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# Financial Mathematics H.W.

1. $d^{(4)} = 8\%$

Quarterly Effective Discount Rate $= \frac{d^{(4)}}{4} = \frac{8\%}{4} = 2\%$

ii.) $V = (1-d) = \left[\frac{1}{1+i}\right]$

$$\left\{1 - \frac{d^{(4)}}{4}\right\}^4 = \frac{1}{(1+i)} = \left[1 - 0.02\right]^4 = 0.92236816$$

$$\therefore (1+i) = \frac{1}{0.92236816} = 1.0841657$$

$\therefore i = 0.0841657$

Ans: $i = 8.41657\%$

ii.) $(1+i) = e^{\delta}$

$\frac{1}{1-d} = e^{\delta}$ $\therefore 1-d = \frac{1}{e^{\delta}} = e^{-\delta}$

$\therefore \boxed{d = 1 - e^{-\delta}}$

$i = e^{\delta} - 1$

→ $\log(1+i) = \delta \cdot \log e = \delta$

→ $\log\left(\frac{1}{1-d}\right) = \delta$

→ $\log\left(\frac{1}{0.92236816}\right) = 0.0808108829 = \delta$

$\delta = 8.08108 29\%$

iii.) $\left[1 - \frac{d^{(4)}}{4}\right]^4 = (1-d) = \left[1 - \frac{d^{(12)}}{12}\right]^{12}$

4 Quarters = 1 yearly = 12 Monthly

$\therefore (1 - 2\%)^{\cancel{4}} = \left[1 - \frac{d^{(12)}}{12}\right]^{\cancel{12}\,3}$

$(1 - 0.02)^{1/3} = \left(1 - \frac{d^{(12)}}{12}\right)$

$\frac{d^{(12)}}{12} = (1 - 0.993288)$ $\therefore \boxed{\frac{d^{(12)}}{12} = 0.006712 = 0.6712\%}$

$\therefore d^{(12)} = 0.0805393945$

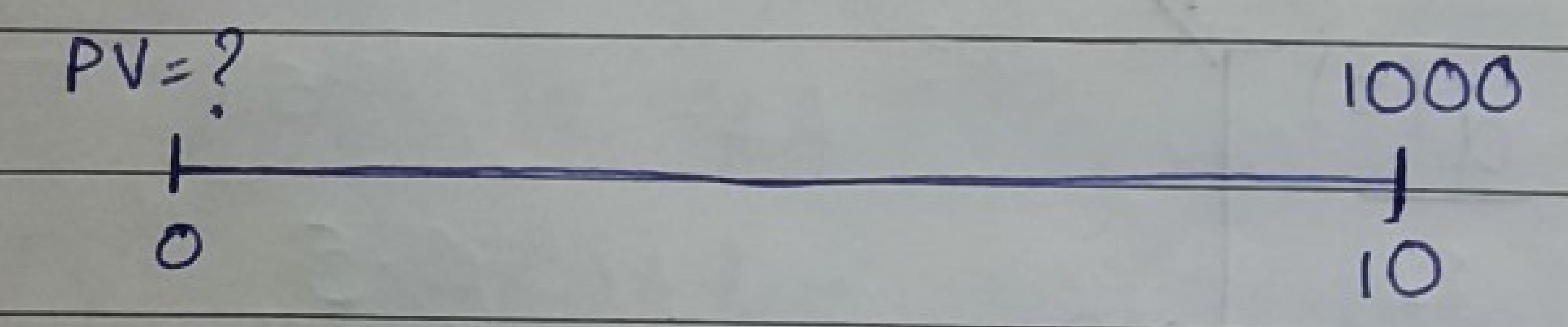
**Ans:** $\therefore \boxed{d^{(12)} = 8.0539395\,\%}$

Hence, I

- Equivalent Force of Interest = 8.0810829 %
- Effective Rate of Interest p.a. = 8.41657 %
- Equivalent Nominal Rate of ~~Interest~~ Discount = 8.05393%

**Q.2** $\delta(t) = 0.004t + 0.0002t^2$ for all 't'

i.)



Present Value = Amount × Discounting Factor

$= 1000 \times e^{-\int_{t_1}^{t_2} \delta(t)\, dt}$

$PV = C \times V(0,10) = C \times \frac{1}{A(0,10)} = \frac{C}{e^{\delta}}$

$= \frac{1000}{e^{\int_0^{10} 0.004t + 0.0002t^2\, dt}}$

$e^{\delta} = e^{\left[\overset{0.002}{\cancel{0.004}}\frac{t^2}{\cancel{2}} + 0.0002\frac{t^3}{3}\right]_0^{10}}$

$= e^{0.002(100-0) + \frac{0.0002}{3}[1000-0]}$

$= e^{\left[0.20 + \frac{0.20}{3}\right]} = e^{\left(\frac{0.60+0.20}{3}\right)} = e^{\left(\frac{0.80}{3}\right)} = e^{0.26667}$

$\therefore PV = 1000 \times \frac{1}{e^{\delta}} = \frac{1000}{e^{0.26667}} = 765.9257853$

**Ans:** PV at $t=0$, is ₹756.9257

ii.) $AV = C \times AF(0,10)$ $\therefore 1000 = 756.9257 \times (1+i)^{10}$

$= AV = C \times \cancel{A}(1+i)^{t_2 - t_1}$ $\quad 1.3056057 = (1+i)^{10}$

~~$(1.3211336)^{1/10} = 1.02824042 = (1+i)$~~

$\therefore$ ~~$i = 0.028240427$~~ $(1+i) = (1.3056057)^{1/10} = 1.027025449$

**Ans:** $\therefore$ Annual Effective Int Rate = ~~2.8240427 %~~ p.a

= 2.702545 %

Q.3

i) ~~LHS~~ = At RHS.

$$i \times v = i \times \frac{1}{(1+i)} = \frac{i}{(1+i)}$$

$$\text{LHS} \rightarrow d = 1 - v = 1 - \frac{1}{(1+i)} = \frac{(\cancel{1}+i)-\cancel{1}}{(1+i)} = \frac{i}{(1+i)} \therefore = \underline{\text{RHS}}$$

As LHS = RHS , $\boxed{d = i.v}$

ii) a. $\frac{d^{(4)}}{4} = 2.25\%$

$$\left[1 - \frac{d^{(4)}}{4}\right]^{\cancel{4}2} = \left[1 - \frac{d^{(2)}}{2}\right]^{\cancel{2}}$$

$$(1 - 2.25\%)^2 = (1 - 0.0225)^2 = \left[1 - \frac{d^{(2)}}{2}\right]$$

$$0.95550625 = 1 - \frac{d^{(2)}}{2}$$

$$\therefore \frac{d^{(2)}}{2} = 1 - 0.95550625 \quad \therefore d^{(2)} = 0.0889875$$

$$\therefore \underline{d^{(2)} = 8.89875\ \%}$$

b.) $\frac{d^{(0.5)}}{0.5} = \frac{5\%}{0.5} = 10\%$ p. year effective:

$$\left[1 - \frac{d^{(0.5)}}{0.5}\right]^{0.\cancel{5}} = \left[1 - \frac{d^{(2)}}{2}\right]^{\cancel{2}4}$$

$$(1 - 10\%)^{1/4} = \left[1 - \frac{d^{(2)}}{2}\right] = 0.97400374$$

$$\therefore d^{(2)} = 2\left[1 - 0.97400374\right] = 2\left[0.02599626\right]$$

$$\therefore d^{(2)} = 0.051992507 = 5.1992507\ \%$$

= ~~5.~~

$$\underline{d^{(2)} = 5.1992507\ \%}$$

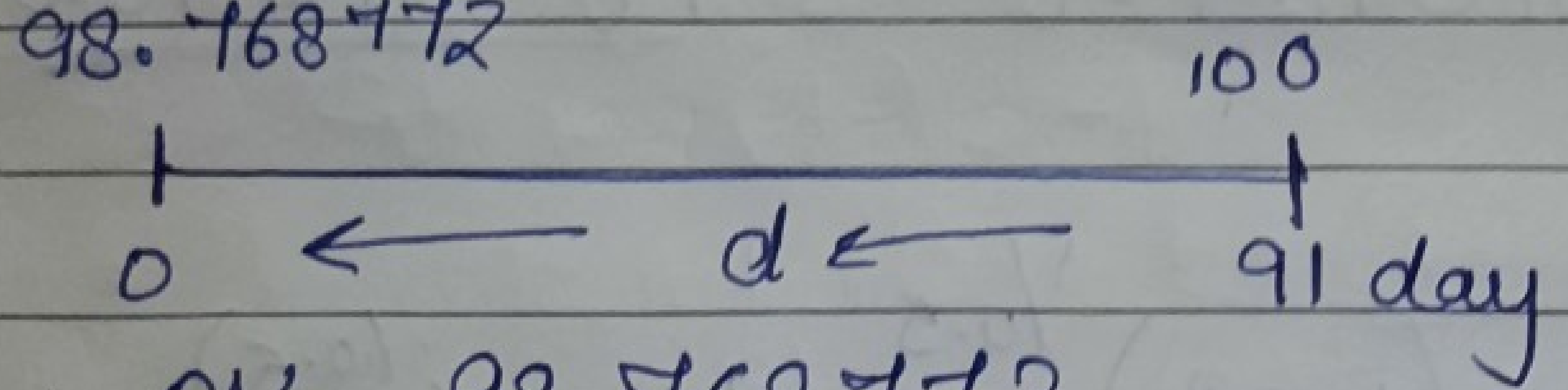
iii) Let Accumulated Value after 91 days Bill = 100 @5% p.a.

AV = Invested Amount × Accumulating Factor

AV = $C \times (1+ni)$

$$100 = C \times \left[1 + \left(\frac{91}{365} \times \frac{\cancel{5}}{\underset{20}{\cancel{100}}}\right)\right] = C\left[\frac{7391}{7300}\right] = 100$$

$$\boxed{\text{Cor Value Invested} = 98.768772}$$



∴ PV = 98.768772

PV = Fund Value × Discounting Factor

$$98.768772 = 100 \times \left[1 - n.d\right]$$

$$\frac{98.768772}{100} = 1 - \left(\frac{91}{365} \times d\right)$$

$$\therefore \frac{91}{365} d = 0.01231228 \qquad d = 0.04938441$$

$$\boxed{d = 4.938441\%}$$

∴

Q4 $i = 0.08$

a) $d = \frac{i}{1+i} = \frac{0.08}{1.08} = \frac{0.04}{0.54} = \frac{2}{27} = 0.0740740$

∴ Formula : $(1-d) = \left[1 - \frac{d^{(p)}}{p}\right]^p$

$$(1-d) = (1 - 0.0740740) = \left[1 - \frac{d^{(12)}}{12}\right]^{12}$$

$$0.9936071086 = \left[1 - \frac{d^{(12)}}{12}\right] \qquad \therefore d^{(12)} = 0.076714696$$

$$d^{(12)} = 7.6714696\%$$

b) By formula, $(1+i) = \left[1 + \frac{i^{(p)}}{p}\right]^p$

$$(1+0.08) = \left[1 + \frac{i^{(365)}}{365}\right]^{365}$$

$$(1.000210874) = 1 + \frac{i^{(365)}}{365}$$

$\therefore i^{(365)} = 0.07696915541$ $\therefore i^{(365)} = 7.69615541\ \%$

c.) $(1+i) = e^{\delta}$ Taking log on BS.

$\log(1+i) = \delta \log e$ $\ln(1+0.08) = \delta$

$\ln(1.08) = 0.7696104114 = \delta$ $\therefore \boxed{\delta = 7.6961041\ \%}$

d. $i^{(0.5)}$

$(1+i) = \left[1 + \frac{i^{(0.5)}}{0.5}\right]^{0.5}$ , $(1.08)^{1/0.5} - 1 = \frac{i^{(0.5)}}{0.5}$

$\therefore i^{(0.5)} = 0.0832$ $\boxed{i^{0.5} = 8.32\%}$

Q.5 $d^{(4)} = 6\%$

Effective Rate of Discount Quarterly: $\frac{d^{(4)}}{4} = \frac{6}{4} = 1.5\%$

i) $(1+i) = \frac{1}{(1-d)}$

$$\left[1+\frac{i^{(2)}}{2}\right]^2 = \frac{1}{\left[1-\frac{d^{(4)}}{4}\right]^4}$$

$$= \frac{1}{(1-0.015)^4} = 1/0.9413365506.$$

~~$\left[1+\frac{i^{(2)}}{2}\right] = (0.9413365506)^{1/2} = 0.970225$~~

~~$\therefore \frac{i^{(2)}}{2} = 1$~~

$$\left[1+\frac{i^{(2)}}{2}\right]^2 = 1.062319315$$

$$\therefore \left(1+\frac{i^{(2)}}{2}\right) = (1.062319315)^{1/2} = 1.0306887$$

$$\therefore \frac{i^{(2)}}{2} = 0.0306887, \quad \frac{i^{(2)}}{2} = 3.06887\%$$

$$\boxed{i^{(2)} = 6.1378\%}$$

ii) $\left[1-\frac{d^{(12)}}{12}\right]^{\cancel{1/2}\,3} = \left[1-\frac{d^{(4)}}{4}\right]^{\cancel{1/4}}$

$$= (1-0.015) = 0.985$$

$$\left[1-\frac{d^{(12)}}{12}\right] = (0.985)^{1/3} = 0.99497478.$$

$$\therefore \boxed{\frac{d^{(12)}}{12} = 0.00502532} = \cancel{0.5}$$

$$d^{(12)} = 0.06030384$$

$$\therefore d^{(12)} = 6.030384\%$$

ii. 2,00,000 — 2,20,000

15 Apr, 05 — 12 Month — 15 Apr, 06

$AV = C \times AF(0,1)$

$$\cancel{220,000} = {}^{10}\cancel{200,000} \times (1+i)^1$$

$$\frac{11}{10} = (1+i) \qquad i = \frac{1}{10} = 0.10 \qquad = 10\% \text{ p.a}$$

$$AV = 2,00,000 \times (1+i)^{91/365}$$
$$= 2,00,000\,[1.1]^{91/365}$$
$$= 2,00,000 \times 1.024046836$$

→ $AV = 2,04,809.3672$

Q.6 i.) Let the 'Retail Price' be 100

a.) Cash Payment: 30% disc. — 100

$$PV = \cancel{80}\ 100 - \frac{30}{100} \times 100 = 70.$$

$PV = 70$ ~~AC~~ $= 100$

$70 = 100\,[1-d]$ $\boxed{d = 30\%}$

b.) $(1-d) = \left[1 - \frac{d^{(12)}}{12}\right]^{12}$ $(0.70)^{1/2}$

$$1 - 0.9707144 = \frac{d^{(12)}}{12} \qquad \therefore d^{(12)} = 35.142\%$$

$$\frac{1}{(1-0.70)} = \left[1 + \frac{i^{(2)}}{2}\right]^2 \qquad \therefore \frac{i^{(2)}}{2} = 39.04572187\%$$

$$(1-d) = e^{-\delta}$$
$$\ln(1-d) = -\delta$$
$$\ln(0.7) = +35.66749 = +\delta \qquad \boxed{\delta = 35.66750\%}$$

Q.7 N = 17 months

i.) AV = C × AF

$$26\cancel{000} = \overset{10}{\cancel{20000}} \times \left[1+\frac{i^{(4)}}{4}\right]^{\frac{\cancel{4}\times 17}{\cancel{12}\,3}}$$

$$\left(\frac{13}{10}\right)^{3/17} = \left[1+\frac{i^{(4)}}{4}\right]$$

$$(1.047388136) = \left\{1+\frac{i^{(4)}}{4}\right\}$$

$$\frac{i^{(4)}}{4} = 0.047388136 \qquad \therefore i^{(4)} = 0.18955254$$

$$i^{(4)} = 18.955254\%$$

ii.

$$26\cancel{000}\,\cancel{x} = 20\cancel{000} \times \left[1+\frac{i^{(2)}}{2}\right]^{\frac{\cancel{2}\times 17}{\cancel{12}\,6}}$$

$$\left(\frac{13}{10}\right)^{6/17} = 1+\frac{i^{(2)}}{2}$$

$$1.0970219 = 1+\frac{i^{(2)}}{2}$$

$$\frac{i^{(2)}}{2} = 0.0970219 \rightarrow i^{(2)} = 0.1940438165$$

$$i^{(2)} = 19.40438165\%$$

iii.) $AV = C \times [1+ni]$

$26000 = 20,000\left[1+\frac{17}{12}\times i\right]$

$\left(\frac{13}{10}-1\right)=\frac{17}{12}i=\frac{3}{10}$

$i=\frac{3\times 12}{17\times 10}$, $i = 0.2117647059$

$\therefore i = 21.1764706\ \%$

iv.) Rate of Interest Increases as the no. of times interest charged per year reduces.

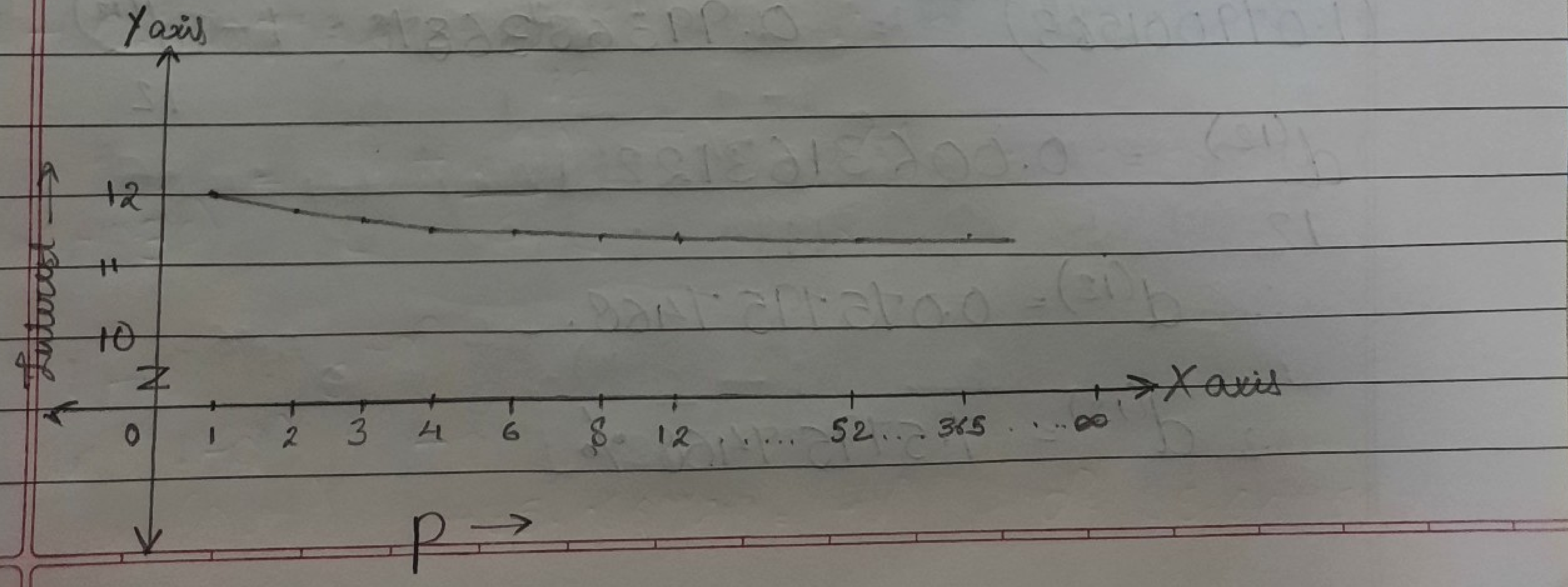
~~Q.8~~

ii.) $20000=26000\left[1-\frac{d^{(2)}}{2}\right]^{\frac{17}{6}}$ $\therefore d^{(2)}=17.688\%$

Q.8. $p= i^{(p)} = p\left[(1+i)^{1/p}-1\right]$

$i^{(1)} = 0.12$
$i^{(2)} = 0.1166$
$i^{(3)} = 0.1155$
$i^{(4)} = 0.1149$
$i^{(6)} = 0.1144$
$i^{(8)} = 0.1141$
$i^{(12)} = 0.1139$
$i^{(52)} = 0.1135$
$i^{(365)} = 0.11346$
$i^{(\infty)} = 0.113329$

- As 'p' decreases, value of equivalent nominal interest rate decreases.
- While going from monthly, weekly, daily, change is less significant.



Q.9

i.) Force of Interest is the instantaneous change in fund value expressed as an Annualised Percentage of Current Fund Value.

ii) $i^{(2)} = 0.0775 = 7.75\%$

~~$i^{(2)}$~~ $\frac{i^{(2)}}{2} = \frac{7.75}{2} = 3.875\%$

$AF = (1+i) = e^{\delta}$

$(1+i) = e^{\delta} \rightarrow \left[1 + \frac{i^{(2)}}{2}\right]^2 = e^{\delta}$

Taking log BS.

~~$\ln[1+$~~

$\ln\left(1 + \frac{i^{(2)}}{2}\right)^2 = e^{\delta}$

$2 \cdot \ln[1 + 0.03875] = \delta \cdot \log e$

$2[0.03801806] = \delta$

$\therefore$ ~~$\delta = 0.06$~~.

$\delta = 0.076036134$

$\therefore \delta = 7.6036134\%$

iii.) $d^{(12)} = ?$

$\left[1 + \frac{i^{(2)}}{2}\right]^2 = \frac{1}{\left(1 - \frac{d^{(12)}}{12}\right)^{12}}$

$(1 + 0.03875)^2 = \left(1 - \frac{d^{(12)}}{12}\right)^{-12}$

$(1.079001563)^{-1/12} = 0.9936836878 = 1 - \frac{d^{(12)}}{12}$

$\frac{d^{(12)}}{12} = 0.0063163122$

$\therefore d^{(12)} = 0.0757957468.$

$\therefore d^{(12)} = 7.57957468\%$

iv. $i^{(0.5)} = ?$

$$\left[1+\frac{i^{(2)}}{2}\right]^{2} = \left[1+\frac{i^{(0.5)}}{0.5}\right]^{0.5}$$

$$\left(1+0.03875\right)^{\overset{4}{\cancel{2}/\cancel{0.5}}} = \left\{1+\frac{i^{(0.5)}}{0.5}\right\}$$

$$(1.164244372) = 1+\frac{i^{(0.5)}}{0.5}$$

$$\frac{i^{(0.5)}}{0.5} = 0.164244372$$

$$\therefore \quad i^{(0.5)} = 0.08212218594$$

$$i^{(0.5)} \rightarrow 8.212218594\ \%$$

Q.10 When $t \rightarrow 0 \le t \le 5$ :

$$AF = e^{\int_{t_1}^{t_2} \delta(t)\, dt} \rightarrow e^{\int_0^5 0.08\, dt} = e^{[0.08t]_0^5}$$

$$AF = e^{0.40} \quad — ①.$$

When $t \rightarrow 5 < t \le 10$

$$AF = e^{\int_{t_1}^{t_2} \delta(t)\, dt} \rightarrow e^{\int_5^{10} 0.06\, dt} = e^{[0.06t]_5^{10}}$$

$$AF = e^{0.06(5)} = e^{0.30} \quad — ②$$

When $t \rightarrow t \ge 10$

$$AF = e^{\int_{10}^{t} 0.04\, dt} = e^{[0.04]_{10}^{t}} = e^{0.04t - 0.40}$$

$PV = C \times V(t)$

Present Value = Fund Value × Discounting Factor.

$$PV = C \times \frac{1}{A(t_1, t_2)}$$

$$\rightarrow PV = 1 = C \times \frac{1}{e^{\delta_1} \times e^{\delta_2} \times e^{\delta_3}}$$

$$1 = \frac{1}{(e^{0.40}) \times (e^{0.30}) \times (e^{0.04t - 0.40})} = 1$$

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$$1 = C \times V(t)$$

$$V(t) = \frac{1}{C} = \frac{1}{(1+i)} = \frac{1}{e^{\delta}}$$

$$V(t) = \frac{1}{C} \times \frac{1}{(e^{0.40}).(e^{0.30}) \times (e^{0.04t - 0.40})}$$

$$v(t) = e^{-0.40} \times e^{-0.30} = \frac{1}{e^{0.30} . e^{0.04t}} = e^{-0.04t - 0.30}$$

Q.11

a. PV=? 50,000

0 d=18%. 9 Month

Present Value = Fund Value × Discounting Factor

$$PV = 50{,}000 \times \left[1 - \frac{9}{\cancel{12}\,2} \times \frac{\cancel{18}\,3}{100}\right]$$

$$= \overset{250}{\cancel{50{,}000}} \times \left\{\frac{200-27}{\cancel{200}}\right\}$$

$$= 250 \times 173$$

$$\boxed{PV = 43{,}250}$$

$$PV = 50{,}000 \times (1 - 0.18)^{9/12}$$

$$= 50{,}000 \times (0.8617085246)$$

$$\boxed{PV = 43{,}085.42623}$$

b.) i.) $\delta = 6\%$

$$\frac{1}{1-d} = e^{\delta}$$

$$\frac{1}{\left\{1 - \frac{d^{(12)}}{12}\right\}^{12}} = e^{\delta} = e^{6\%}$$ ~~Taking log on BS.~~

$$\cancel{\ln \{} \quad \cancel{1} \quad \left\{1 - \frac{d^{(12)}}{12}\right\}^{12} = e^{0.06}$$

$$1-\frac{d^{(12)}}{12} = (1.061836547)^{-1/12} = 0.9950124792$$

$$\frac{d^{(12)}}{12} = 0.0049875208.$$

$$d^{(12)} = 0.05985024969.$$

$$\therefore d^{(12)} = 5.985024969\ \%$$

iii.) $d^{(4)} = 6\%$. $\frac{d^{(4)}}{4} = 1.5\%$.

$$\left[1+\frac{i^{(12)}}{12}\right]^{\cancel{12}} = \frac{1}{\left[1-\frac{d^{(4)}}{4}\right]^{4}} = (1-0.015)^{-\cancel{4}/\cancel{12}\,3} = 1.005050591$$

$$\therefore i^{(12)} = 12[1.005050591-1]$$

$$i^{(12)} = 0.060607088 \quad \boxed{i^{(12)} = 6.060708\%}$$

$$\left[1+\frac{i^{(4)}}{4}\right]^{\cancel{4}} = \left(1-\frac{d^{(4)}}{4}\right)^{\cancel{-4}-1} = (0.985)^{-1} = 1.015228426$$

$$\therefore i^{(4)} = 4[1.015228426-1]$$

$$\therefore i^{(4)} = 0.06091370558.$$

$$\therefore \boxed{i^{(4)} = 6.091370558\ \%}$$

c.) $d < \delta < i^{(4)}$ : As 'd' is lowest value, and 'i' the highest value, force of interest is what that comes from both i & d when compounded annually. As $i^{(4)}$ is paid per quarter, it is less than 'i'

d.) On RHS, 'v' is discounting factor,
RHS is $i \times v$. Discounting factor is reciprocal of Accumulating factor. RHS $= \frac{i}{1+i}$.

LHS, d in terms of 'i' is given by $\frac{i}{1+i}$

As both are same, $d = iv$ is true..

'd' is equal to discounting the interest earned from 0 to 1 for same amount of accumulation & principle.

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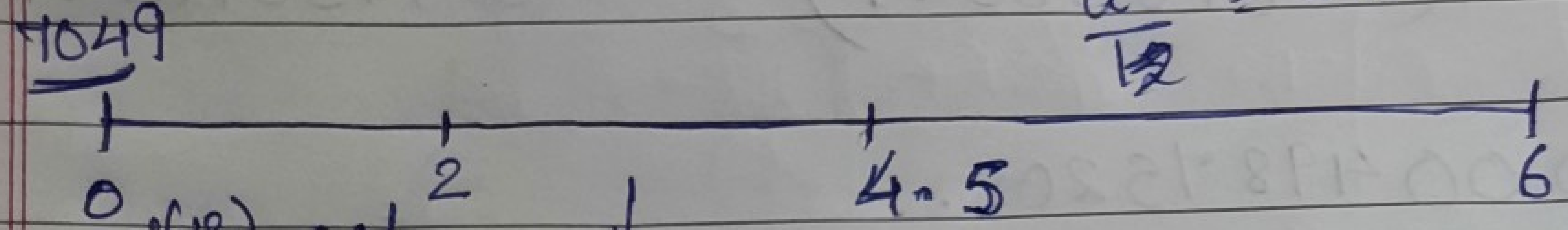
Q.12



$\frac{d^{(12)}}{12} = 6\%$ $\qquad \frac{d^{(12)}}{12} = \frac{6\%}{12} = 0.5\%$

$i^{(12)} = 6\%$

simple i
$= 2.1:5\%$

$\frac{i^{(12)}}{12} = \frac{6\%}{12} = 0.5\%$

$\frac{i^{(4)}}{4} = 1.5\% \rightarrow 6\%$ for Year

$AV(0,2) = C \times \underline{AF(0,2)}$

$AF(2,4.5) = [1 + ni] = \left[1 + \frac{2.5 \times 6}{100}\right]$

$= (1 + 0.15) = \cancel{1.015}\ (1.15)$

~~AV~~ $AF(0,2) = \left[1 + \frac{i^{(12)}}{12}\right]^{12 \times 2} = [1 + 0.005]^{24}$

$= (1.005)^{24} = 1.127159776$

$AF(4.5,6) = \frac{1}{v(4.5,6)} = \frac{1}{\left\{1 - \frac{d^{(12)}}{12}\right\}^{\cancel{12} \times \frac{18}{\cancel{12}}}}$

$\rightarrow \frac{1}{(1 - 0.005)^{18}} = 1.094421325$

$\therefore AV(0,6) = C \times AF(0,2) \times AF(2,4.5) \times AF(4.5,6)$
$= (7049) \times [1.127159776] \times [1.\cancel{0}15] \times [1.094421325]$

~~AV(0,6) = 8825.993061~~

$AV(0,6) = 9999.893616$

Q.13 Let Invest Amt = $x$ Accumulated Value = $2x$

i. $AV = C \times AF(0,t)$

$2\cancel{x} = \cancel{x} \times [1+i]^t$

$2 = (1+0.10)^t = (1.1)^t$

Taking log on BS.

$\ln(2) = t \cdot \ln(1.1)$

$0.6931471806 = t \times (0.0953101798)$

$t = 7.27254$ ~~≈~~ ~~7 and~~

$t \approx 7.25$

$t \rightarrow$ 7 years and 1 Quarter

$t = 7.27254 \times 365.25$

★ $\cancel{t} = 2656.295235$ days.

ii. $d^{(12)} = 10\%$

$$\frac{d^{(12)}}{12} = \frac{\overset{5}{\cancel{10}}}{\underset{6}{\cancel{12}}}\% = \frac{d^{(12)}}{12} = 0.00833333$$

$PV = C \times V(0,t)$

$$x = 2x \times \left[1 - \frac{d^{(12)}}{12}\right]^{12 \times t}$$

$$\left(\frac{1}{2}\right) = [1 - 0.008333]^{12 \times t}$$

$$\left(\frac{1}{2}\right) = [0.9044620224]^{t}$$

Taking log BS

$\ln(0.5) = t \times \ln[0.9044620224]$

$$\frac{-0.6931471806}{-0.1004149624} = t \quad , \boxed{t = 6.902827667}$$

$\therefore t = 6.902827667 \times 365.25$

★ $\therefore t = 2521.257805$ days