PQ-2

Inequalities, Modulus Function, Quadratics

1) $4x+15 < 23$

$4x < 23-15$

$4x < 8$

$x < 2$

2) $x^2 - 3 > 25$

$x^2 > 28$

$x \in (\sqrt{28}, \infty)$ $\quad (\sqrt{28}, \infty)$

3) If $-7 \leq x \leq 6$, $-5 \leq y \leq 8$ Max or Min

+ve -ve

of xy

$xy = -56 \quad (x=-7, y=8)$

$xy = 48 \quad (x=6, y=8)$

) $5y+3 \geq -7y+13$

$12y \geq 10$

$y \geq \frac{10}{12}$

5) $|r| < 1$, $a = 5(1-r)$

$-1 < r < 1$ $\Rightarrow$ $1 > -r > -1$

$a = 5(1-r)$ $\Rightarrow$ $2 > 1-r > 0$

$10 > 5(1-r) > 0$

$10 > a > 0$

$0 < a < 10$

6) $x^2 - 6x + 12$

$b^2 - 4ac = 36 - 48$

$= -12$

- For Real and Distinct $(b^2 - 4ac > 0)$
- For Real and Coinciding $(b^2 - 4a = 0)$
- For Complex $(b^2 - 4ac < 0)$

$\rightarrow$ Complex

8) $ax^2 + bx + c = 0$

where $a \neq 0$

$$\boxed{ax^2 + bx = 0}$$

$ax^2 + c = 0$

$ax^2 = 0$

$ax^2 + bx + c = 0$

b

$a = 0$

a) $x^2 - x^2 + 12x - 8x = 0$

b) Cubic

c) $x^2 + \sqrt{x} - 3 = 0$

$(x^2 - 3)^2 = x$

$x^4 - 6x^2 + 9 - x = 0$

d) ~~$x^2 - 4 = 0$~~ $x^2 - 4 = 0$

9) $(x-1)(x-2) = x(x+1)$ is a linear eqn.

10) Any Quad. Eqn. with roots α, β can be expressed as

$$(x-\alpha)(x-\beta)=0$$
$$(x-1)(x+3)=0$$
$$x^2-x+3x-3=0$$
$$x^2+2x-3=0$$

11) $|2x-5| > |x+1|$

4 cases

→	+	+
→	+	-
→	-	+
→	-	-

(4) $|2x-5| = 2x-5$

$2x-5>0$

$x > \frac{5}{2}$

$|x+1| = x+1$

$x+1>0$

$x > -1$

$2x-5 > x+1$

$x > 6$

Soln $\Rightarrow$ $\boxed{x>6}$

2) $2x-5>0$

$x>\frac{5}{2}$

$x+1<0$

$x<-1$

not possible

3) $2x-5<0$

$x<\frac{5}{2}$

$x+1>0$

$x>-1$

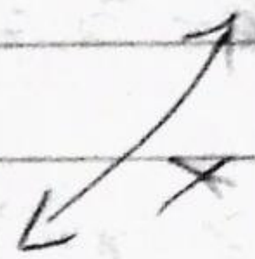
$\boxed{x\in(-1,\frac{5}{2})}$

$-(2x-5)>x+1$

$2x+5+x+1<0$

$\Rightarrow 3x+6<0$

$\boxed{x<-2}$



Do not intersect

4) $2x-5<0$

$x<\frac{5}{2}$

$x+1<0$

$\boxed{x<-1}$

$-(2x-5)<-(x+1)$

$2x-5>x+1$

$\boxed{x>6}$

Do not Intersect

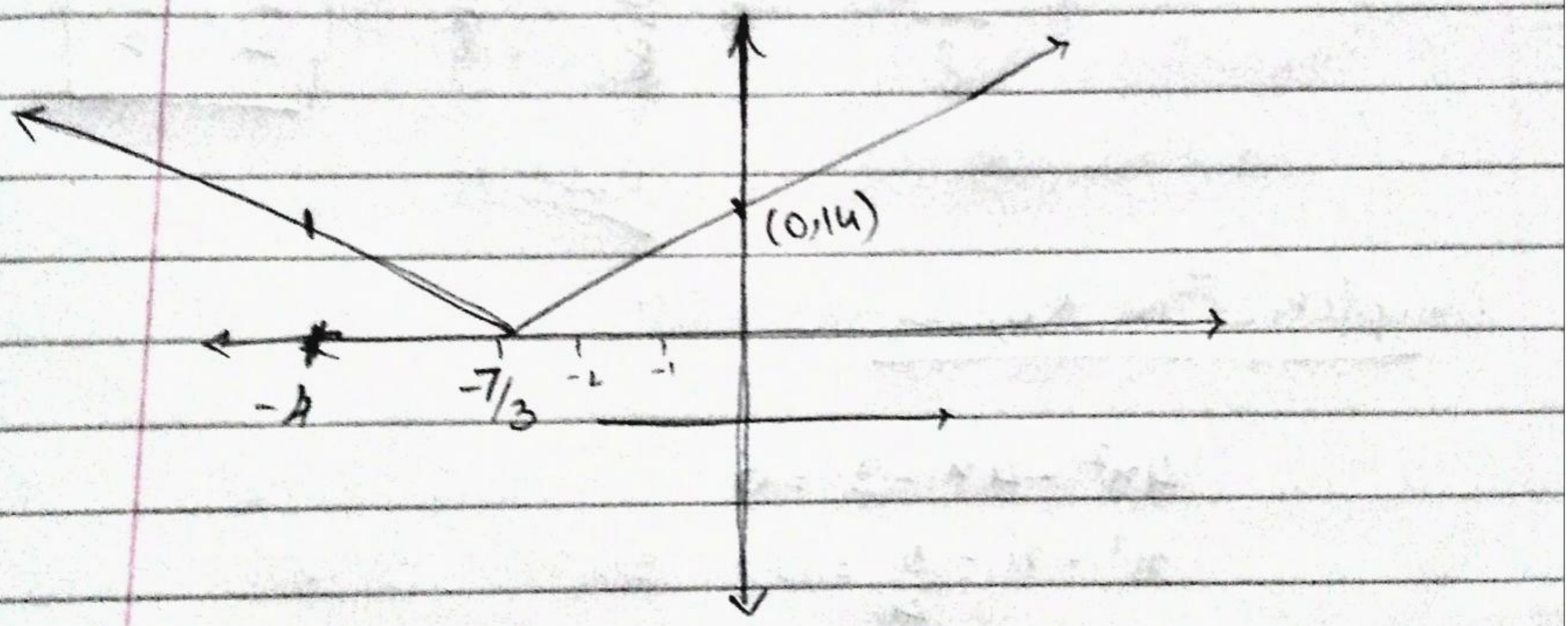
(2)

12) $f(x) = 2|3x+7|$

$f(x) = 2(3x+7) \Rightarrow 3x+7 \geq 0 \Rightarrow \boxed{x \geq -\frac{7}{3}}$

$f(x) = -2(3x+7) \Rightarrow 3x+7 < 0 \Rightarrow \boxed{x < -\frac{7}{3}}$

$$f(x) = \begin{cases} 6x+14 & x \geq -\frac{7}{3} \\ -6x-14 & x < -\frac{7}{3} \end{cases}$$



13) Factorisation

$4x^2 - 4x - 3$

$4x^2 - 6x + 2x - 3 = 0$

$2x(2x-3) + (2x-3) = 0$

$(2x+1)(2x-3) = 0$

$\boxed{x = -\frac{1}{2}}, \boxed{x = \frac{3}{2}}$

Formula

$$-\frac{b \pm \sqrt{b^2-4ac}}{2a} = \frac{4 \pm \sqrt{16+48}}{8}$$

$$= \frac{4 \pm 8}{8} = \frac{12}{8}, \frac{-4}{8} = \boxed{\frac{3}{2}, \frac{-1}{2}}$$

Same

Completion of Square

$4x^2 - 4x - 3 = 0$

$x^2 - x - \frac{3}{4} = 0$

$x^2 - 2 \cdot x \cdot \frac{1}{2} - \frac{3}{4} = 0$

$x^2 - 2x \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - \frac{3}{4} = 0$

$$\Rightarrow \left(x-\frac{1}{2}\right)^2 = 1$$

$$\Rightarrow x - \frac{1}{2} = \pm 1$$

$$\Leftrightarrow \boxed{x = \frac{3}{2}, -\frac{1}{2}}$$

14) Let the no. be x & y.

$$xy = 9 \qquad -(1)$$

$$x - y = 6$$

$$x = y + 6$$

$$y(y+6) = 91$$

$$y^2 + 6y - 91 = 0$$

$$y^2 + 13y - 7y - 91 = 0$$

$$(y-7)(y+13) = 0$$

$$y = 7, 13$$

$$x = 13, -7$$

15) $x^2 - 6x - 16 \leq 0$

$x^2 - 6x + 9 - 9 - 16 \leq 0$

$(x-3)^2 - 25 \leq 0$

$(x-3)^2 \leq 25$

$-5 \leq x-3 \leq 5$

$\boxed{-2 \leq x \leq 8}$

16) $x^4 + 2x^2 - 3 = 0$, only Real value of x.

$x^2 = t$

$t^2 + 2t - 3 = 0$

$t^2 + 3t - t - 3 = 0$

$(t+3)(t-1) = 0$

$t = -3, t = 1$

$x^2 = -3, 1$

↑

✗

So, $x = \pm 1$ $x = 1$

17)

a) $\frac{1}{x^2+2x-8} > 0$

$\frac{1}{x^2+4x-2x-8} > 0$

$\frac{1}{(x-2)(x+4)} > 0$

Both +ve

$x-2>0$ and $x+4>0$

$x>2$ and $x>-4$ $\Rightarrow x>2$

Both -ve

$x-2<0$ and $x+4<0$

$x<2$ and $x<-4$

$\Rightarrow x<-4$

$x<-4$ or $x>2$