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MON TUE WED THR FRI SAT SUN

$\boxed{\text{Calculus}}$

Assignment - II

Q1.

$$\int_{1/50}^{2} \frac{e^{2/w}}{w^2}\, dw$$

Let $t = 2/w$

$$\frac{dt}{dw} = \frac{-2}{w^2}$$

$$\frac{-dt}{2} = \frac{dw}{w^2}$$

$$\Rightarrow -\frac{1}{2}\int_{1/50}^{2} e^{t}\, dt$$

$$\Rightarrow -\frac{1}{2}\left[e^{t}\right]_{1/50}^{2}$$

$$\Rightarrow -\frac{1}{2}\left[e^{2/w}\right]_{1/50}^{2} \Rightarrow -\frac{1}{2}\left[e - e^{100}\right]$$

$$\Rightarrow \frac{e^{100} - e}{2} = 1.344058571 \times 10^{43}$$

Q2: $$\int \frac{2t^3+1}{(t^4+2t)^3}\,dt$$

let $\cancel{2}t^4+2t=u$

$$\frac{du}{dt} = 4t^3+2$$

$$\frac{du}{2} = (2t^3+1)\,dt$$

$$\Rightarrow \int \frac{du}{2u^3} \Rightarrow \frac{1}{2}\int u^{-3}\,du$$

$$\Rightarrow \left[\frac{1}{2} \times \frac{u^{-2}}{-2}\right] + C = \frac{-1}{4(t^4+2t)^2} + C$$

Q3. $$f'(x) = x^4+3x-9$$

$$f(x) = \frac{x^5}{5} + \frac{3}{2}x^2 - 9x + C$$

Q4. $$f(x) = \begin{cases} 6, & x>1 \\ 3x^2, & x\leq 1 \end{cases}$$

$$\int_{-2}^{3} f(x)\,dx$$

$$\Rightarrow \int_{-2}^{1} 3x^2\,dx + \int_{1}^{3} 6\,dx$$

$$\Rightarrow \left[x^3\right]_{-2}^{1} + \left[6x\right]_{1}^{3}$$

$$\Rightarrow 1-(-8) + 18-6$$

$$\Rightarrow 1+8+18-6 \Rightarrow \underline{\underline{21}}$$

Q5. $$\int 3(8y-1)\, e^{4y^2-y}\, dy$$

Let $4y^2 - y = t$

$$\frac{dt}{dy} = 8y-1$$

$$dt = (8y-1)\,dy$$

$$\Rightarrow 3\int e^{t}\, dt$$

$$\Rightarrow 3e^{t} + C \Rightarrow \underline{\underline{3e^{(4y^2-y)} + C}}$$

Q6. $f''(x) = 15\sqrt{x} + 5x^3 + 6$

$f(1) = -5/4$

$f(4) = 404$

$$f'(x) = 10x^{3/2} + \frac{5}{4}x^4 + 6x + C_1$$

$$f(x) = 4x^{5/2} + \frac{1}{4}x^5 + 3x^2 + C_1x + C_2$$

$$f(1) = 4(1)^{5/2} + \frac{(1)^5}{4} + 3(1)^2 + C_1 + C_2 = \frac{-5}{4}$$

$$C_1 + C_2 + 7 = \frac{-6}{4}$$

$$C_1 + C_2 = \frac{-34}{4} = \frac{-17}{2} \rightarrow (1.)$$

$$f(4) = 4(4)^{5/2} + \frac{(4)^5}{4} + 3(4)^2 + 4C_1 + C_2 = 404$$

$$4C_1 + C_2 = 404 - 128 - 256 - 48$$

$$4C_1 + C_2 = -28 \rightarrow (2.)$$

From eq. (1.) & (2.)

$$\Rightarrow \quad 4C_1 + \cancel{C_2} = -28$$

$$-C_1 \pm \cancel{C_2} = \mp 17/2$$

$$3C_1 = \frac{17}{2} - 28$$

$$\underline{C_1 = -6.5}$$

$$\text{eq. (1.)} \Rightarrow C_1 + C_2 = \frac{-17}{2}$$

$$C_2 = \frac{-17}{2} + 6.5$$

$$\underline{C_2 = -2}$$

$$\Rightarrow \quad f(x) = 4x^{5/2} + \frac{x^5}{4} + 3x^2 - 6.5x - 2$$

Q8. $\frac{dr}{d\theta} = \frac{r^2}{\theta}$

$r(1) = 2$

$\int \frac{dr}{r^2} = \int \frac{d\theta}{\theta}$

$\frac{-1}{r} = \log(\theta) + C$

$r = \frac{-1}{\log(\theta) + C}$

$2 = \frac{-1}{\log(1) + C}$

$2 = \frac{-1}{C}$

$C = -1/2$

$\Rightarrow r = \frac{-1}{\log(\theta) - 1/2} = \frac{-2}{2\log(\theta) - 1}$

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Q9. $\frac{dv}{dt} = 9.8 - 0.196v$

$\Rightarrow \int \frac{dv}{9.8-0.196v} = \int dt$

Let $a = 9.8 - 0.196v$

$-\frac{da}{0.196} = dv$

$\Rightarrow -\int \frac{da}{0.196a} = t + c$

$\Rightarrow -\frac{\log(a)}{0.196} = t + c$

$\Rightarrow -\frac{\log(9.8-0.196v)}{0.196} = t + c$

$\Rightarrow -\log(9.8-0.196v) = 0.196t + c$

Q11. $y' = \frac{xy^3}{\sqrt{1+x^2}}$, $y(0) = -1$

$\Rightarrow \frac{dy}{dx} = \frac{xy^3}{\sqrt{1+x^2}}$

$\int \frac{dy}{y^3} = \int \frac{x\,dx}{\sqrt{1+x^2}}$

Let $1+x^2 = t$

$\frac{dt}{2} = x\,dx$

$\Rightarrow \frac{y^{-2}}{-2} = \left(\frac{1}{\cancel{2}} \times 2\right) t^{1/2} + c$

$\Rightarrow -\frac{1}{2y^2} = \sqrt{1+x^2} + c$

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$$\Rightarrow \frac{-1}{2(-1)^2} = \sqrt{1+(0)^2} + c$$

$$\Rightarrow -\frac{1}{2} - 1 = c$$

$$\Rightarrow c = -3/2$$

$$\Rightarrow \cancel{f(x)} \frac{-1}{2y^2} = \sqrt{1+x^2} - \frac{3}{2}$$

$$\cancel{f(x)} \sqrt{1+x^2} + \frac{1}{2y^2} - \frac{3}{2} = 0$$

$$\boxed{2\sqrt{1+x^2} + \frac{1}{y^2} - 3 = 0}$$

Q12. $f(x) = x^3 - 10x^2 + 6$ for $x = 3$

$$f(x) = x^3 - 10x^2 + 6 \rightarrow f(3) = (3)^3 - 10(3)^2 + 6 = -57$$
$$f'(x) = 3x^2 - 20x \rightarrow f'(3) = 3(3)^2 - 20(3) = -33$$
$$f''(x) = 6x - 20 \rightarrow f''(3) = 6(3) - 20 = -2$$
$$f'''(x) = 6 \rightarrow f'''(3) = 6$$

$$\Rightarrow x^3 - 10x^2 + 6 = -57 + (x-3)(-33) + (x-3)^2(-2) + (x-3)^3$$
$$x^3 - 10x^2 + 6 = (x-3)^3 - 2(x-3)^2 - 33(x-3) - 57$$

Q13. Maclaurin series for $(1+x)^u$

$$f(x) = (1+x)^u \rightarrow f(0) = 1$$
$$f'(x) = u(1+x)^{u-1} \rightarrow f'(0) = u$$
$$f''(x) = u(u-1)(1+x)^{u-2} \rightarrow f''(0) = u(u-1)$$
$$f'''(x) = u(u-1)(u-2)(1+x)^{u-3} \rightarrow f'''(0) = u(u-1)(u-2)$$

$$f(x) = 1 + ux + u(u-1)\frac{x^2}{2} + u(u-1)(u-2)\frac{x^3}{6} + \dots$$

$$\Rightarrow f(x) = 1 + \sum_{i=1}^{x} \frac{ux^x}{x!}(u-x)$$

Q14. $f(x) = \sqrt{1+x}$, Maclaurin series

$f(x) = \sqrt{1+x} \rightarrow f(0) = 1$

$f'(x) = \dfrac{1}{2\sqrt{1+x}} \rightarrow f'(0) = \dfrac{1}{2}$

$f''(x) = \dfrac{-1}{4(1+x)^{3/2}} \rightarrow f''(0) = \dfrac{-1}{4}$

$f'''(x) = \dfrac{3}{8(1+x)^{5/2}} \rightarrow f'''(0) = \dfrac{3}{8}$

$$\Rightarrow f(x) = 1 + \frac{x}{2} + \frac{x^2}{8} + \frac{x^3}{16} + \dots$$

Q15. Taylor series for $f(x) = e^{-6x}$ for $x = -4$

$f(x) = e^{-6x} \rightarrow f(-4) = e^{24}$

$f'(x) = -6e^{-6x} \rightarrow f'(-4) = -6e^{24}$

$f''(x) = 36e^{-6x} \rightarrow f''(-4) = 36e^{24}$

~~$f'''(x)$~~

$$\Rightarrow f(x) = e^{24} - 6(x+4)e^{24} + 36(x+4)^2 e^{24} + \dots$$

Q16. Taylor series for $f(x) = \ln(3+4x)$ for $x = 0$

$f(x) = \ln(3+4x)$ $\quad f(0) = \ln(3)$

$= 1.0986122289$

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$$f'(x) = \frac{4}{3+4x} \rightarrow f'(0) = 4/3$$

$$f''(x) = \frac{4 \times 4 \times -1}{(3+4x)^2} = \frac{-16}{(3+4x)^2} \rightarrow f''(0) = -16/9$$

$$f'''(x) = \frac{32}{(3+4x)^3} \rightarrow f'''(0) = 32/27$$

$$\Rightarrow f(x) = \ln(3) + \frac{4x}{3} - \frac{\cancel{16}^{8}}{9}\left(\frac{x^2}{\cancel{2}}\right) + \frac{\cancel{32}^{16}}{27}\left(\frac{x^3}{\cancel{6}\,3}\right) + \ldots$$

$$\Rightarrow f(x) = \ln(3) + \frac{4x}{3} - \frac{8x^2}{9} + \frac{16x^3}{81} + \ldots$$