)^{10}$

$(1+i)^{10} = 1.305605172.$

Raising by $1/10$

$(1+i) = 1.027025404.$

$i = 0.027025404.$

$i = 2.7025404\,\%.$

3.) i) $1+i = \frac{1}{1-d}$ $1-d = \frac{1}{1+i}$ $d = \frac{1+i-i}{1+i}$ $d = \frac{i}{1+i}$

$(1-0.01)^{1/4} = 1 - \frac{d^2}{2}$ $d = iv$

$$\frac{d^2}{2} = 0.025996237.$$

$(d)^{12} = 0.05199250715$

$d^{(12)} = 5.199250715$

ii) a) $\frac{d^{(4)}}{4} = 2.25\% = 0.0225$

$d^{(12)} = ?$

$$\left(1 - \frac{d^4}{4}\right)^4 = \left(1 - \frac{d^2}{2}\right)^2.$$

Raising both sides by 1/12.

$(1-0.0225)^2 = 1 - \frac{d^2}{2}$

$\frac{d^2}{2} = 0.04449375$

$d^2 = 0.0889875$

$d^2 = 8.89875\%.$

iii) $i = 5\%$ $n = 91$ days.

SP = ? (annual)

let Bill amt be £100

$PV = AV\ (1+i)$

$= 100\ (1+i)^{91/365}$

$= 100\ (1+0.05)^{91/365}$

$= 101.2238407$

$100 = 101.2238407.$

~~$i = 0.04$[illegible]~~

b) $d^{(1/2)} = 5\% = 0.05$

$d^{12} = ?$

$$\left(1 - \frac{d^{(1/2)}}{1/2}\right)^{1/2} = \left(1 - \frac{d^{(2)}}{2}\right)^2$$

$(1 - 0.05 \times 2)^{1/2} = \left(1 - \frac{d^2}{2}\right)^2.$

Raising both sides by 1/2 100 =

[illegible]

$d = 4.8494618\%.$

Q.4. A) i) a.) $i = 0.08$.

$d^{(12)} = ?$

$$\left(1 - \frac{d^{12}}{12}\right)^{12} = \frac{1}{1+0.08}$$

Raising both sides by 1/12

$$1 - \frac{d^{12}}{12} = \left(\frac{1}{1.08}\right)^{1/12}.$$

$$1 - \frac{d^{(12)}}{12} = 0.993607102.$$

$$d^{(12)} = 0.0767147764$$

$$d^{(12)} = 7.671477614\%.$$

b) $i^{(365)} = ?$ $i = 0.08$.

$$\left(1 + \frac{i^{(365)}}{365}\right)^{365} = 1+0.08.$$

Raising both sides by 1/365

$$\left(1 + \frac{i^{(365)}}{365}\right)^{365} = (1.08)^{1/365}$$

$$i^{(365)} = 0.07696915541$$

$$i^{(365)} = 7.696915541\%.$$

c.) $\delta = \ln(1+i)$

$\delta = \ln(1+0.08)$

$\delta = 0.0769610414$

$\delta = 7.69610414\%.$

d.) $i^{(0.5)} = ?$ $i = 0.08$.

$$\left(1 + \frac{i^{(0.5)}}{0.5}\right)^{0.5} = (1+0.08)$$

Squaring both sides

$$\left(1 + \frac{i^{(0.05)}}{0.5}\right) = (1.08)^2$$

$$\frac{i^{(0.5)}}{0.5} = 0.1664$$

$$i^{(0.5)} = 0.0832$$

$$i^{(0.5)} = 8.32\%.$$

Q5.) A.) i.) $d^{(4)} = 6\%$.

a) $i^{(2)} = ?$

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \frac{1}{\left(1 - \frac{d^4}{4}\right)^4}.$$

Raising both by 1/2

$$\frac{1+i^{(2)}}{2} = \frac{1}{\left(\frac{1-0.06}{4}\right)^2}$$

$$\frac{1+i^2}{2} = \frac{\cancel{1}}{\cancel{1}-\cancel{0}.\cancel{0}6} \quad 1.030688768.$$

$(i)^2 = 0.0613775152.$

$(1)^2 = 6.137755152$

B.) $(d)^{12} = ?$ $d^{(4)} = 6\%$.

$$\left(1 - \frac{d^4}{4}\right)^4 = \left(1 - \frac{d^{12}}{12}\right)^{12}$$

Raising both sides by 1/12.

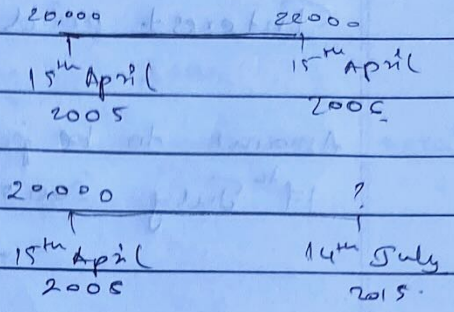
$$\left(1 - \frac{d^{4)}}{4}\right)^{1/3} = \frac{1 - d^{12}}{12}.$$

$$\left(\frac{1 - 0.06}{4}\right)^{1/3} = \frac{1-d^{(12)}}{12}.$$

$d^{(12)} = 0.06030252528.$

$d^{(12)} = 6.030252528\%.$

ii)



April	May	June	July
15	31	30	14

93days.

if paid on 15th April 2006

$$\frac{22000}{2,00,000} = (1+i)$$

$i = 0.1$ or 10%.

PAGE No.
DATE / /

Interest paid for 93 day = $\frac{10}{100} \times \frac{93}{365}$ = 0.0284794

Amount to be paid the contrasl is terminates on 17th July 2005 = 2,00,000 + [2,00,000 × 0.02547945

= 2,00,000 + [5095.8904]

= 205905.8904

Q6.A.

a) let the retail price of the toy be £100.

Cash payment = 30% below the recommended retail price.

Cash payment = $100 - \frac{30}{100} \times 100$

$= 100 \cdot 30 = 70.$

b.) Six month credit = 25% below the recomended retail price

Payment after 6 months = $100 - \frac{25}{100} \times 100$

$= 100 - 25 = 75$

Therefore the objective rate of discount is:

$70 = 75(1-d)^{0.5}$

$\frac{70}{75} = (1-d)^{0.5}$

Squaring both sides.

$\left(\frac{70}{75}\right)^2 = 1-d.$

$d = 0.1288889$

$d = 12.88889\%$

ii) $d^{(12)} = ?$

$1 - d = \left(1 - \frac{d^{(12)}}{12}\right)^{12}.$

Raising both sides by 1/12.

$(1 - 0.1288889)^{1/12} = 1 - \frac{d^{12}}{12}.$

$d^{12} = 0.137195439$

$d^{12} = 13.7195439.$

$i^{(2)} = ?$

$$\left(1 + \frac{i^{2}}{12}\right)^{2} = \frac{1}{1-d}$$

Square Rooting both sides.

$$1 + \frac{i^{2}}{2} = \left(\frac{1}{1-0.12889}\right)^{1/12}$$

$$\frac{i^{2}}{2} = 0.07142857143$$

$$i^{2} = 0.1428571429$$

$$i^{(2)} = 14.28571429\%$$

$$\delta = \ln(1+i)$$

$$\delta = \ln\left(\frac{1}{1-d}\right)$$

$$\delta = \ln\left(\frac{1}{1-0.128889}\right)$$

$$\delta = 0.1379857 43$$

$$\delta = 13.7985743\%$$

n = 17 months.

9.) 20,000 26000

i) $AV = PV\left(1+\frac{i^4}{4}\right)^4$

$26000 = 20000\left(1+\frac{i^4}{4}\right)^{\frac{1}{\cancel{4}}\times\frac{17}{\cancel{12}_3}}$

$\frac{26000}{20000} = \left(1+\frac{i^4}{4}\right)^{17/13}$

Raising both sides by 3/7

$\left(\frac{13}{10}\right)^{3/17} = 1+\frac{i^{(4)}}{4}.$

$i^4 = 0.18955525456.$

$i^4 = 18.955525456\%.$

ii) $\left(1-\frac{d^2}{2}\right)^2 = \left(\frac{1}{1+\frac{i^4}{4}}\right)^4$

Raising both sides by 1/2 2,

$\left(1-\frac{d^2}{2}\right) = \left(\frac{1}{1.047388136}\right)$

$\frac{d^2}{2} = 0.0884417655.$

$d^2 = 0.176882353\,1$

$d^2 = 17.68823531\%.$

iii) $26000 = 20000(1+i)$

$\frac{26000}{20000} = 1+\frac{17}{12}i$

$0.3 = \frac{17i}{12.}$

$i = 0.211764 7059$

$i = 21.17647059\%.$

iv) Interest rates keep getting lower as & when the frequency get highe shaps the lowest numerical value & is the highest.

The relationship between i^{p} and p can be represented as:

$$i^{p} = p[(1+i)^{1/p} - 9]$$

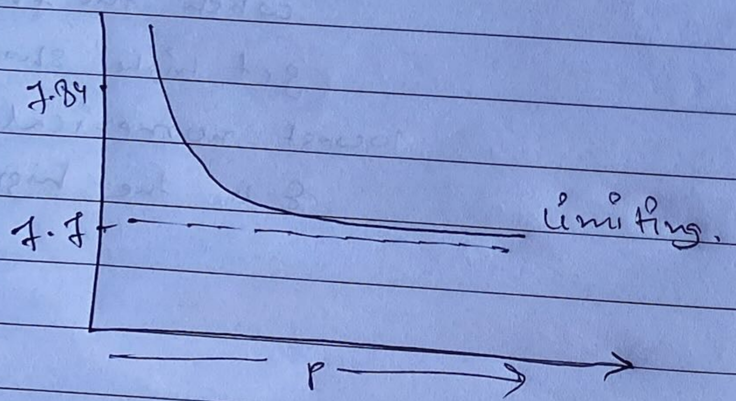
$i = 8\%$.

$p = 1, 2, 3, 4, 5, 10, 50. 365$

p	$i^{(p)}$
1	0.08
2	0.7846096908
3	0.07795670402.
4	0.07770618763
5	0.07785639199
10	0.07725795243
50	0.07702030156
365	0.07696915541

As p increases, the value of i decreases. The change is as significant, in going weekly, monthly daily there is as limit to compounding. The limiting case is when $p \to \infty$, there is isntataneous change and it keeps compounding this delta (δ)

↳ limiting interest rate where. $p \to \infty$



9.) Define force of interest.

i.) Force of interest - Force of interest can be expressed as $\delta(t)$. there is a limiting factor to $i^{(p)}$ that limiting factor is side 4a. In the limiting interest rate where 'p' tends to infinity $\lim_{p \to \infty} i^{(p)} = \delta$

ii.) $i^2 = 0.0775$

$$\left(1 + \frac{i^2}{2}\right) = 1 + i$$

$$\left(\frac{1 + 0.0775}{2}\right)^2 = 1 + i$$

$1 = 0.79001563 = 1 + i.$

$i = 0.079001562.$

$i = 7.9001563\%$

$$\delta = \ln\left(1 + \frac{i^2}{2}\right)^2$$

$$\delta = 2\ln\left(\frac{1 + 0.0775}{2.}\right)$$

$$\delta = 2\ln(1.03875)$$

$\delta = 0.07602613438.$

$\delta = 7.60313438\%.$

$d^{(12)} = ?$

$$\left(1 + \frac{d^{12}}{12}\right)^{12} = \frac{1}{\left(i + \frac{i^2}{2}\right)^2.}$$

Raising both sides by 1/12.

$$\left(1 - \frac{d^{12}}{12}\right) = \left(\frac{1}{1.03878}\right)^{1/12}$$

$$1 - \frac{d^{12}}{12} = 0.9936836878.$$

$d^{12} = 0.0757952408.$

$d^{12} = 7.5795478\%$

$i^{(1/12)} = ?$

$$\left(1 + \frac{i^2}{2}\right)^2 = \left(1 + \frac{i^{1/2}}{1/2}\right)^{1/2}$$

Raising both sides by 2

$$\left(\frac{1 + 0.0775}{2}\right)^7 = \left(1 + \frac{i^{1/2}}{1/2}\right)$$

$$0.1642443719 = \frac{i^{(1/2)}}{1/2.}$$

$i^{(1/2)} = 0.05212218594$

$i^{(1/2)} = 8.212218594$

Q9.A.) $\delta(t) = \begin{cases} 0.08 & \text{for } 0 \leq t \leq 5 . \\ 0.06 & \text{for } 5 \leq t \leq 10 . \\ 0.04 & \text{for } t \geq 10 . \end{cases}$

$$v(t) = e^{-\delta} \text{ or } \frac{1}{A(t)}$$

$$= \frac{1}{A(0,5) \times A(5,10) \times A(0,t)}$$

$$= e^{-\int_0^5 0.05 dt} \times e^{-\int_5^{10} 0.06 dt} \times e^{\int_{10}^{t} 0.04 dt}$$

$$= e^{-(-0.08t)_0^5} \times e^{-(0.06)_5^{10}} \times e^{-(0.04)_{10}^{t}}$$

$$= e^{-(0.08(5)} \times e^{-(0.66(10 - 0.06(5))} \times e^{-(0.04 + -}$$

$$= e^{-0.4} \times e^{-0.3} \times e^{-0.04 + 0.4}$$

$$= e^{-(0.3 + 0.04t)}$$

$$v(t) = e^{-(0.3 + 0.04t)}$$

10.) 2 $a=18\%$. 50,000.

0 $= 9$ months

$$PV = AV\,(v(t))$$

$$PV = C\,(1-d)^n$$

$$= 50{,}000\,(1-0.18)^{9/12}$$

$$= 50{,}000\,(0.82)$$

$$PV = 43085.42623$$

$\delta = 6\%$. $d^{(12)} = ?$

$$\delta = \ln\left(\frac{1}{1-d}\right)$$

$$0.06 = \ln\left(\frac{1}{1-\frac{d^{12}}{12}}\right)^{12}.$$

$$e^{0.06} = \left(\frac{1}{1-\frac{d^{(12)}}{12}}\right)^{12}.$$

Raising both sides by 1/12.

$$1-\frac{d^{12}}{12} = 7/\left(e^{0.06}\right)^{1/12.}$$

$$\frac{d^{12}}{12} = 0.0094987620807$$

$$d^{12} = 0.05985024969$$

$$d^{12} = 5.985024969\%.$$

$\delta(ii) = d^{(4)} = 6\% = 0.06$

$i^{(12)} = ?$

$$(1+i) = \frac{1}{1-d}.$$

$$\left(1+\frac{i^{(12)}}{12}\right)^{12} = \left(\frac{1}{1-\frac{d^{(4)}}{4}}\right)^{4}.$$

Raising both sides by 1/12

$$1+\frac{i^{12}}{12} = \left(\frac{1}{1-\frac{0.06}{4}}\right)^{1/3}.$$

$$i^{(12)} = 0.0050500721$$

$$i^{12} = 0.06060708865$$

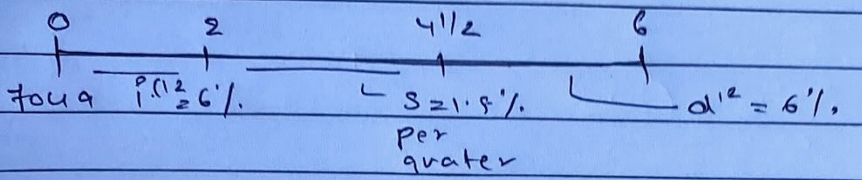
$$i^{12} = 6.060708865\%.$$

c) i, d, i^{4}, δ.

$d, \delta < i^{(4)} < i$.

The order is purely based on the payment of interest d is payable at the begning of the year, immediate payment. Hence the numeric percentage would be less smaller than i which is interest payable at end. Similarly δ and i^{4} is also less than i as δ(force of interest is at exat due to force of interest is nominal interest convertible quaterly.

12.)



Accumulated value = $7049 \times A(0,2) \times A(2,4\frac{1}{2}) \times A(4\frac{1}{2},6)$

$$= 7049\left(1+\frac{i^{(12)}}{12}\right)^{24} \times (1+10\times i) \times \left(\frac{1}{1-\frac{d^{(12)}}{12}}\right.$$

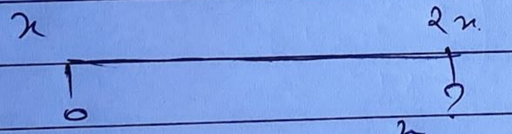
$$= 7049 \times \left(1+\frac{0.06}{12}\right)^{24} \times (1+0.15) \times \left(\frac{1}{1-\frac{0.06}{12}}\right.$$

$$= 7049 \times (1.127159776) \times (1.15) \times 1.0904421325$$

AV = 9999.893613.

13.) let the initial investment be x Rs.

AV = $2(x) = 2x$.



$i = 10\%$

$2x = x(1+i)^n$

$2 =$ ~~(1+ (0.~~ $(1+0.01)^n$.

$2 = (1.01)^n$

taking log on both sides

$n(\ln(1.01)) = \ln 2$

$n = 7.2725409402$ years.

ii $d^{(12)} = 10\%$

$$2x = 2x(1-d)^{n}$$

$$x = 2x\left(1 - \frac{d^{12}}{12}\right)^{12n}$$

$$\frac{1}{2} = \left(1 - \frac{d^{12}}{12}\right)^{12n}$$

taking log on both sides

$$12n \ln\left(1 - \frac{0.1}{12}\right) = \ln 1/2$$

$$12n \ln 0.99166667 = \ln 1/2$$

$$12n = 82.83060475$$

$n = 6.902580392$ years.