Calculus

Ans-1 $\int_{1/50}^{2} \frac{e^{2/w}}{w^2} dw$

[Using $t = e^{2/w}$

$\therefore \frac{dt}{dx} \frac{-2e^{2/w}}{w^2}$]

$= \int_{1/50}^{2} \frac{tw^2}{-2w^2 e^{2/w}} dt$

$= \frac{-1}{2} \int_{1/50}^{2} dt$

$= \frac{-1}{2} [\cancel{t}]_{1/50}^{2}$

$= \frac{-1}{2} [2 - 0.02\cancel{8}]$

$= -0.99$

Ans-2 $\int \frac{2t^3+1}{(t^4+2t)^3} dt$

Using $x = (t^4+2t)^3$

$\frac{dx}{dt} = 3(t^4+2t)^2(4t^3+2)$

$\Rightarrow \int \frac{2t^3+1}{x(3)(t^4+2t)^2} \frac{\cancel{dx}}{(4t^3+2)} dx$

Multplying and dividing by 2

$\Rightarrow \int \frac{4t^3+2}{6x(4t^3+2)} x^6$

$\Rightarrow \frac{1}{6} \int x^{-7} \; dx$

$\Rightarrow \frac{1}{6} \times \frac{x^{6}}{6} + C$

$= \frac{x^6}{36} + C$

$= \frac{(t^4+2t)^{18}}{36} + C$

Ans-3 $f'(x) = x^4 - 3x - 9$

$f(x) = \int f'(x) \; dx$

$= \int x^4 - 3x - 9 \; dx$

$= \frac{x^5}{5} - \frac{3x^2}{2} - 9x + C$

Ans-4 $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$

$\int_{-2}^{3} f(x) \, dx = \int_{-2}^{1} f(x) \, dx + \int_{1}^{3} f(x) \, dx$

$= \int_{-2}^{1} 3x^2 \, dx + \int_{1}^{3} 6 \, dx$

$= [x^3]_{-2}^{1} + [6x]_{1}^{3} = \underline{\underline{21}}$

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Ans-5 $\int 3(8y-1)e^{4y^2-y}\,dy$

Using $x = e^{4y^2-y}$

$\frac{dx}{dy} = (e^{4y^2-y})(8y-1)$

$\int \frac{3(8y-1)(e^{4y^2-y})}{(e^{4y^2}-y)(8y-1)}\,dx$

$\int 3 \quad dx$

$3x + c$

$\Rightarrow 3e^{4y^2-y} + C$

Ans-6 $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f'(x) = \frac{15x^{3/2}}{3/2} + \frac{5x^4}{4} + 6x + k$

$f(x) = 4x^{5/2} + \frac{x^4}{4} + 3x^2 + kx + c$

$f(1) = -5/4$ & $f(4) = 404$

$\therefore -\frac{5}{4} = 4 + \frac{1}{4} + 3 + k + c$

$-\frac{6}{4} - 7 = k + c$

$\boxed{k + c = -\frac{17}{2}}$ —— (1)

$\therefore 404 = 4 \times (4)^{5/2} + \frac{4^4}{4} + 3\,4^2 + 4k + c$

$\boxed{164 = 4k + c}$ —— (2)

Putting $c = -\frac{17}{2} - k$ from eq^n (1) in eq^n (2)

$$164 = 4k - \frac{17}{2} - k$$

$$164 = -3k - \frac{17}{2}$$

$$-3k = \frac{345}{2}$$

$$k = -\frac{345}{6}$$

$$\circ \quad k = -\frac{115}{2} = \underline{\underline{-57.5}}$$

$$\circ \quad C = \underline{\underline{-66}}$$

Ans-15

$$f(x) = e^{-6x} \qquad f(-4) = e^{24}$$

$$f'(x) = e^{-6x}(-6x)' \qquad f'(-4) = 6e^{24}$$

$$f''(x) = e^{-6x}(-6)^2 \qquad f''(-4) = e^{24}(6)^2$$

$$f'''(x) = e^{-6x}(-6)^3 \qquad f'''(-4) = e^{24}(6)^3$$

$$\circ \quad e^{-6x} = e^{24} + \frac{(x-4)^1}{\angle 1}(e^{24})(6) + \frac{(x-4)^2}{\angle 2}(e^{24})(6)^2 + \frac{(x-4)^3}{\angle 3}(e^{24})(6)^3 \ldots$$

Ans-16 $f(x) = \ln(3+4x)$ $\qquad f(0) = \ln(3)$

$f'(x) = \frac{4}{3+4x}$ $\qquad f'(0) = \frac{4}{3}$

$f''(x) = (-4)(4x+3)^{-2}(4)$ $\qquad f''(0) = (-4)(4)(3)^{-2}$

$f'''(x) = (8)(4x+3)^{-3}(4)^2$ $\qquad f'''(0) = (3)^{-3}(8)(4)^2$

$\therefore \ln(3+4x) = \ln(3) + \frac{x}{\angle 1}\frac{4}{3} + \frac{x^2}{\angle 2}(-4)(4)(3)^{-2} + \frac{x^3}{\angle 3}(3)^{-3}(8)(4)^2 \ldots$

Ans-14 $f(x) = \sqrt{1+x}$ $\qquad f(0) = 1$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$ $\qquad f'(0) = \frac{1}{2}$

$f''(x) = -\frac{1}{4}(1+x)^{-3/2}$ $\qquad f''(0) = -\frac{1}{4}$

$f'''(x) = \frac{-3}{8}(1+x)^{-5/2}$ $\qquad f'''(0) = -\frac{3}{8}$

$\therefore \sqrt{1+x} = 1 + \frac{x}{\angle 1}\left(\frac{1}{2}\right) + \frac{x^2}{\angle 2}\left(\frac{-1}{4}\right) + \frac{x^3}{\angle 3}\left(-\frac{3}{8}\right) \ldots$

Ans-13

$f(x) = (1+x)^{\mu-1}$

$f'(x) = \mu(1+x)^{\mu-1}$

$f''(x) = (\mu)(\mu-1)(1+x)^{\mu-2}$

$f'''(x) = (\mu)(\mu-1)(\mu-2)(1+x)^{\mu-3}$

- $f(x) = f(0) + \frac{x}{\angle 1} f'(0) + \frac{x^2}{\angle 2} f''(0) + \frac{x^3}{\angle 3} f'''(0) \ldots$

- $(1+x)^{\mu-1} = 1 + (\mu) + (\mu-1)\mu + (\mu)(\mu-1)(\mu-2) + \ldots$