Financial Mathematics

Assignment-1

1. $i^{(4)} = 8\%$

$$i = \left(1 + \frac{i^{(p)}}{p}\right)^p - 1$$

$$= \left[1 + \frac{8\%}{4}\right]^4 - 1 = 8.2432160\%$$

(i.) $\delta = \ln(1+i) = \ln(1 + 0.08243216 0)$

$= 7.9210509\%$

(ii.) $i = 8.2432160\%$

(iii.) $d^{(p)} = p\left[1 - (1-d)^{1/p}\right]$

$$d = \frac{i}{1+i} = \frac{8.2432160\%}{1 + 8.2432160\%} = 7.6154574\%$$

$$d^{(12)} = 12\left[1 - (1 - 7.6154574\%)^{1/12}\right]$$

$= 7.8949654\%$

2. $\delta(t) = 0.004t + 0.0002t^2$, from $t_1 = 0$ to $t_2 = 10$

$C = 1000 \quad \int_{t_1}^{t_2} \delta(t)\,dt$

(i.) $PV_0 = C \cdot e^{\int_{t_1}^{t_2} 0.004t + 0.0002t^2\,dt}$

$= 1000 \cdot e^{\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]_0^{10}}$

$= 1000 \cdot e^{\left[0.2 + \frac{0.2}{3}\right]}$

$= 1000 \times 1.30560517 2$

$= 1305.605172$

(ii.) $\delta = \ln(1+i)$

$$i = \frac{A(n) - A(n-1)}{A(n-1)} \times \frac{1}{n}$$

$$= \frac{1305.605172 - 1000}{1000} \times \frac{1}{10}$$

$$= 3.0560517\%$$

$$\delta = \ln(1 + 3.0560517\%) = 3.010284569\%$$

③ (i) To prove $\Rightarrow d = i.v$

$$R.H.S. = d = \frac{i}{1+i} = i.\frac{1}{1+i} = i.v = L.H.S.$$

$$\Rightarrow d = i.v$$

(ii.) $d^{(2)} = ?$

(a.) $d^{(4)} = 2.25\%$

$$\frac{d^{(4)}}{4} \quad \cancel{i \neq 1} \quad \cancel{\frac{0.0225}{1+0.0225}}$$

$$d = 1 - \left(1 - \frac{d^{(4)}}{4}\right)^4$$

$$= 1 - \left[1 - 0.0225\right]^4$$

$$= 8.7007806\%$$

$$d^{(p)} = p\left[1 - (1-d)^{1/p}\right]$$

$$d^{(2)} = 2\left[1 - (1 - 8.7007806\%)^{1/2}\right]$$

$$= 8.8987500\%$$

(b.) $d^{(0.5)} = 5\%$

$$d = 1 - \left(1 - \frac{d^{(0.5)}}{0.5}\right)^{0.5}$$

$$= 1 - \left(1 - \frac{0.05}{0.5}\right)^{0.5} = 5.1316702\%$$

$$d^{(p)} = p\left[1 - (1-d)^{1/p}\right]$$

$$d^{(2)} = 2\left[1 - (1 - 5.13167002\%)^{1/2}\right]$$

$$= 5.19925070\%$$

~~(iii) $d = 15\%$, $t = 91/365$~~

④ (i) Same as ③(i)

(ii) $i = 8\% = 0.08$

(a) $d^{(12)} = ?$

$$d = \frac{0.08}{1+0.08} = 7.40740740\%$$

$$d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$$

$$= 7.67147776\%$$

(b.) $$i^{(365)} = 365\left[(1+i)^{1/365} - 1\right]$$

$$= 7.69691155\%$$

(c.) $$\delta = \ln(1+i) = \ln(1.08)$$

$$= 7.69610410\%$$

(d.) $$i^{(0.5)} = 0.5\left[(1+0.08)^{1/0.5} - 1\right]$$

$$= 8.32\%$$

⑤ $d^{(4)} = 6\%$

(a.) $i^{(2)} = ?$

$$d = 1 - \left[1 - \frac{0.06}{4}\right]^{1/4} = 5.86634449\%$$

$$i = \frac{d}{1-d} = 6.2319315\%$$

$$i^{(2)} = 2\left[(1+i)^{1/2} - 1\right] = 6.1377516\%$$

(b.) $$d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$$
$$= 6.030252 5\%$$

(ii.) $PV_0 = 200000$

$AV_1 = 220000$

$$i = \frac{AV_1 - PV_0}{PV_0} = 10\%$$

$$AV_2 = PV_0\, A(0, 93/365)$$

$$= 200000 \times (1+i)^{93/365}$$

$$= 200000 \times (1.1)^{93/365}$$

$$= 204916.3564$$

(7.) $PV_0 = 20000$

$AV_{17} = 26000$

$A(0,17) = \frac{26K}{20K} = 1.3$

$i^{(12)} = 12(\sqrt[17]{1.3} - 1) = 18.66634785\%$

$A(0,17) = (1+i)^{17/12}$

$i = (1.3)^{12/17} - 1$

$i = 20.3457067\%$

(i) $i^{(4)} = 4\left[(1+i)^{1/4} - 1\right]$

$= 18.9552546\%$

(ii) $d^{(12)} = 2\left[1 - (1+i)^{-1/2}\right]$

$= 17.6882358\%$

(iii) $1.3 = 1 + \frac{17}{12} i$

$i = 0.3 \times \frac{12}{17}$

$i = 21.1764706\%$

(9) (i) Force of interest refers to nominal interest rate or discount rate compounded continuously, per time period.

(ii) $i^{(2)} = 7.75\%$

$i = \left(1 + \frac{i^{(2)}}{2}\right)^2 - 1 = 7.9001563\%$

$\delta = \ln(1+i) = 7.6036134\%$

$d^{(12)} = 12\left[1 - (1+i)^{-1/12}\right] = 7.5795747\%$

$$i^{(0.5)} = 0.5\left[(1+i)^{1/0.5} - 1\right]$$
$$= 8.2122186\%$$

(10.) $\delta(t) = 0.08$ for $0 \le t \le 5$
0.06 ; $5 \le t \le 10$
0.04 ; $t \ge 10$

$v(t) = e^{-\delta}$
$= e^{-0.08} = 0.923116346$, $0 \le t \le 5$
$= e^{-0.06} = 0.941764534$, $5 \le t \le 10$
$= e^{-0.04} = 0.960789439$, $t \ge 10$

(11) (a) $AV_9 = 50000$
$d = 18\%$
$PV_0 = 50000\,(1-0.18)^{9/12}$
$= 43085.42623$

(b)

(i) $\delta = 0.06$
$d = 1 - e^{0.06}$
$= 0.05823466$
$d^{(12)} = 12\left[1 - (1-d)^{1/12}\right]$
$= 5.985025\%$

(ii) $d^{(4)} = 6\%$
$d = 1 - \left[1 - \frac{d}{4}\right]^{4}$
$= 5.8663449\%$

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$$i = \frac{d}{1-d} = 0.062319315 = 6.2319315\%$$

$$i^{(12)} = 12\left[(1+i)^{1/12} - 1\right] = 6.0607088\%$$

$$i^{(4)} = 4\left[(1+i)^{1/4} - 1\right] = 6.0918705\%$$

(c.) $i, d, i^{(4)}, \delta$

$$i > i^{(4)} > \delta > d$$

(12) $PV = 7049$

$i^{(12)} = 6\%$

$$AV_2 = 7049\left[1 + \frac{i^{(12)}}{12}\right]^{24}$$

$$= 7945.349262$$

$$AV_{4.5} = 7945.349262\left[1 + 10(0.015)\right]$$

$$= 9137.151652$$

$$AV_6 = 9137.151652\left[1 - \frac{i^{(12)}}{12}\right]^{-18}$$

$$= 9999.899619$$

(13) $PV_0 = 1$

$AV_n = 2$

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(i) $i = 0.1$

$A(0,n) = (1.1)^n$

$AV_n = 1 \times (1.1)^n$

$2 = (1.1)^n$

$\log 2 = n \log(1.1)$

$n = 7.272540897$ years

(ii) $d^{(12)} = 0.1$

$A(0,n) = (0.99)^{-n}$

$2 = 1.(0.99)^{-n}$

$\log 2 = -n \log(0.99)$

$n = 68.96756394$ years