

Assignment-2

Q. 1 i. Loan Amount = 20000
Monthly Payments = 427.9
Time = 5 years

$$20000 = 427.9 (12) a_{\overline{5}|i}^{(12)} @ i\%$$

$$\Rightarrow 20000 = 427.9 (12) \left(\frac{1 - (1+i)^{-5}}{((1+i)^{1/12} - 1)12} \right)$$

$$\text{At } i = 10\%$$

$$\text{R.H.S} = 20341$$

$$i = 11\%$$

$$\text{R.H.S} = 19916$$

$\therefore$ By interpolating, $i = 10.8\%$

$$\therefore \text{APR} = 10.8\%$$

ii. $PV_1 = 427.9 (12) a_{\overline{4}|i}^{(12)} @ i = 10.8\%$

$$= 427.9 (12) \left(\frac{1 - (1.108)^{-4}}{((1.108)^{1/12} - 1)12} \right)$$

$$= 427.9 (41.18260914) = 17622.03895$$

$$= 427.9 (39.205369) = 16775.97728$$

$$16775.97728 = 274.49 (12) a_{\overline{t}|i}^{(12)} @ 10.8\%$$

$$\frac{16775.97728}{274.49} = \frac{(1 - (1.108)^{-t})}{((1.108)^{1/12} - 1)12} (12)$$

$$1 - (1.108)^{-t} = 0.52456676$$

$$-t = -2.499 - 7.2499$$

$$t = 2.499 + 7.2499$$

$$\text{iii. Total interest of initial loan} \Rightarrow 427.9 \times 60 = 25674$$

$$= 25674.3763.22$$

$$\text{Total interest of restricted loan} \Rightarrow 7.2499 \times 274.49 \times 12$$

$$= 16775.97728$$

$$= 7104.3233$$

$\therefore$ They will pay 3341.43 more interest.

Q.2 i. b. Discounted Payback Period

- Payback period is the number of years necessary to recover funds invested in the project without considering the interest, both on the borrowing and investing.

a. Discounted Payback Period

- Discounted Payback Period is the number of years after which the cumulative discounted cash inflows cover the initial investment.

ii. Unlike the NPV, neither the DPP nor the PP give any indication of how profitable a project is, as they ignore cash flows after the accumulated value of zero is reached.

The PP can give misleading results, as it does not take into account the time value of money.

Hence, NPV is a superior measure as compared to DPP or PP.

iii a. Project A

$$PV \text{ of outlay} = -170 - 20V - 10V^2$$

$$PV \text{ of inflay} = 20V + 20V^2 + 200V^3$$

$$NPV = -170 - 20V - 10V^2 + 20V + 20V^2 + 200V^3$$

$$0 = -170 + 10V^2 + 200V^3$$

$$IRR \Rightarrow 0 = -170 + 10V^2 + 200V^3$$

$$170 = 10(1+i)^{-2} + 200(1+i)^{-3}$$

$$\text{At } i = 0.07, \text{ LHS} = 171.99$$

$$i = 0.08, \text{ LHS} = 167.33$$

$\therefore$ By interpolating, $i = 7.4\%$

Project B

$$PV \text{ of outlay} = -200$$

$$PV \text{ of inflay} = 14a_{\overline{6}|i} + 200V^6$$

$$NPV = -200 + 14 \left(\frac{1 - (1+i)^{-6}}{i} \right) + 200(1+i)^{-6}$$

$$IRR \Rightarrow 0 = -200 + 14 \left(\frac{1 - (1+i)^{-6}}{i} \right) + 200(1+i)^{-6}$$

$$200 = 14 \left(\frac{1 - (1+i)^{-6}}{i} \right) + 200(1+i)^{-6}$$

$$\text{At } i = 0.07, \text{ LHS} \sim 200$$

$$i = 0.08, \text{ LHS} =$$

By interpolating

 $i = 7\%$

b. $NPV_A = -170 + 10v^2 + 200v^3$

At $d = 6\%$

$$NPV_A = -170 + 10(1-0.06)^2 + 200(1-0.06)^3$$

$$NPV_A = \underline{4.9528}$$

$$NPV_B = -200 + 14a_6 + 200v^6$$

$$= -200 + 14 \left(\frac{1 - (0.94)^6}{0.06} \right) + 200(0.94)^6$$

$$NPV_B = \underline{5.99585091}$$

c. For Project A, the interest rate should be below 7.4%, if above this then the project is unprofitable.

For Project B, the interest rate should be below 7%, if above this then the project is unprofitable.

Other factors:

- Project A (No net income during FY) - Comparison time period
- lower IRR for Project B for a longer period of both projects.

Q.3

$$\text{PV of annuity 1} = 200 \ddot{a}_{\overline{22}|}^{(3/12)}$$

$$= (1.08)^{-1/4} (200) (10.2007) \frac{(0.08)}{(0.074074)}$$

$$= 2161.365534$$

$$\text{PV of annuity 2} = 320 \ddot{a}_{\overline{16.25}|}^{(4)} \sqrt{1/2}$$

$$= 320 \left(\frac{1 - (1.08)^{-16.25}}{0.08} \right) (1.049519) (1.08)^{-1/8}$$

$$= 2996.048870$$

$$\text{PV of annuity 3} = \left(15 \ddot{a}_{\overline{226}|} @ d_{\overline{12}|}^{(12)} = 0.006392917 \right)$$

$$= 15 \left(\frac{1 - (1.08)^{-226/12}}{0.006392917} \right)$$

$$= 1795.651419$$

$$\text{Total PV} = 6953.065823$$

$$\text{Revised annuity} = 6953.065823 = X \ddot{a}_{\overline{295}|}^{(2)} @ d_{\overline{2}|}^{(2)}$$

$$6953.065823 = X \left(\frac{1 - (1.08)^{-21.5}}{0.08} \right) (1.059615)$$

$$X = 649.0135657$$

Q.4

To Derive : $(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}$

Proof:

$$PV = 1 + 2v + 3v^2 + \dots + nv^{n-1} \quad \text{--- (1)}$$

$$(1-d)(PV) = v + 2v^2 + 3v^3 + \dots + (n-1)v^{n-1} + nv^n \quad \text{--- (2)}$$

$$\text{①} - \text{②}$$

$$PV - (1-d)PV = 1 + v + v^2 + v^3 + \dots + v^{n-1} - nv^n$$

$$d(PV) = \ddot{a}_{\overline{n}|} - nv^n$$

$$\boxed{(I\ddot{a})_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{d}}$$

Q.5

For Property Fund

$$1+i = \frac{13.1}{12.4} \times \frac{14.8}{13.1} \times \frac{15.8}{14.8} \times \frac{16.4}{15.8}$$

$$TWR = i = \underline{32.2580645\%}$$

For Equity fund

$$1+i = \frac{9.2}{12.1} \times \frac{10.3}{9.2} \times \frac{13.1}{10.3} \times \frac{15.5}{13.1}$$

$$TWR = i = \underline{28.0991736\%}$$

ii.

$$a. \quad 12.4(1+i) + 13.1(1+i)^{3/4} + 14.8(1+i)^{1/2} + 15.8(1+i)^{1/4} = 4(16.4)$$

$$\underline{i = 29.32\%}$$

iii.

$$a. \quad (12.1)(1+i) + 9.2(1+i)^{3/4} + 10.3(1+i)^{1/2} + 13.1(1+i)^{1/4} \times 15.5$$

$$\underline{i = 67.31\%}$$

Q.6 i. $160000 = X a_{\overline{10}|8\%}$

$$X = \frac{160000}{6.7101} = \underline{23844.65201}$$

$$ii. \quad PV_4 = 23844.65201 a_{\overline{7}|8\%}$$

$$= 23844.65201(4.6229)$$

$$= 110231.4422$$

$$110231.4422 = X' a_{\overline{7}|10\%}$$

$$X' = \frac{110231.4422}{4.8553} = \underline{25309.72428}$$

$$iii. \quad PV_7 = 25309.72428 a_{\overline{3}|10\%}$$

$$= 25309.72428(2.4869)$$

$$= \underline{62942.75331}$$

$$62942.75331 = X'' a_{\overline{31}|9\%}$$

$$X'' = \frac{62942.75331}{2.5313} = \underline{24865.78174}$$

Q.8 i. PV of outlay = $2.7 + 0.2a_{\overline{31}|}$

$$PV \text{ of inlay} = \bar{a}_{\overline{10}|} (0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3$$

$$0 = NPV = \bar{a}_{\overline{10}|} (0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3 - 2.7 - 0.2a_{\overline{31}|}$$

$$\therefore 0 = \frac{1 - (v^{10})}{-\ln(v)} (0.25v + 0.2v^2 + 0.1v^3) + 0.1v^3 - 2.7 - 0.2 \left(\frac{1 - v^{31}}{v - 1} \right)$$

$$\underline{i = 9.3\%}$$

ii. $NPV = 0.1v^3 + 0.85\bar{a}_{\overline{10}|} v^3 - 2.7 - 0.2a_{\overline{31}|} @ 10\%$

$$= 0.1(1.1)^{-3} + 0.85(1.1)^{-3} \left(\frac{1 - (1.1)^{-10}}{\ln(1.1)} \right) - 2.7 - 0.2 \left(\frac{1 - (1.1)^{-31}}{0.1} \right)$$

$$= \underline{1.0089}$$

Q.9 i. $(1+i)^3 = \frac{33}{30.5} \times \frac{38.5}{33.6} \times \frac{41.05 - 4.5}{38.5} \times \frac{45}{41.05} \times \frac{45.6}{45} \times \frac{47}{45.6 - 2.5}$

$$(1+i)^3 = 1.228313$$

$$\underline{i = 7.0951\%} = \text{TWRR}$$

$$ii. \quad 30.5(1+i)^3 + 6(1+i)^{2.25} + 4.5(1+i)^{1.5} - 2.5(1+i)^{0.5} = 47$$

$$At \quad i = 7\%, \quad RHS = 46.74$$

$$i = 7.5\%, \quad RHS = 47.37$$

By interpolation, $i = 7.208\% = \text{MWR}$

ii. (linked rate of return

2011 $\Rightarrow$

$$30.5(1+i) + 6(1+i)^{0.25} = 37.5$$

$$i = \underline{6.266\%}$$

2012 $\Rightarrow$

$$37.5(1+i) + 4.5(1+i)^{0.5} = 45$$

$$i = \underline{4.92\%}$$

2013 $\Rightarrow$

$$45(1+i) + 2.5(1+i)^{0.5} = 47$$

$$i = \underline{10.276\%}$$

$$(1+i)^3 = 1.06266 \times 1.0492 \times 1.10276$$

$$i = \underline{7.13\%}$$

iii. TWRR eliminates the effect of the cashflow amount and timing, and therefore gives a fairer basis for assessing the investment performance for the fund.

MWR is sensitive to the amount and timing of the net cashflows.

Q.10 i. $PV = 180000 a_{\overline{15}|} + 20000 (Ia)_{\overline{15}|} @ 12\%$

$$= 180000 (6.8109) + 20000 (40.7313)$$

$$= \underline{2040588}$$

$\therefore$ Total cost of house = $\frac{2040588}{0.8} = 2550735$

ii. $PV_9 = 340000 a_{\overline{7}|} + 20000 (Ia)_{\overline{7}|} @ 12\%$

$$= 340000 (4.5638) + 20000 \left(\frac{5.1114 - 7 \times 0.45235}{0.12} \right)$$

$$= 1875850.33$$

Year	Loan	Payment	Interest	Capital Repayment
9	1875850.33	360000	225102.04	134897.96
10	1740952.37	380000	208914.28	171085.72
11	1569866.65			

iii. $1569866.65 = X a_{\overline{5}|} @ 10\%$

$$X = \frac{1569866.65}{3.790786} = 414126.87$$

Total installments less loan outstandings at the end of 10th year = 630133.35

$$\text{Extra Payment made} = 5(414126.87) + 0.01(1569866.65) - 1569866.65$$

$$= \underline{516466.35}$$

Q.11 i. $TWRR = (1+i)^2 = \frac{500}{460} \times \frac{550-50}{500+40} \times \frac{600}{550} \times \frac{X}{600}$

$$1.44 = \frac{500}{460} \times \frac{500}{540} \times \frac{X}{550}$$

$$X = \underline{786.9312}$$

ii. $MWRR \Rightarrow 460(1+i)^2 + 40(1+i)^{\frac{21}{12}} - 50(1+i) = 786.9312$

$$i = \underline{30.909\%}$$

Q.12 i. Discounted Payback Period (DPP) is the number of years after which the cumulative discounted cash inflows cover the initial investment.

ii. a. $-800(1+i)^t - 50(1+i)^{t-1} + 100 \frac{1}{1+i} = 0 \quad @ \quad i = 7\%$

$$-800(1.07)^t - 50(1.07)^{t-1} + 100 \left(\frac{(1.07)^{t-2} - 1}{\ln(1.07)} \right) = 0$$

$$\therefore t = \underline{17.767}$$

b. $100 \frac{1}{4.233} @ 6\%$

$$= 100 \left(\frac{(1.06)^{4.233} - 1}{\ln(1.06)} \right) = 480$$

iii. $-800(1+i)^t - 50(1+i)^{t-1} + 102.971 \frac{1}{1+i} @ 7\% = 0$
 $\Rightarrow t = 18$

$$-800(1+i)^{18} - 50(1+i)^{17} + 102.971 \frac{1}{1+i} @ 7\% = 9.7714$$

$$\Rightarrow 9.7714(1+i)^4 + 100 \frac{1}{4} @ 6\%$$

$$= \underline{462.79} = \text{Accumulated amount after } \underline{22 \text{ years}}$$

Q.13 i. $988000 = X(12) a_{\overline{25}|}^{(12)} @ 7\%$

$$988000 = 12X \left(\frac{1 - (1.07)^{-25}}{(1.07)^{1/12} - 1} \right)_{12}$$

$X = 6848$, Monthly repayment = 6848

ii. $X a_{\overline{34/3}|}^{(12)} = 6848 \left(a_{\overline{34/3}|}^{(12)} @ 7\% \right) = 648600$

iii. PV after 10th September's payment = $X a_{\overline{83/6}|}^{(12)} @ 7\%$

PV after 10th October's payment = $X a_{\overline{13.75}|}^{(12)} @ 7\%$

$$\begin{aligned} \text{Capital repaid} &= X a_{\overline{13.75}|}^{(12)} - X a_{\overline{83/6}|}^{(12)} \\ &= X \left(\frac{1 - (1.07)^{-13.75}}{(1.07)^{1/12} - 1} - \frac{1 - (1.07)^{-83/6}}{(1.07)^{1/12} - 1} \right) @ 7\% \\ &= \underline{26.86} \end{aligned}$$

iv. Capital repayment in these 12 instalments =

$$\begin{aligned} &= X \left(a_{\overline{22/3}|}^{(12)} - a_{\overline{19/3}|}^{(12)} \right) @ 7\% \\ &= 516.2 \end{aligned}$$

∴ Total interest in these 12 instalments =

$$12X - 516.20 = 30556$$

$$12X - 51620 = 30556$$