

LAGRANGES INTERPOLATION

classmate

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1) $y = f(x)$

x	7	8	9	10
$f(x)$	3	1	1	9

$x_k = 9.5$, find $f(9.5)$

using Lagranges Interpolation

$$f(9.5) = \frac{(1.5)(0.5)(-0.5)(3)}{(-1)(-2)(-3)} + \frac{(2.5)(0.5)(-0.5)(1)}{(1)(-1)(-2)}$$

→

$$+ \frac{(2.5)(1.5)(-0.5)(1)}{(2)(1)(-1)} + \frac{(2.5)(1.5)(0.5)(9)}{(3)(2)(1)}$$

$$\Rightarrow \frac{3}{16} - \frac{5}{16} + \frac{15}{16} + \frac{45}{16} = \frac{58}{16}$$

$$\Rightarrow 3.625$$

2)	x	3	7	9	10
	$f(x)$	168	120	72	63

Find $f(x)$ when $x = 6$.

$$f(6) = \frac{(-1)(-3)(-4)(168)}{(-4)(-6)(-7)} + \frac{(3)(-3)(-4)(120)}{(4)(-2)(-3)} \\ + \frac{(3)(-1)(-4)(72)}{(6)(2)(-1)} + \frac{(3)(-1)(-3)(63)}{(7)(3)(1)}$$

$$= 12 + 180 + 27 - 72 \\ = 147.$$

3)	x	1	2	7	8
	$f(x)$	4	5	5	4

Find $f(6)$ using LT.

$$f(6) = \frac{(4)(-1)(-2)(4)}{(-1)(-6)(-7)} + \frac{(5)(-1)(-2)(5)}{(1)(-5)(-6)} \\ + \frac{(5)(4)(-2)(5)}{(6)(5)(-1)} + \frac{(5)(4)(-1)(4)}{(7)(6)(1)}$$

$$\Rightarrow \frac{-32}{42} + \frac{5}{3} - \frac{20}{3} + \frac{-40}{21}$$

$$\Rightarrow \frac{-23}{3} = -7.6667.$$

4)

x	1	2	3	4	5
$f(x)$	2	4	8	16	32

to find $f(5)$ using LT.

$$\Rightarrow f(5) = \frac{(3)(2)(1)(-2)(2)}{(-1)(-2)(-3)(-6)} + \frac{(4)(3)(2)(-2)(4)}{(1)(-1)(-2)(-5)}$$

$$\Rightarrow + \frac{(4)(3)(1)(-2)(8)}{(2)(1)(-1)(-4)} + \frac{(4)(3)(2)(-2)(16)}{(3)(2)(1)(-3)}$$

$$\Rightarrow + \frac{(4)(3)(2)(1)(128)}{(6)(5)(4)(3)}$$

$$\Rightarrow \frac{-2}{3} + \frac{32}{5} - 24 + \frac{128}{3} + \frac{128}{15}$$

$$\Rightarrow 32.93$$

5)

x	10	20	30	40	50
$f(x)$	0.25	0.2	0.15	0.10	0.05

And $f(15)$

$$\Rightarrow f(15) = \frac{(-5)(-1)(-1)(0.25)}{(-10)(-4)(-6)} + \frac{(5)(1)(-1)(0.2)}{(10)(6)(4)}$$

$$+ \frac{(5)(-5)(-1)(0.15)}{(4)(-6)(-2)} + \frac{(5)(-5)(1)(0.1)}{(6)(-4)(2)}$$

$$\Rightarrow \frac{-1}{192} - \frac{1}{240} + \frac{5}{64} + \frac{5}{96} = 29$$

$$\Rightarrow 0.120833$$

1) $x^3 + x - 1 = 0$ HW - Regular Falsi Method

x	$y = x^3 + x - 1$
0.5	-0.375
0.6	-0.184
0.7	0.043

$$x_n = \frac{(0.6)(0.43) - (-0.184)(0.7)}{0.043 - (-0.184)}$$

$$x_n = 0.68105$$

$$y_n = -0.0030418$$

$$x_1 = 0.68105$$

$$y_1 = -0.0030418$$

$$x_2 = 0.7$$

$$y_2 = 0.043$$

$$x_n = \frac{(0.043)(0.68105) - (0.7)(-0.0030418)}{(0.043 - (-0.0030418))}$$

$$x_n = 0.6823019$$

$$y_n = 0.00006195$$

∴ root of $x^3 + x - 1$ is 0.6823019

$$2) \quad x^3 - 2x + 0.5 = 0 \quad \text{b/w } 0 \text{ \& } 1$$

$$x \quad y = x^3 - 2x + 0.5$$

$$0 \quad 0.5$$

$$0.1 \quad -10.301$$

$$0.2 \quad 0.108$$

$$0.3 \quad -0.073$$

$$x_1 = -0.073$$

$$x_2 = 0.2$$

$$y_1 = -0.073$$

$$y_2 = 0.108$$

$$x_n = \frac{(0.3)(0.108) - (0.2)(-0.073)}{0.108 - (-0.073)}$$

$$= 0.2596685$$

$$y_n = -0.001828157$$

$$x_1 = 0.2596685$$

$$x_2 = 0.2$$

$$y_1 = -0.001828157$$

$$y_2 = 0.108$$

$$x_n = \frac{(0.2596685)(0.108) - (0.2)(-0.001828157)}{0.108 - 0.001828157}$$

$$x_n = 0.25867528$$

$$y_n = -0.0000418485$$

So root of $x^3 - 2x + 0.5$ is 0.25867528

$$3) \quad x^3 + 2x - 20 = 0$$

$$y = x^3 + 2x - 20$$

$$x \quad -8$$

$$2$$

$$2.1 \quad -6.539$$

$$2.2 \quad -4.952$$

$$2.3 \quad -3.233$$

$$2.4 \quad -1.376$$

$$2.5 \quad 0.625$$

$$x_1 = 2.4$$

$$x_2 = 2.5$$

$$y_1 = -1.376$$

$$y_2 = 0.625$$

$$x_n = \frac{(2.4)(0.625) - (2.5)(-1.376)}{(0.625 - (-1.376))}$$

$$x_n = 2.468766$$

$$y_n = -0.015827$$

$$x_1 = 2.468766$$

$$x_2 = 2.5$$

$$y_1 = 0.015827$$

$$y_2 = 0.625$$

$$x_n = 2.4693574$$

$$y_n = -0.00016722$$

$$\text{Root of } x^3 + 2x - 20 \text{ is } 2.4693574$$

x)

$$3x^2 - \cos x - 1 = 0.$$

 x

$$y = 3x^2 - \cos x - 1$$

0

-2

0.5

-0.499619

0.6

-0.1994517

0.7

0.10007463

$$x_2 = \frac{(0.10007463)(0.6) - (0.7)(-0.1994517)}{(0.10007463 - (-0.1994517))}$$

$$= 0.666589$$

$$y_2 = -0.0001652$$

So root of $3x^2 - \cos x - 1$ is 0.666589

5)

$$x^2 - 3x - 5 = 0$$

 x

$$y = x^2 - 3x - 5$$

 x -3 2.1 -2.039 2.2 -0.952 2.3 0.267 2.4

$$x_n = \frac{(0.267)(2.2) - (-0.952)(2.3)}{0.267 - (-0.952)}$$

$$= 2.2780968$$

$$y_n = 0.0115944$$

$$x_1 = 2.2780968$$

$$x_2 = 2.3$$

$$y_1 = 0.0115944$$

$$y_2 = 0.267$$

$$x_n = 2.2789316$$

$$y_n = -0.001958206$$

$$x_1 = 2.27893$$

$$x_2 = 2.3$$

$$y_1 = -0.0019582$$

$$y_2 = 0.267$$

$$x_n = 2.279016$$

$$y_n = -0.0006335$$

so root of $x^2 - 3x - 5$ is 2.279016 .

Q

$$x \log x - 1.2 = 0$$

$$y = x \log_{10} x - 1.2$$

$$0.1 \quad -1.2$$

$$2 \quad -0.59794$$

$$3 \quad 0.23136$$

$$x_n = \frac{x_1(2)(0.23136) - (-0.59794)(3)}{0.23136 + 0.59794}$$

$$x_n = 2.72102$$

$$y_n = -0.017086$$

x	y
2.72102	-0.017086
3	0.23136

$$x_n = \frac{(2.72102)(0.23136) - (3)(-0.017086)}{(0.23136) - (-0.017086)}$$

$$x_n = 2.740206$$

$$y_n = -0.0003$$

$$\text{root of } x \log_{10} x - 1.2 = 0 \text{ is } 2.740206$$

$$1) \quad x^3 - 4x + 1 = 0$$

 x
 0
 0.1
 0.2
 0.3

$$y = x^3 - 4x + 1$$

$$-2.601$$

$$0.208$$

$$-0.173$$

$$x_1 = 0.3$$

$$y_1 = -0.173$$

$$x_2 = 0.2$$

$$y_2 = 0.208$$

$$x_n = \frac{0.3 \times 0.208 - 0.2(-0.173)}{0.208 + 0.173}$$

$$x_n = 0.25459$$

$$y_n = -0.00187056$$

$$x_1 = 0.25459$$

$$x_2 = 0.2$$

$$y_1 = -0.00187056$$

$$y_2 = 0.208$$

$$x_n = \frac{(0.208)(0.25459) - (0.2)(-0.00187056)}{0.208 + 0.00187056}$$

$$= 0.254103$$

$$y_n = 0.00006671$$

$$\text{So root of } x^3 - 4x + 1 \text{ is } 0.254103$$

$$8) \quad x^3 + 2x^2 - 8 = 0$$

x	y
0	-8
1	-5
2	8

$$x_1 = \frac{8 - (-10)}{8 - (-5)} = \frac{18}{13} = 1.384615$$

$$y_1 = -1.15151$$

x	y
1.384615	-1.15151
2	8

$$x_2 = \frac{(1.384615)(8) - (-1.15151)(2)}{8 - (-1.15151)}$$

$$= 1.4823886$$

$$y_2 = -0.347534$$

x	y
1.4823886	-0.347534
2	8

$$x_3 = 1.5039388$$

$$y_3 = -0.074684$$

x	y
1.5039388	-0.0746846
2	8

$$x_4 = \frac{(8 \times 1.5039388 - (-0.0746846)(2))}{(8 - (-0.07468))}$$

$$= 1.50852698$$

$$y_4 = -0.0158077$$

x	y
1.50852698	-0.0158077
2	8

$$x_5 = \frac{8(1.50852698) - (-0.0158077)(2)}{8 - (-0.0158077)}$$

$$= 1.509496$$

$$y_5 = -0.00333645$$

x	y
1.509496	-0.00333645
2	8

$$x_6 = 1.5097006$$

$$y_6 = 0.000704268$$

9) $x^3 - 9x + 1 = 0$

$y = x^3 - 9x + 1$

x	y
0	1
1	-7
2	-9

x	y
1	-7
0	1

$$x_1 = \frac{1}{8} = 0.125$$

$$y_1 = -0.12304$$

x	y	x	y
-0.223		0.125	-0.12304
		0	1

$$x_2 = \frac{0.125}{1 - (-0.12304)} = 0.111304$$

$$y_2 = 0.0003602$$

Root of $x^3 - 9x + 1$ is 0.111304

$$10) \quad x^3 - x - 4 = 0$$

x	$y = x^3 - x - 4$
0	-4
1	-4
2	2

$$x_1 = \frac{(2)(1) - (-4)(2)}{(2 - (-4))} = \frac{2+8}{6} = \frac{10}{6} = \frac{5}{3}$$

$$= 1.66667$$

$$y_1 = -1.03703$$

x	y
1.66667	-1.03703
2	2

$$x_2 = \frac{(2)(1.66667) - (-1.03703)(2)}{(2 - (-1.03703))}$$

$$= 2.788 \quad 1.780489$$

$$y_2 = -0.1360834$$

x	y
1.780489	-0.1360834
2	2

$$x_3 = \frac{(2)(1.780489) - (2)(-0.1360834)}{(2 - (-0.1360834))}$$

$$= 1.7944733$$

$$y_3 = -0.016027$$

$$\begin{array}{cc} x & y \\ 1.7944733 & -0.01602727 \\ 2 & 2 \end{array}$$

$$x_4 = \frac{(2)(1.7944733) - (2)(-0.01602727)}{(2 - (-0.01602727))}$$

$$= 1.79610723$$

$$y_4 = -0.00186324$$

$$\begin{array}{cc} x & y \\ 1.79670723 & -0.00186324 \\ 2 & 2 \end{array}$$

$$x_5 = \frac{(2)(1.79670723) - (0.00186324)(2)}{(2 - 0.00186324)}$$

$$x_5 = \cancel{1.79670723} 1.7968964$$

$$y_5 = 0.0049889$$

$$\begin{array}{cc} x & y \\ 1.7968964 & 0.0049889 \\ 2 & 2 \end{array}$$

$$x_6 = 1.796388$$

$$y_6 = 0.000578039$$

the root of $x^3 - x - 4$ is ~~0.000578039~~ 1.796388