Financial Engineering Assignment - 2

Roll No - 434

(1) Given →

$S_0 = £65 \qquad \sigma = 25\%$ p.a

$r = 2\%$ pa. $\qquad k = £55$

$T = 6$ months / 0.5 year $\qquad C_0 = ?$

(i) $C_t = S_0 \Phi(d_1) - ke^{-rT} \Phi(d_2)$

$$d_1 = \frac{\ln\left(\frac{S_0}{k}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$\Rightarrow \frac{\ln\left(\frac{65}{55}\right) + \left(2\% + \frac{(20\%)^2}{2}\right)(0.5)}{\sigma\sqrt{0.5}}$$

$$\Rightarrow \boxed{d_1 = 1.08996}$$

$d_2 = d_1 - \sigma\sqrt{T}$

$\boxed{d_2 = 0.91318}$

$\therefore C_t = 65\,\Phi(1.09) - 55e^{-0.02 \times 0.5}\,\Phi(0.91318)$

$\boxed{C_t = 11.9696}$

(ii) $\Delta = \dfrac{\text{change in price of call option}}{\text{Change in price of underlying}}$

(iii) $\Delta_c = \Phi(d_1)$

$\Delta_p = -\Phi(d_1)$

$\Delta_c = 0.86214$

(iv) $\Delta_p = (-0.13786)$

(2) (i) $\Delta = \frac{\text{Change in price of derivative}}{\text{Change in price of underlying asset}}$

rate of change of option price w.r.t change in price of underlying asset.

$\vartheta = \frac{\text{change in price of option}}{\text{change in volatility of underlying asset}}$

rate of change of option price w.r.t change in volatility of underlying asset.

ii) Put call parity →

$$c - p = S - Ke^{-rT}$$

$$\frac{d}{d\sigma}(c-p) = \frac{d}{d\sigma}(S - Ke^{-rT})$$

$V_c - V_p = 0$ (because S & k } are constants.)

$\boxed{\Rightarrow V_c = V_p.}$

$\sigma = 25\%$ p.a

$r = 5\%$

$T = 1$

$$d1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$\Rightarrow \frac{\ln\left(\frac{55}{50}\right) + \left(5\% + \frac{(25\%)^2}{2}\right)}{25\%}$$

$$\boxed{d_1 = 0.70624}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$\boxed{d_2 = 0.4562}$$

$$C_t = S_0 \Phi(d_1) - Ke^{-rT} \Phi(d_2)$$

$$= \cancel{550} \quad 55(0.76115) - 50\, e^{-0.05}(0.67$$

$$\boxed{C_t = \$9.65}$$

$$C - P = S - ke^{-rT}$$

(iv) Delta Hedging → A portfolio for which t overall delta is '0' is described as d neutral/delta hedged. The portfolio is imm to changes in the price of underlying a

Vega Hedging → A portfolio for which th overall vega is equall to '0' is describ as vega hedged or Vega neutral. The portfolio is immune to small changes in assumed level of volatility.

(v) X units of call option
Y units of put option
Z units of forward

	C.F at PV	Δ	ν
Call	C = 9.65	Δ_c	V_c
Put	P = 2.211	Δ_p	V_p
Forward	—	1	0

Δ of forward = 1
ν of forward = 0

Vega Hedging - $xV_c + yV_p + zV_F = 0$

$$\Rightarrow xV_c + yV_p = 0$$

$$\therefore x + y = 0 \quad \text{Or} \quad \boxed{x = -y}$$

<u>Delta Hedging</u> - $x\Delta_c + y\Delta_p + z\Delta_F = 0$

$$\Rightarrow x\Delta_c + y\Delta_p + z = 0$$
(from (i), $(y = -x)$, $\Delta_p = \Delta_c - 1$)

$$\Rightarrow x\Delta_c - x(\Delta_c - 1) + z = 0$$
$$\Rightarrow x + z = 0$$
$$\boxed{(x = -z)} \quad \text{— (ii)}$$

From (i) & (ii),

$$x = -y = -z$$

portfolio = 1000

$$x \cdot C + y \cdot P + z \cdot F = 1000 \quad (F = 0)$$

$$xC + yP = 1000$$
$$x(9.6525) + y(2.2190) = 1000$$
$$\Rightarrow x(9.6525 - 2.2190) = 1000$$

$$x = \frac{1000}{7.4385}$$
$$\Rightarrow \boxed{x = 134.44}$$

$$\boxed{y = z = -134.44}$$

Therefore, Portfolio -

- Long Position on 134 call option.
- Short Position on 134 put option.
- Short Position on 134 ~~fo~~ forward.

(3) (i) $C_t = \mathbb{E}\left(e^{-r(T-t)} C_T / F_t\right)$

$F_t \rightarrow$ Filtration at $t > 0$
$C_T \rightarrow$ Payoff under derivative at T
$C_t \rightarrow$ Derivative value at time 't'.

(ii) $S_0 = £50$
$X = £49$
$r = 5\%$ p.a
$\sigma = 25\%$ p.a
$T = 0.5$

$$C_t = S_0 \Phi(d_1) - Ke^{-rT} \Phi(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$\boxed{d_1 = 0.399}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$\boxed{d_2 = 0.1673}$$

$\Phi d_1 = 0.6346$

$\Phi d_2 = 0.5669$

$$C_T = S_0\,\Phi(d_1) - Ke^{-rT}\,\Phi(d_2)$$

$$= 50 \times 0.6346 - 49 \times e^{-0.05\times 30} \times (0.5669)$$

$$\boxed{C_T = 4.66}$$

(iii) European call $= 4.66$

(iv) $S_t + p = C_t + Ke^{-rT}$

$$p = 4.66 + 49e^{-0.05\times 35} - 50$$

$$\boxed{p = 2.45}$$

v) • If the stock pays a divident then, the price of the underlying would fall. This will result in a fall in price of an european call and a rise in ~~call~~ price of a european put.

• Due to the potential early exercise opportunity, the American call will be placed higher than the European call.

1) $Z_t \sim SBM$ under P

$\gamma_t \rightarrow$ prereversible process

$\therefore$ There exists a measure of equivalent to P where,

$$\tilde{Z}_t = Z_t + \int_0^t \gamma_s \, ds$$

$\tilde{Z}_t \rightarrow$ SBM under Q

If Z_t is a SBM under P & if Q is equivalent to P then there exists a prevorsible process γ_t such that

$$\tilde{Z}_t = Z_t + \int_0^t \gamma_s \, ds$$

$\tilde{Z}_t \rightarrow$ SBM under Q

(ii) The discounted value of asset prices are martingales.

(5) (i) $\boxed{\Delta = \Phi(d_1)}$

(ii) $$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$\Rightarrow 0.3 = \frac{\ln\left(\frac{40}{45.91}\right) + (0.02 + 0.5 \times \sigma^2)5}{\sigma\sqrt{5}}$$

Simplifying,

$2.5\sigma^2 - 0.070835 - 0.0378$

=) $\boxed{\sigma = 32\%}$

(iii) $R_0 = \$30$

$\sigma = \sqrt{15\%}$ p.a

Option payoff = c at time T if

$\frac{S_1}{S_0} > ks, \quad \frac{R_1}{R_0} < R$

$$\text{payoff} = Ce^{-rT} Q\left(\frac{S_1}{S_0} < k_S\right) Q\left(\frac{R_1}{R_0} < k_R\right)$$

(iv) Perfect Corr. →

$$\left(\frac{R_1}{R_0} = \frac{S_1}{S_0}\right)$$

price ⇒ $Ce^{-rT} Q(R_1/R_0 < k_S) = Ce^{-rT} Q(S_1/S_0 < k_S)$

$$\text{Price} = Ce^{-rT} \cdot Q\left(\frac{R_1}{R_0} < \min(k_S, k_R)\right)$$

$$= Ce^{-rT} Q\left(\frac{S_1}{S_0} < \min(k_S, k_R)\right)$$

(v) $$\text{Price} = Ce^{-rT} Q\left(\frac{S_1}{S_0} < k_S\right) Q\left(\frac{R_1}{R_0} < k_R\right)$$

$$= 50e^{-0.02\times 1} Q(S_1 < 32) \cdot Q(R_1 < 18)$$

$$= \$1.61$$

6 (i) (a) $\Delta = -\Phi(-d_1)$
$= \Phi(d_1) - 1$

(b) $\Delta = 0$

$100{,}000\Delta + 24{,}830 = 0$

$\Delta = \dfrac{-24830}{100{,}000}$

$$\boxed{\Delta = -0.2483}$$

(ii) $\Phi(-d_1) = 0.2483$

$1 - \Phi(d_1) = 0.2483$

$\Rightarrow \Phi(d_1) = 0.7517$

$\Rightarrow d_1 = 0.68$

$$0.68 = \frac{\ln\left(\frac{0.64}{0.63}\right) + \left(0.03 + \frac{1}{2}\sigma^2\right)}{\sigma}$$

$$\boxed{\therefore \sigma = 7.1\%}$$

(iii). (a) $Xe^{-rT}\Phi(-d_2) - S_0[\Phi(-d_1)] = 0.2983$

$\Rightarrow 0.63 \times e^{-0.03} \times \left(1 - \Phi(0.609)\right) - 0.68 \times 0.2983$

$\Rightarrow \boxed{6.894}$

(b) Total cash holding

$= 24830 \times 0.69 + 100,000 \times 0.6894$

$= £165,806$

Q7. (i) $\Delta = \frac{df}{ds} = \frac{df}{ds}(t, S_t)$

$f \rightarrow$ derivative
$S \rightarrow$ underlying asset

$\gamma = \frac{d^2 f}{ds^2}$

$V = \frac{df}{d\sigma_s}$

(ii) $X = 50$; $C = 17.91$; $S_0 = 60$
$\sigma = 25\%$ p.a ; $T = 3$ years ; $r = 3\%$ p.a
$V = \$29$

$\boxed{\Delta = 0.801}$

$\Rightarrow 17.91 + 29 \times 0.02$

$\Rightarrow \underline{18.49}.$

Q8 $\Delta = \dfrac{df}{ds}$

(ii) $S_0 = \$100$; $r = 3\%$; $X = 109.42$

$t = 1$; $\Delta = 0.42074$

$\Rightarrow \Phi(d_1) = 0.42074$

$d_1 = (-0.2)$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

$$\Rightarrow (-0.2) = \frac{\ln\left(\frac{100}{109.42}\right) + \left(0.03 + \frac{\sigma^2}{2}\right)}{\sigma}$$

$\Rightarrow -0.2\sigma + 0.09 - 0.03 - \dfrac{\sigma^2}{2} = 0$

$\Rightarrow \dfrac{-\sigma^2}{2} - 0.2\sigma + 0.06 = 0$

$\boxed{\sigma = 0.2}$

Q. 9 (i) PDE for g -

$$\frac{dg}{dt} + (r-q)S + \frac{dg}{dS_T} + \frac{1}{2}\sigma^2 S_t^2 \frac{d^2 g}{dS_t^2} = rg$$

Boundary Condition:

$$D_T = g(T, S_T) = f(S_T)$$

(ii) Price of derivative at time 't' & value of μ

$$D_T = g(t, S_t) = \frac{S_t^n}{S_0^{n-1}} e^{\mu(T-t)} \quad (n>1)$$

Then, $D_T = g(T, S_T)$

$$\Rightarrow f(S_T) = \frac{S_T^n}{S_0^{n-1}}$$

$$\frac{dg}{dt} = -\mu \frac{S_t^n}{S_0^{n-1}} e^{\mu(T-t)} = -\mu g$$

$$\frac{dg}{dS_t} = \frac{n S_t^{n-1}}{S_0^{n-1}} e^{\mu(T-t)} = \frac{n}{S_t} g$$

$\Rightarrow -\mu g + (r-q) S_t \frac{h}{S_t} g + \frac{1}{2}\sigma^2 S_t^2 \frac{h(h-1)}{S_t^2} g = rg$

$\Rightarrow -\mu + (r-q)h + \frac{1}{2}\sigma^2 h (h-1) = r$

$\mu = (r-q)h - r + \frac{1}{2}\sigma^2 h(h-1)$

If $q=0$,

$\mu = rh - r + \frac{1}{2}\sigma^2 h(h-1)$

$= \left(r + \frac{1}{2}\sigma^2 h\right)(h-1)$

Q10 (i) Put call parity for stocks paying dividends

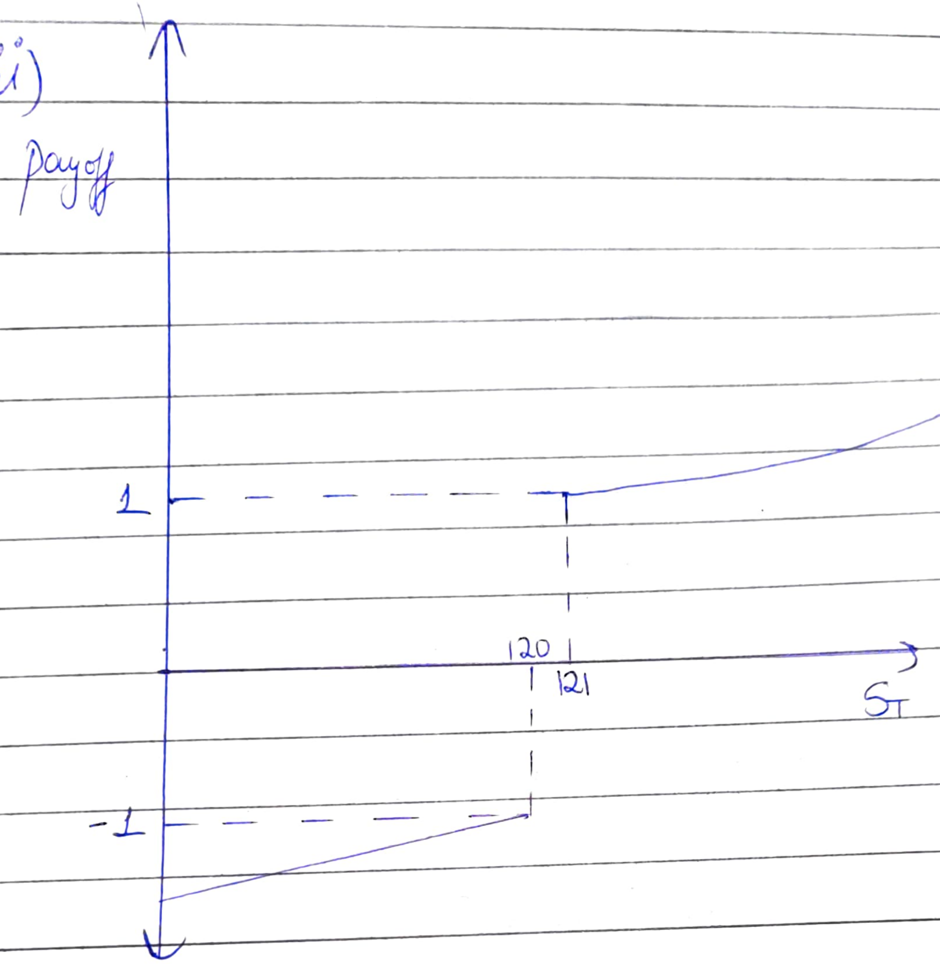
$C_t + Ke^{-r(T-t)} = S_t + P_t$

(ii) $X = 120$; $T = 1$; $C = 10.09$; $r = 2\%$ p.a
$S_0 = 990$

$C = S_0 \Phi(d_1) - Ke^{-rT} \Phi(d_2)$

$\boxed{d_2 = d_1 - \sigma}$

(iii)



(iv) $S_1 - 121 \leq D \leq S_1 - 120$

$\Rightarrow S_0 - 121e^{-r} \leq V \leq S_0 - 120e^{-r}$

Q11 $X = 150$

$r = 2\%$ p.a

$S_0 = 117.98$

Q12. $S_0 = 8$

$X = 9$

$r = 2\%$ p.a

$\sigma = 20\%$ p.a

$T = 3/12$