Calculus

1) $\int_{1/50}^{2} \frac{e^{2/w}}{w^2} dw$

Let $t = 1/w$ $w = 2 : t = 1/2$

$\frac{dt}{dw} = \frac{1}{w^2}$ $w = 1/50$
$t = 50$

$dt = \frac{\cdot dw}{w^2}$

∴ on subsitution we get

$\int_{50}^{-1/2} e^{2t} dt$

$= \frac{-1}{2} (e^{2t})_{50}^{1/2}$

$= \frac{-1}{2} (e^{1} - e^{100})$

$= \frac{1}{2} (e^{100} - e^{1})$

$= 1.34405 \times 10^{43}$

2) $\int \frac{2t^3 + 1}{(t^4 + 2t)^3} dt$

Let $w = (t^4 + 2t)^3$

$\frac{dw}{dt} = 4t^3 + 2$

on subsitution we get

$\frac{1}{2}\int \frac{4t^3+2}{(t^4+2t)^3}\, dt$

$\frac{1}{2}\int \frac{1}{w^3}\, dt$

$\frac{1}{2}\ \frac{w^{-2}}{-2}$

$= \frac{-1}{4} w^2$

$= \frac{-1}{4(t^4+2t)^2} + c$

3) $f'(x) = x^4 + 3x - 9$

$f(x) = \int f'(x)$

$= \int x^4 + 3x - 9$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + c$

4) $f(x) = \begin{cases} 6 & x > 1 \\ 3x^2 & x < 1 \end{cases}$

Evaluate : $\int_{-2}^{3} f(x)\, dx$

$\int_{-2}^{3} f(x) = \int_{-2}^{1} 3x^2\, dx + \int_{1}^{3} 6\, dx$

$= \left[\frac{3x^3}{3}\right]_{-2}^{1} + (6x)_1^3$

$= (x^3)_{-2}^{1} + (6x)_1^3$

$= 9 + 12$

$= 21$

5) $\int 3(8y-1)e^{4y^2-y}\, dy$

let $4y^2 - y = w$

$\therefore \frac{dw}{dy} = 8y - 1$

$\therefore dw = (8y-1)dy$

$= \int 3e^w\, dw$

$= 3e^w + c$

$3e^{4y^2-y} + c$

6) $f''(x) = 15\sqrt{x} + 5x^3 + 6$; $f(1) = -5/4$,

$f(4) = 404$

$f'(x) = \int f''(x)\, dx$

$= \int 15\sqrt{x} + 5x^4 + 6$

$= 10x^{3/2} + \frac{5x^4}{4} + 6x + c_1$

$f(x) = \int f'(x)$

$\therefore f(x) = \int 10x^{3/2} + \frac{5x^4}{4} + 6x + c_1\, dx$

$= 4x^{5/2} + \frac{x^5}{4} + 3x^2 + c_1x + c_2$

$\because f(1) = -5/4$

$\therefore 4(1)^{5/2} + \frac{(1)^5}{4} + 3(1)^2 + c_1(1) + c_2 = -\frac{5}{4}$

$\therefore c_1 + c_2 = -\frac{5}{4} - 4 - \frac{1}{4} - 3$

$\therefore c_1 + c_2 = -\frac{17}{2}$

$\therefore c_2 = -\frac{17}{2} - c_1$

also $f(4) = 404$

$\therefore 4(4)^{5/2} + \frac{4^5}{4} + 3(4)^2 + 4c_1 + c_2 = 404$

$\therefore 4c_1 + c_2 = 404 - 4^3\sqrt{4} - \frac{4^5}{4} - 3(4)^2$

$\therefore \cancel{4c_1} - \cancel{\frac{17}{2}} - \cancel{c_1} =$

$\therefore 3c_1 = \frac{-56 + 17}{2}$; $\therefore c_1 = -\frac{13}{2}$; $c_2 = -2$

$\therefore f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - \frac{13}{2}x - 2$

7) → Doubt

8) $\frac{dr}{d\theta} = \frac{r^2}{\theta}$, $r(1) = 2$ ∴

$\therefore \frac{dr}{r^2} = \frac{d\theta}{\theta}$

$\therefore \int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$

$\therefore \frac{-1}{r} = \log\theta + c$

$\therefore r = \frac{-1}{\log\theta + c}$

$\because r(1) = 2$

$\therefore \frac{-1}{r} = \log\theta + c$

$\therefore \frac{-1}{2} = \log 1 + c$

$\therefore c = \frac{-1}{2}$

$$\boxed{\therefore \frac{-1}{r} = \log\theta - \frac{1}{2}}$$

9) $\frac{dv}{dt} = 9.8 - 0.196v$

$\therefore \frac{dv}{dt} + 0.196v = 9.8$

$I.F = e^{\int P\,dx} = e^{\int 0.196v\,dv} = e^{0.196v}$

$ve^{0.196v} = 9.8e^{0.196v}$

10) Find solution to following initial value problem (IVP)

$$ty^t + 2y = t^2 - t + 1$$

$$y(1) = 1/2$$

Sol^n:

$$t\frac{dy}{dt} + 2y = t^2 - t + 1$$

$$t\frac{dy}{dt} = t^2 - t + 1 - 2y$$

Let $x = t^2 - t + 1 - 2y$

$$\therefore \frac{dx}{dt} = 2t - 1 - 2\frac{dy}{dt}$$

$$\therefore \frac{2dy}{dt} = 2t - 1 - \frac{dx}{dt}$$

$$\therefore \frac{dy}{dt} = t - \frac{1}{2} - \frac{1}{2}\frac{dv}{dt}$$

$\because$ $y = 1/2$ at $t = 1$

$$\therefore t\frac{dy}{dt} + 2y = t^2 + t + 1$$

$$\therefore t\left(t - \frac{1}{2} - \frac{1}{2}\frac{dx}{dt}\right) = x$$

$$\therefore \int t^2 - \frac{t}{2} - \frac{t}{2}\, dx = \int x\, dt$$

$$\therefore \frac{t^3}{3} - \frac{t^2}{4} - \frac{tx}{2} = xt + c$$

$$\therefore \frac{t^3}{3} - \frac{t^2}{4} = \frac{3xt}{2} + c$$

$$\therefore \frac{t^3}{3} - \frac{t^2}{4} = \frac{3t}{2}\left(t^2 - t + 2 - 2y\right) + c$$

$$\therefore c = \frac{t^3}{3} - \frac{t^2}{4} - \frac{3t^3}{2} + \frac{3t^2}{2} - \frac{3t}{2} + 3$$

$$\therefore c = \frac{-7}{6} + \frac{10}{8} - \cancel{\frac{3}{2}} + \cancel{\frac{3}{2}}$$

$$\boxed{\therefore c = \frac{1}{12}}$$

12) Find Taylor series for $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$x = 3 \quad f(3) = -57$

$f'(x) = 3x^2 - 20x \qquad f'(3) = -33$
$f''(x) = 6x - 20 \qquad f''(3) = -2$
$f'''(x) = 6 \qquad f'''(3) = 6$
$f''''(x) = 0 \qquad f''''(3) = 0$

$$f(x) = F(3) + (x-3)' f'(3) + \frac{(x-3)^2}{2} f''(3) + \frac{(x-3)^3}{3} f'''(3) + \frac{(x-3)^4}{4} f''''(3)$$

$$= -57 + (x-3)(-33) + \frac{(x-3)^2}{2}(-2) + \frac{(x-3)^3}{3} 6$$

$$f(x) = -57 - 33(x-5) - (x-3)^2 + (x-3)^3$$

13) Find Mclaurian series for $(1+x)^{u}$

$f(x) = (1+x)^u$
$f'(x) = u(1+x)^{u-1} \qquad f'(0) = u$
$f''(x) = u \cdot \frac{d}{dx}(1+x)^{u-1} + (1+x)^{u-1} \cdot \frac{d}{dx} u \qquad f''(0) = u^2 - u$

$f'''(0) = u(u-1)(u-2)(1+0)^{u-3}$
$= u(u-1)(u-2)$

Mclaurian series :

$$f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} \cdots\cdots$$

$$= 1 + ux + \frac{u^2 - u}{2} x^2 + \frac{u(u-1)(u-2)}{6} \cdots\cdots\cdots$$

14) Determine the Mclaurian series for $f(x) = \sqrt{1+x}$

$f(x) = \sqrt{1+x}$ $\qquad f(0) = \sqrt{1}$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$ $\qquad f'(0) = \frac{1}{2}$

$f''(x) = \frac{-1}{4}(1+x)^{-3/2}$ $\qquad f''(0) = \frac{-1}{4}$

$f'''(x) = \frac{3}{8}(1+x)^{-5/2}$ $\qquad f'''(0) = \frac{3}{8}$

$\therefore$ Mclaurian series :

$f(0) + f'(0)\frac{x}{1!} + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} \cdots\cdots$

$= \sqrt{1} + \frac{x}{2} + -\frac{1}{8}x^2 + \frac{x^3}{16} \cdots\cdots\cdots$

15) Taylor series for $f(x) = e^{-6x}$ for $x = -4$

$f(x) = e^{-6x}$ $\qquad f(0) = e^{24}$

$f'(x) = -6e^{-6x}$ $\qquad f'(0) = -6e^{24}$

$f''(x) = 36e^{6x}$ $\qquad f''(0) = 36e^{24}$

$f'''(x) = -216e^{6x}$ $\qquad f'''(0) = -216e^{24}$

$\therefore f(x) = f(-4) + \frac{x+4}{1!}f'(-4) + \frac{(x+4)^2}{2!}f''(-4) + \frac{(x+4)^3}{3!}f'''(-4)$

$= e^{24} + -6e^{24}(x+4) + 18e^{24}(x+4)^2 - 36e^{24}(x+4)^3$

b) Taylor series for $f(x) = \ln(3+4x)$ for $x = 0$

$f(x) = \ln(3+4x)$ $\qquad$ $f(0) = \ln 3$

$f'(x) = \dfrac{4}{3+4x}$ $\qquad$ $f'(x) = \dfrac{4}{3}$

$f''(x) = -16(3+4x)^{-2}$ $\qquad$ $f''(x) = \dfrac{-16}{9}$

$f'''(x) = \dfrac{32(3+4x)^{-3}}{3}$ $\qquad$ $f'''(x) = \dfrac{\cancel{3}82}{81}$

$$f(x) = f(0) + x \cdot f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0)$$

$$= \ln 3 + \frac{4x}{3} - \frac{8x^2}{9} + \frac{16x^3}{243}$$