Numerical Methods & Algebra

1) $\sqrt{7\sqrt{7\sqrt{7\ldots}}}$

$= 7^{1/2} \cdot 7^{1/4} \cdot 7^{1/8} \cdot 7^{1/16} \ldots$

$= 7^{\left(1/2 + \frac{1}{4} + \frac{1}{8} + \ldots\right)}$

$\downarrow$

$1/2 + 1/4 + 1/8 + 1/16 + \ldots$

(Infinite G.P)

Sum of an infinite G.P $= \frac{a}{1-r}$

$= \frac{1/2}{1-1/2}$

$= 1$

$\therefore 7^{(1/2+1/4+1/8+\ldots)} = 7^{(1)}$

$= \underline{7}$

2) $7x^2 - 6y^2 = -11x^2 - 4y^2$

$18x^2 = 2y^2$

$\frac{x^2}{y^2} = \frac{2}{18} \Rightarrow \frac{x}{y} = \sqrt{\frac{1}{9}}$

3. $x+y=4$ —(i)

$x^2+y^2-4y=12$ —(ii)

$x=4-y$ [from (i) in (ii)]

$(4-y)^2+y^2-4y=12$

$16+y^2+y^2-8y-4y=12$

$16+2y^2-12y=12$

$2y^2-12y+4=0$

$y^2+6y+2=0$

$$y=\frac{6\pm\sqrt{36-8}}{2}$$

$$=\frac{6\pm\sqrt{28}}{2}$$

$$=3\pm\sqrt{7}$$

$x=4-y$

$x=4-3+\sqrt{7}$ or $4-3-\sqrt{7}$

$x=1+\sqrt{7}$ or $1-\sqrt{7}$

the ordered pairs are

$(1+\sqrt{7}, 3-\sqrt{7})$ and $(1-\sqrt{7}, 3+\sqrt{7})$

4. $d = 2r^2 - 16r + 34$ (Density)

(i) $r = 3.5$
$d = 2.5$ million particles/foot^3

(ii) $d = 2r^2 - 16 + 34 = 0$

On differentiation,

$4r - 16 = 0$ —(i)
$r = 4$ (400 RPM)

differentiation of (i) [Second Order]

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Hence minima at $r = 4$
$\therefore$ minimum value →

$d = 2(4)^2 - 16_{(4)} + 34 = 0$

$d = -32 + 34$

$d = 2$

(iii) $d = 100$

$$100 = 2r^2 - 16r + 34$$
$$50 = r^2 - 8r + 17$$

$$r^2 - 8r - 33 = 0$$
$$(r-11)(r+3) = 0$$

Since r cannot be negative,

$r = 11$.

Hence, the speed should be 11 RPM.

5. Let x be the number of km(s) travelled.

$\therefore$ (i) $C_1 = 1500 + 25(x)$

(ii) $C_2 = 2100 + 22(x)$

(iii) $C_1 > C_2$

$$1500 + 25x > 2100 + 22x$$

(iv) $3x > 600$
$x > 200$

Hence, when distance > 200 km, C_2 is cheaper.

6. $x^3 - 7x^2 + 14x - 6 = 0$

(a) $a = 0, \quad b = 1$

a	b	a+b/2	f(a)	f(b)	$f\left(\frac{a+b}{2}\right)$
0	1	0.5	-6	2	-0.625
0.5	1	0.75	-0.625	2	0.9844
0.5	0.75	0.625	-0.625	0.9844	0.2598

$x = 0.625$

(b) $a = 1, \quad b = 3.2$

a	b	a+b/2	f(a)	f(b)	$f\left(\frac{a+b}{2}\right)$
1	3.2	2.1	2	-0.112	1.791
2.1	3.2	2.65	1.791	-0.112	0.5521
2.65	3.2	2.925	0.5521	-0.112	0.08582

$x = 2.925$

(c) $a = 3.2, \quad b = 4$

a	b	$\frac{a+b}{2}$	f(a)	f(b)	$f\left(\frac{a+b}{2}\right)$
3.2	4	3.6	-0.112	2	0.336
3.2	3.6	3.4	-0.112	0.336	-0.016
3.4	3.6	3.5	-0.016	0.336	0.125
3.4	3.5	3.45	-0.016	0.125	0.046125
3.4	3.45	3.425	-0.016	0.0461	0.0130

$x = 3.425$

7. $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$

$f'(x) = 920x^3 + 54x^2 + 18x - 221$

(a)

i	x_i	$f(x)$	$f'(x)$	$x_{i+1} = x_i - \frac{f(x)}{f'(x)}$
1	-1	156.1398	-1105	-0.608121
2	-0.6081	43.994	-418.898	-0.2354
3	-0.23541	1.534	-234.246	-0.04759
4	-0.04759	0.0004	-221.704	-0.04066
5	-0.04066	$\cong 0$	-221.70443	-0.04066
6	-0.04066	$\cong 0$	-221.70443	-0.04066

$x = -0.04060$

(b)

i	x_i	$f(x_i)$	$f'(x_i)$	$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$
1	0.9	-36.585	509.62	0.97179
2	0.97178	6.37668	691.80	0.9625
3	0.9625	0.11523	666.87	0.9624
	0.9624	0.3267	666.411	0.9624
	0.9624			

$x = 0.9624$

8. a Average $= \frac{70+86+81+83+x}{5}$

$= \frac{320+x}{5}$

b Average = B

$\therefore 80 < \frac{320+x}{5} < 90$

c $80 < \frac{320+x}{5} < 90$

$400 < 320 + x < 450$

$\boxed{80 < x < 130}$

9. $x_1 = 0.04$ $x_2 = 0.2$ $x = 0$

$f(x_1) = 8.54$ $f(x_2) = -11.81$ $f(x) = 2.$

$\frac{x - x_1}{x_2 - x_1} = \frac{f(x) - f(x_1)}{f(x_2) - f(x_1)}$

$\frac{-0.04}{0.16} = \frac{f(x) - 8.54}{-20.35}$

$f(x) = 13.6275$

10. $R(x) = 32x - 0.21x^2$

$C(x) = 195 + 12x$

for profit, $R(x) > C(x)$

$\therefore 32x - 0.21x^2 > 195 + 12x$

$20x - 195 - 0.21x^2 > 0$

$0.21x^2 - 20x + 195 < 0$

$0.21x^2 - 20x + 195 = 0$

$$x = \frac{20 \pm \sqrt{212.8}}{0.42}$$

$x = 82.38$ or 12.88

$(x - 82.38)(x - 12.88) < 0$

$x \in (12.88, 82.38)$