1) i) 4, 7, 9, 9, 11, 15, 18, 18, 18, 25

a) Mean $= \bar{x} = \dfrac{\sum x_i}{N} = \dfrac{4+7+9+9+11+15+18+18+18+25}{10}$

$$= \frac{134}{10}$$

$$\bar{x} = 13.4$$

b) Median $= \left(\frac{N+1}{2}\right)^{th}$ term $= \left(\frac{10+1}{2}\right)^{th}$ term $= 5.5^{th}$ term $= \dfrac{11+15}{2} = \dfrac{26}{2} = 13$

c) Mode = 18

d)

x_i	4	7	9	11	15	18	25	
$\lvert x_i-\bar{x}\rvert$	-9.4	-6.4	-4.4	2.4	1.6	4.6	11.6	
$\sum n\lvert x_i-\bar{x}\rvert$	-9.4	-6.4	8.8	2.4	1.6	13.8	11.6	$\sum n\lvert x_i-\bar{x}\rvert = 54.2$

Standard deviation $= \sigma = \dfrac{\sum n\lvert x_i-\bar{x}\rvert}{N} = \dfrac{54.2}{10}$

$$\sigma = 5.42$$

e) $Q_1 = \left(\frac{N+1}{4}\right)^{th}$ term $= \left(\frac{11}{4}\right)^{th}$ term $= (2.75)^{th}$ term $= \dfrac{(7+9)}{2} = \dfrac{16}{2} = 8$

$Q_3 = 3\left(\frac{N+1}{4}\right)^{th}$ term $= 3(8.25)^{th}$ term $= \dfrac{18+18}{2} = \dfrac{36}{2} = 18$

Inter-Quartile Range $= Q_3 - Q_1 = 18 - 8 = 10$

ii)

Number of households (x_i)	Number of People (f_i)	~~$f_i\lvert x_i-\bar{x}\rvert$~~ $x_i f_i$	cf	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
0	5	0	5	2	10
1	11	11	16	1	11
2	26	52	42	0	0
3	15	45	57	1	15
4	3	12	60	2	6
	$\Sigma f_i = 60$	$\Sigma x_i f_i = 120$			$\Sigma f_i\lvert x_i-\bar{x}\rvert = 42$

a) Mean $= \dfrac{\Sigma x_i f_i}{\Sigma f_i} = \dfrac{120}{60} = \underline{2} = \bar{x}$

b) Median $= \left(\dfrac{61}{2}\right)^{th}$ term $= 30.5^{th}$ term $= \underline{2}$

c) Mode $= \underline{2}$

d) Standard Deviation $= \sigma = \dfrac{\Sigma f_i \lvert x_i-\bar{x}\rvert}{\Sigma f_i} = \dfrac{42}{60} = \underline{0.7}$

e) $Q_1 = \left(\dfrac{61}{4}\right)^{th}$ term $= (15.25)^{th}$ term $= 1$

$Q_3 = 3\left(\dfrac{61}{4}\right)^{th}$ term $= (45.75)^{th}$ term $= 3$

Inter-Quartile Range $= Q_3 - Q_1 = 3-1 = \underline{2}$

2) $X \sim Poi(0.5)$

i) Probability that no claims arise in a single year = $P(x=0)$

$$= \frac{e^{-0.5}\, 0.5^{0}}{0!}$$

$$P(x=0) = \underline{0.95123}$$

ii) Probability of less than one claim in 1 year = $P(x<1) = P(x=0)$

~~$P(x<1) = 0.99899$~~

So, Probability of one or more claims = $P(x \geq 1) = 1 - P(x=0)$

$= 1 - 0.95123 = 0.04877$

Let $P(x \geq 1) = p$ and $P(x=0) = q$

Probability that there is one or more claims in one of the years and no claims in each of the other two years must follow Binomial.

So, $Y \sim Bin(3, 0.04877)$

$$P(Y=1) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

$$= {}^{3}C_{1}\,(0.04877)^{1}\,(0.95123)^{2}$$

$$= 3(0.04877)(0.9048385573)$$

$$P(Y=1) = \underline{0.13239}$$

iii) P(more than 2 years before next claim) = $P(x=0) \cdot P(x=0)$

$= (0.95123)(0.95123)$

P(more than 2 years before next claim) = $\underline{0.90484}$

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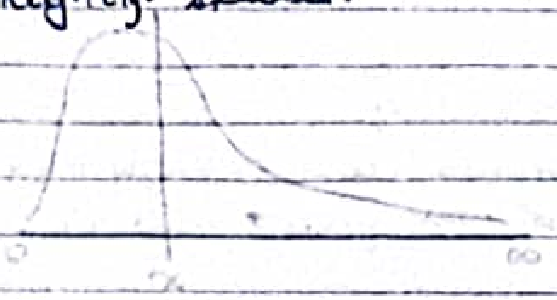
3) $X \sim Gamma(2, 1/2)$, $0 < x < \infty$

i) Mean $= E(X) = \alpha/\lambda = \dfrac{2}{1/2} = \underline{4}$

ii) Variance $= \sigma^2 = V(X) = \alpha/\lambda^2 = \dfrac{2}{(1/2)^2} = 4 \times 2 = \underline{8}$

Standard Deviation $= \sigma = \sqrt{V(X)} = \sqrt{8} = \underline{2\sqrt{2}}$

ii) There is a deviation of $2\sqrt{2}$, so, the shape must be Rightly-skewed.



iii) $$f_X(x) = \frac{\lambda^{\alpha} e^{-\lambda x} x^{\alpha-1}}{\Gamma(\alpha)}$$

$$= \frac{(1/2)^2 e^{-2x} x^{2-1}}{(2-1)!}$$

$$= \frac{(1/4) e^{-2x} x}{1!}$$

$$f_X(x) = \frac{x e^{-2x}}{4} \quad \cancel{\geq 0 \Rightarrow xe^{-2x} \geq 0 \Rightarrow e^{-2x} \geq 0 \Rightarrow -2x \geq \ln 0}$$

$$F_X(x) = \int_0^{\infty} f_X(x)\, dx$$

$$= \frac{1}{4}\int_0^{\infty} x \cdot e^{-2x}\, dx$$

$$= \frac{1}{4}\left[x \int e^{-2x} dx - \int\left(1 \int e^{-2x} dx\right) dx\right]_0^{\infty}$$

$$= \frac{1}{4}\left[x\, e^{-2x}/-2 - \frac{1}{2}\int e^{-2x} dx\right]_0^{\infty}$$

$$= \frac{1}{4}\left[x\, e^{-2x}/-2 + \frac{1}{2}\left(e^{-2x}/-2\right)\right]_0^{\infty}$$

$$F_X(x) = \underline{\frac{-e^{-2x}}{8}\left(x + \frac{1}{2}\right)}, \quad 0 < x < \infty$$

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4) $P(M) = \frac{10}{18} = \frac{5}{9}$, $P(F) = \frac{8}{18} = \frac{4}{9}$,

$\cancel{P(QM) = \frac{6}{10} = \frac{3}{5}}$, $\cancel{P(QF) = \frac{7}{8}}$

Possible selections are FF, FM, MF, MM

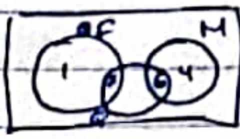
So, Probability of atleast 1 female = FF + FM + MF

$$\cancel{= (\tfrac{4}{9})(\tfrac{3}{8}) + (\tfrac{4}{9})(\tfrac{5}{9}) +}$$

$$= (\tfrac{8}{18})(\tfrac{7}{17}) + (\tfrac{8}{18})(\tfrac{10}{17}) + (\tfrac{10}{18})(\tfrac{8}{17})$$

$$= \frac{56 + 80 + 80}{306}$$

$$= \frac{216}{306}$$



$P(Y)$ $\cancel{P(1F)}$ $\cancel{P(F \geq 1)}$ $= \frac{12}{17}$

Probability of two qualified females given that it contains at least one female $= P(QF/Y)$

$$= \frac{P(QF \cap Y)}{P(Y)}$$

$$= \frac{7/8}{21/17}$$

$$= \frac{7}{8} \times \frac{17}{21}$$

$\cancel{P(QF}$ Ans. $= \frac{17}{24}$

5) $P(D1) = 40\% = 0.4$, $P(D2) = 50\% = 0.5$, $P(D3) = 10\% = 0.1$

Let $P(L)$ be the probability of the package being late

$P(L/D1) = 20\% = 0.2$, $P(L/D2) = 40\% = 0.4$, $P(L/D3) = 10\% = 0.1$

Probability that the ~~po~~ late package was sent using ~~tot~~ level 2 service $= P(D2/L)$

$$= \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{3} P(E_i) \cdot P(A/E_i)} \quad \left[\text{Baye's Theorem}\right]$$

$$= \frac{P(D2) \times P(L/D2)}{P(D1) \times P(L/D1) + P(D2) \times P(L/D2) + P(D3) \times P(L/D3)}$$

$$= \frac{(0.5)(0.4)}{(0.4)(0.2) + (0.5)(0.4) + (0.1)(0.1)}$$

$$= \frac{0.2}{0.08 + 0.2 + 0.01}$$

$$= \frac{0.2}{0.29}$$

$$P(D2/L) = \frac{20}{29}$$

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6) $X \sim U(1,2)$, $Y = \frac{3}{6-x}$

$$Y = \frac{3}{6-x} = G(x)$$

$$6 - x = \frac{3}{Y}$$

$$x = 6 - \frac{3}{Y} = \cancel{G(x)}\ G^{-1}(Y)$$

$$x = \frac{6Y-3}{Y^2}$$

Diff. wrt 'y'

$$\frac{dx}{dy} = \frac{Y(6) - (6Y-3)}{Y^2}$$

$$\frac{dx}{dy} = \frac{3}{Y^2}$$

We know that $f_Y(y) = F'_Y(y)$

$$F_Y(y) = P(Y<y) = P(G(x) < y)$$

$$= P(X < G^{-1}(y))$$

$$F_Y(y) = F_X(G^{-1}(y))$$

$$F'_Y(y) = f_Y(y) = F'_X(G^{-1}(y))$$

$$= \frac{dx}{dy}$$

$$f_Y(y) = \frac{3}{Y^2}$$

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7) ~~ii) $F(3) = P(X \le 3) = 0.05 + 0.15 + 0.35$~~

i) $F(3) = \cancel{0.25}\ \underline{0.35}$

ii) $E(X) = \sum_{i=1}^{5} x \cdot P(X=x)$

$= (1)(0.05) + (2)(0.15) + (3)(0.35) + (4)(0.4) + (5)(0.05)$

$= 0.05 + 0.3 + 1.05 + 1.6 + 0.25$

$E(X) = \underline{3.25}$

iii) $V(X) = E(x^2) - [E(x)]^2$

$= \sum_{i=1}^{5} x^2 \cdot P(X=x) - \left[\sum_{i=1}^{5} x \cdot P(X=x)\right]^2$

$= \left[(1)^2(0.05) + (2)^2(0.15) + (3)^2(0.35) + (4)^2(0.4) + (5)^2(0.05)\right]$

$- (3.25)^2$

$= 0.05 + 0.6 + 3.15 + 6.4 + 1.25 - 10.5625$

$V(X) = \underline{0.8875}$

iv) Skewness $(X) = E(x_i - \bar{x})^3$

~~$= E[(x_i - \bar{x})^2 \times P(x_i - \bar{x})]$~~

$= \sum_{i=1}^{5} (x_i - \bar{x})^3 \cdot P(X=x)$

$= \sum_{i=1}^{5} [(x_i - \bar{x})^2 \cdot P(X=x)](x_i - \bar{x})$

$= \sum_{i=1}^{5} V(x) \cdot (x_i - \bar{x})$

$= V(X) \sum_{i=1}^{5} (x_i - \bar{x})$

$= 0.$

Coefficient of skewness $(x) = V(x) = \underline{0.8875}$.

8) p = ~~Proba~~ Claim exceeds 1000 $= 0.2$

$q = 1 - 0.2 = 0.8$

$n = 15$, $r < 3 \Rightarrow r = 2$ [Because no. of claims cannot be decimals]

$P(X<3) = {}^{15}C_3 \quad P(X=2) = {}^{15}C_2\, p^2 q^{15-2} = \frac{15!}{(15-2)!\,2!} \times (0.2)^2 \times (0.8)^{13}$

$= 105 \times 0.04 \times 0.054975581$

$P(X<3) = \underline{\cancel{2.}\,0.2309}$

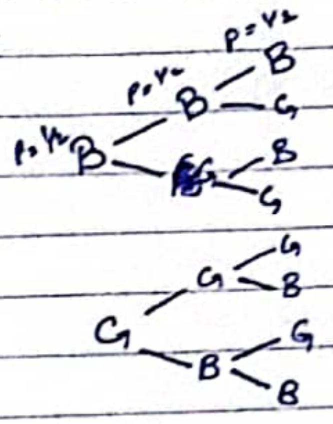
9) p = Probability of winning raffle = 0.01
$q = 1 - 0.01 = 0.99$
$n = 7 \quad , x = 3$

Probability that he wins his third prize in the 7th week = ~~P~~ $P(\cancel{7})(W)$

$$P(W) = {}^{7}C_{3}\, p^{3} q^{7-3}$$
$$= \frac{7!}{(7-3)!\,3!} \times (0.01)^{3} (0.99)^{4}$$
$$= (35)(0.000001)(0.96059601)$$
$$P(W) = \underline{\underline{0.000033621}}$$

10) If the average is of 2 claims in every 5 days, then there cannot be more than 2 claims in one day. The maximum no. of claims in one day is 2. ~~The claims~~.
Hence $P(\text{claims} > 2 \text{ per in one day}) = \underline{0}$.

11) $p =$



So, Probability that next 3 children are Boys $= (1/2)(1/2)(1/2)$
$$\underline{\underline{P(BBB) = 1/8}}$$

12) $X \sim Poi(10)$, 1

$$P(X=0.1) = \frac{e^{-10}\,10^{0.1}}{10!}$$

$$= \frac{(0.0000454)(1.258925412)}{3628800}$$

$$= \frac{0.0000571\ldots}{3628800}$$

$=$

$P(X=10) = 0.00050$

$P(X=0) = 0.000005$

$1 = 0.000045$

$0.1 = y$

$y = (0.1)(0.000045)$

$y = 0.0000045$

$P(X=0.1) = P(X=0) + 0.1$

$= 0.000005 + 0.0000045$

$P(X=0.1) = \underline{0.000095}$

13) $X \sim U(75, 120)$, $P(80 < X < 100) = ?$

$$f_X(x) = \frac{1}{b-a} = \frac{1}{100-80} = \frac{1}{20}$$

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14) $X \sim \text{Gamma}(5, 0.4)$, $\lambda = 0.4, \alpha = 5$

$$P(X < 22.89) = \int_0^{22.89} \frac{\lambda^\alpha e^{-\lambda x} x^{\alpha-1}}{\Gamma(\alpha)} dx$$

$$\left[\begin{array}{l} \text{If } X \sim \text{Gamma}(\alpha, \lambda) \\ \text{then} \quad 2\lambda X \sim \chi^2_{2\alpha = 2} \end{array}\right]$$

$0.8X \sim \chi^2_{10}$

$$P(X < 22.89) = P(0.8X < 18.312)$$
$$= P(\chi^2_{10} < 18.312)$$
$$=$$

$P(\chi^2_{10} < 18.5) = 0.9529$

$\underline{P(\chi^2_{10} < 18.0)} = \underline{0.9450}$

$0.5 = 0.0079$

$1 = y$

$0.5y = 0.0079$

$y = \frac{(0.0079)}{0.5}$

$1y = 0.0158$

$0.312 = y(0.312)$

$z = (0.0158)(0.312)$

$z = 0.0049296$

$$P(\chi^2_{10} < 18.312) = 0.9450 + 0.0049296$$
$$= \underline{0.9499296}$$

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15) $\dfrac{k}{(x+5)^4}$, $x > 0$

i) $\cancel{\dfrac{k}{(x+5)^4} = f_x(x)}$

i) $F'_x(x) = \int f_x(x)\,dx$

$F_x(x) = \int_0^\infty k/(x+5)^4 \,dx$

$1 = k \int_0^\infty (x+5)^{-4}\,dx$

$1 = k \left[\dfrac{(x+5)^{-3}}{-3}\right]_0^\infty$

$1 = k \left[\dfrac{(x+5)^{-3}}{-3}\right]_0^\infty$

$\left(\left[\dfrac{(x+5)^{-3}}{-3}\right]_0^\infty\right)^{-1} = k$

$\left[\dfrac{-3}{(x+5)^{-3}}\right]_0^\infty = k$

i) $F'_x(x) = \int_0^\infty f_x(x)\,dx$

$1 = \int_0^\infty f_x(x)\,dx$ [Because cum. freq cannot be > 1]

Diff. w.r.t. 'x'

$0 = f_x(x)\,dx$

$0 = \dfrac{k}{(x+5)^4}$

$0 = k$

$\underline{\underline{k = 0}}$

ii) $\underline{F(10) = 0}$

iii) $\underline{\underline{E(x) = 0}}$.