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# Calculus Assignment 2

1) $I = \int_{\frac{1}{50}}^{2} \frac{e^{2/w}}{w^2} dw$

→ let $t = e^{2/w}$ → ①

$dt = e^{2/w} \cdot \frac{d}{dw}(2/w)$

$= e^{2/w} \cdot 2(-1/w^2)$

$dt = \frac{-2e^{2/w}}{w^2} dw$

$\boxed{\frac{dt}{-2} = \frac{e^{2/w}}{w^2} dw}$

| | | |
|---|---|---|
| $w$ | $\frac{1}{50}$ | $2$ |
| $t$ | $e^{100}$ | $e$ |

... from ①

$\therefore I = \frac{-1}{2} \int_{e^{100}}^{e} dt$

$= \frac{-1}{2} [(t)]_{e^{100}}^{e}$

$= -\frac{1}{2} e^{100} + \frac{1}{2} e$

$= \frac{e}{2} - \frac{e^{100}}{2}$

2) $I = \int \frac{2t^3 + 1}{(t^4 + 2t)^3} dt$

→ let $y = t^4 + 2t$

$dy = (4t^3 + 2) dt$

$= 2(2t^3 + 1) dt$ → ①

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= Putting in I, we get,

$$I = \frac{1}{2}\int \frac{dy}{y^3}$$

$$= \frac{1}{2}\, \frac{y^{-3+1}}{-2} = \frac{-1}{4} \cdot \frac{1}{y^2}$$

$$= \frac{-1}{4y^2}$$

$$= \frac{-1}{4(t^4+2t)^2}$$

3) $f'(x) = x^4 + 3x - 9.$

→ $f(x) = \int f'(x)\,dx$

$= \int (x^4 + 3x - 9)\,dx$

= $f(x) = \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C$

4) $f(x) = \begin{cases} 6 & \text{if } x > 1 \\ 3x^2 & \text{if } x \le 1 \end{cases}$

→ $\int_{-2}^{3} f(x)\,dx$

= $\int_{-2}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx$

= So, as per the question,

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$$= \quad I = \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$= \left[\frac{\cancel{3}x^3}{\cancel{3}}\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$= 1-(-8) + 6(3) - 6(1)$$

$$= 1+8+18-6$$

$$= 21$$

5) $$I = \int 3(8y-1)e^{4y^2-y}\,dy$$

$$= \cancel{\int 3(8} \; 3\int (8y-1)\,e^{4y^2-y}\,dy$$

$= \therefore$ let $t = e^{4y^2-y}$

$$dt = e^{4y^2-y}(8y-1)\,dy$$

$$= \therefore \quad I = 3\int dt = 3t + C$$

$$= \quad = 3(e^{4y^2-y}) + C$$

6) $f''(x) = 15\sqrt{x} + 5x^3 + 6, \quad f(1) = -\frac{5}{4}$ and

$f(4) = 404.$

$$\rightarrow \int f''(x) = f'(x) = \int 15x^{1/2} + 5x^3 + 6\,dx$$

$$= \frac{2\cdot\cancel{15}^{5}x^{3/2}}{\cancel{3}} + \frac{5x^4}{4} + 6x$$

$$= \quad f(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + C$$

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$$= f(x) = \frac{\cancel{10}^{4} x^{5/2}}{\cancel{5/2}} + \frac{\cancel{5}}{4}\frac{x^5}{\cancel{5}} + \frac{\cancel{6}^{3} x^2}{\cancel{2}} + cx + c'$$

$$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + cx + c'$$

$$= \frac{-5}{4} = 4 + \frac{1}{4} + 3 + c + c'$$

$$= \frac{-3}{2} = 7 + c + c'$$

$$= \boxed{c + c' = \frac{-17}{2}.}$$

$$= f(4) = 4\,4^{5/2} + (4)^4 + 3(4)^2 + 4c + c'$$

$$= 404 = 4^{7/2} + 4^4 + 48 + 4c + c'$$

$$= 404 = 128 + 256 + 48 + 4c + c'$$

$$= -28 = 4c + c'.$$

$$= c' = -4c - 28$$

$$= c + (-4c) - 28 = -8.5$$

$$= -3c = 28 - 8.5$$

$$= c = -6.5$$

$$= c' = -8.5 + 6.5 = -2.5$$

$$= c' = -2.5$$

$$= \left[ f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 + \frac{-13}{2}x - \frac{5}{2}. \right]$$

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7) -

8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$ $\quad r(1) = 2.$

$\rightarrow \frac{dr}{r^2} = \frac{d\theta}{\theta}.$

= integrating on both sides,

$$\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}.$$

$= \frac{r^{-2+1}}{-1} = \log\theta + c$

$= -r^{-1} = \log\theta + c$

$= -\frac{1}{r} = \log\theta + c$

= $r(1) = 2.$

$-\frac{1}{r} = \log(1) + c$

$= -\frac{1}{2} = 0 + c$

$= \boxed{c = -1/2}$

$= \therefore -\frac{1}{r} = \log\theta - \frac{1}{2}$

$= \boxed{\log\theta = \frac{1}{2} - \frac{1}{r}.}$

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9) $\frac{dv}{dt} = 9.8 - 0.196v$

$\longrightarrow \frac{dv}{9.8 - 0.196v} = dt$

$= \frac{dv}{\frac{98}{10} - \frac{196v}{1000}} = dt$

$= \int \frac{dv\,(1000)}{9800 - 196v} = \int dt$

$= \frac{\cancel{100}\,50}{\cancel{-196}\,-98} \log(9800 - 196) = t + c$

$= \boxed{-\frac{50}{98} \log(9604) = t + c}$

10) $ty' + 2y = t^2 - t + 1 \qquad y(1) = 1/2.$

$\longrightarrow t\frac{dy}{dt} + 2y = t^2 - t + 1$

$= t\frac{dy}{dt} = t^2 - t + 1 - 2y$

$= x = t^2 - t + 1 - 2y$

$= \frac{dx}{dt} = 2t - 1 - 2\frac{dy}{dt}$

$= 2\frac{dy}{dt} = 2t - 1 - \frac{dx}{dt}$

$= \frac{dy}{dt} = t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}$

$= \boxed{\frac{dy}{dt} = t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}}$

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= $y = 1/2$ at $t = 1$.

= $t\frac{dy}{dt} + 2y = t^2 - t + 1$

= $t\left(t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}\right) = x$

= $t^2 - t/2 - t/2\,\frac{dx}{dt} = x$

= $t^2\,dt - \frac{t}{2}\,dt - \frac{t}{2}\,dx = x\,dt$

= $\frac{t^3}{3} - \frac{t^2}{4} - \frac{tx}{2} = xt + c$

= $\frac{t^3}{3} - \frac{t^2}{4} = \frac{3xt}{2} + c$

= $\frac{t^3}{3} - \frac{t^2}{4} = \frac{3\cancel{x}t(t^2 - t + 1 - 2y)}{2} + c$

= $\frac{t^3}{3} - \frac{t^2}{4} = \frac{3t}{2}(t^2 - t + 1 - 2y) + c$

= $\frac{t^3}{3} - \frac{t^2}{4} - \frac{3t^3}{2} + \frac{3t^2}{2} - \frac{3t}{2} + 3yt = c$

= $-\frac{7t^3}{6} + \frac{10t^2}{8} - \frac{3t}{2} + 3yt = c$

= $-\frac{7}{6} + \frac{10}{8} - \cancel{\frac{3}{2}} + \cancel{\frac{3}{2}} = c$

= $\frac{-28 + \cancel{30} - \cancel{36} + 30}{24} = c$

$c = 2/24$

$c = 1/12$

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(11) $y' = \dfrac{xy^3}{\sqrt{1+x^2}}$ $\quad y(0) = -1$

$\longrightarrow \dfrac{dy}{dx} = \dfrac{xy^3}{\sqrt{1+x^2}}$

$= \displaystyle\int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}}\, dx$

$= \dfrac{y^{-2}}{-2} = \cancel{\dfrac{1}{2}} \dfrac{t^{1/2}}{\cancel{1/2}} + c$

$= \dfrac{-1}{2y^2} = t^{1/2} + c$

$= \dfrac{+1}{2(+1)} = 0 + c$

$= \underline{c = 1/2}$

$= 1+x^2 > 0$ for all $x \in R$.

$=$

$\therefore$ The function is valid for all Numbers.

(12) $y' = \dfrac{xy^3}{1+x^2}$

$\longrightarrow y = 0$ at $x = -1$

$= \therefore \boxed{\dfrac{dy}{dx} = \dfrac{xy^3}{1+x^2}}$

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(12) $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$\rightarrow$ $f'(x) = 3x^2 - 20x$

$f''(x) = 6x$

$f'''(x) = 6$

$f''''(x) = 0$

$\therefore$ $f'(3) = 3(3)^2 - 20(3)$
$= 27 - 60$
$= -33$

$f''(3) = 18$

$f'''(3) = 6$

$f''''(3) = 0$

$f(3) = 27 - 90 + 6 = -57$

$\therefore$ Taylor expansion $= -57 + (x-3)(-33)$
$+ \frac{18}{2!}(x-3)^2 + \frac{6}{3!}(x-3)^3$
$+ 0$

$= -57 + 99 - 33x + 9(x-3)^2 + (x-3)^3$

$= 42 - 33x + 9(x^2 - 6x + 9) +$
$(x^3 - 27 - 9x(x-3))$

$= 42 - 33x + \cancel{9x^2} - 54x + 81 +$
$x^3 - 27 - \cancel{9x^2} + 27x$

$= x^3 - 27x - 33x + 42 + 81 - 27$

$= x^3 - 60x + 123 - 27$

$= x^3 - 60x + 96.$

(15) $f(x) = e^{-6x}$ about $x = -4$

$\rightarrow$ Taylor Expansion :-

$$= f(-4) + (x-(-4))f'(-4) + \frac{(x-(-4))^2}{2!}f''(-4)$$

$$+ \frac{(x-(-4))^3}{3!}f'''(-4) \cdots\cdots$$

$$= f(x) = e^{-6x}$$

$$f'(x) = \frac{-1}{6}e^{-6x}$$

$\therefore$ at $x = -4$

$$f'(-4) = \frac{-1}{6}e^{24}$$

$$f''(-4) = \frac{1}{36}e^{-6x} = \frac{1}{36}e^{24}.$$

$\therefore$ Taylor expansion =

$$= e^{24} + (x+4)\frac{-1}{6}e^{24} + \frac{(x+4)^2}{2!}\frac{1}{36}e^{24}$$

$$- \frac{1}{216}e^{24}\frac{(x+4)^3}{3!} \cdots\cdots$$

(16) $f(x) = \ln(3+4x)$

$\rightarrow$ Taylor expansion about $x=0$,

$= f(0) + (x-0)f'(0) + (x-0)^2 f''(0) + (x-0)^3 f'''(0)$ - - - - -

$\therefore$ $f(x) = \ln(3+4x) = x=0 \Rightarrow \ln(3)$

$f'(x) = \frac{4}{3+4x} \Rightarrow x=0 \Rightarrow \frac{4}{3}$

$= f''(x) = \frac{(4x+3)(0)-(4)(4)}{(3+4x)^2} = \frac{-16}{(3+4x)^2}$, $\Rightarrow x=0$

$= \frac{-16}{9}$.

$= f'''(x) = -16\left[\frac{(3+4x)^2(0) - 1(2)(4x+3)}{(3+4x)^4}\right]$

$= \frac{32}{(3+4x)^3} = \frac{32}{27}$

$= \therefore = \ln(3) + \frac{4x}{3} + (x-0)^2\frac{(-16)}{9(2!)} + \frac{32}{27(3!)}(x-0)^3 + \cdots$

$\cdots - \frac{64}{81(4!)}(x-0)^4 - - - -$

$= \ln 3 + \frac{4x}{3\cancel{(1!)}} + \frac{-16}{9(2!)}x^2 + \frac{32}{27(3!)}x^3 +$

$- \frac{64}{81(4!)}x^4 + \cdots\cdots$

(13) $f(x) = (1+x)^{\mu}$

→ $f(0) = 1^{\mu} = 1$

= $f'(0) = \mu(1+x)^{\mu-1}$

$= \mu(1+0)^{\mu-1}$

$= \mu(1) = \mu$

= $f''(0) = \mu(\mu-1)(1+0)^{\mu-1-1}$

$= \mu^2-\mu(1)$

$= \mu^2-\mu$

= $f'''(0) = \mu(\mu-1)(\mu-2)(1+0)^{\mu-3}$

$= \mu(\mu-1)(\mu-2)\,1$

$= \mu(\mu-1)(\mu-2)$

= ~~Taylor S~~ Mclaurin series →

$$= f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$$

$$= \cancel{\mu \quad 1 \quad \mu^2-\mu)}$$

$$= 1 + \mu x + \frac{(\mu^2-\mu)x^2}{2!} + \frac{\mu(\mu-1)(\mu-2)x^3}{3!}$$

$$+ \frac{\mu(\mu-1)(\mu-2)(\mu-3)x^4}{4!} + \dots$$

(14) $f(x) = \sqrt{1+x}$

→ $f(0) = 1$

= $f'(0) = \frac{1}{2}(1+x)^{-1/2} = \frac{1(1)}{2} = \frac{1}{2}$

= $f''(0) = -\frac{1}{2}\frac{1}{2}(1+x)^{-3/2} = -\frac{1(1)}{4} = -\frac{1}{4}$

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Maclaurin series →

$$= 1 + \frac{1}{2}(x) - \frac{1}{4}\frac{(x^2)}{2!} + \frac{1}{8}\frac{(x^3)}{3!} - \frac{1}{16}\frac{(x^4)}{4!} - - - - \cdots - \frac{1}{2^n}\frac{x^n}{n!}$$

(7) let $z = f(x+iy) + g(x-iy)$

→ differentiating z w.r.t $x$ & $y$ we get,

$= p = z' = f'(x+iy) + g'(x+iy)$

$= m = z' = f'(x+iy)(i) + g'(x-iy)(-i)$

$= q = z'' = f''(x+iy) + g'(x-iy)$ —— ①

$= n = z'' = f''(x+iy)(i^2) + g'(x-iy)(-i)^2$

$n = f''(x+iy)(i^2) + g'(x-iy)(i^2)$

$n = i^{(2)}\{q\}$

$n = i^{(2)}q$

$\boxed{n = i^2 q}$ ...... from ①

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