Moksh Doshi ROLL - 18
Section - 1A

Date ____
Page ____

NMA assignment

Q-1 Original error radius of circle
$= 12.5 \text{ cm} = r_1$

area $= \pi r^2$
$= \pi (12.5)^2$
$= 491.0714286$

radius of circle with error radius
$= 12.65 = r_2$

area of circle $= \pi r_2^2$
$= \pi (12.65)^2$
$= 502.927857$

→ Absolute error = area of circle with errored radius - area of circle with og. radius -

$= 502.92785 - 491.0714286$
$= 11.85642850$

→ Relative error $= \dfrac{\text{absolute error}}{\text{original area}}$
$= 0.024144000$

→ Percentage error = relative error ×100
$= 2.41439998\ \%$

Q2. a)

x	$f(x) \log x$
4	0.60206
6	0.778151

Date ______
Page ______

Interpolation

$(6-4) = (0.7781513 - 0.60206)$

$2 = 0.17609130$

1 cause of change of $= \frac{0.17609300}{2}$

$1 = 0.08804650$

Thus $x = 5$

$\log(5) = \log(4) + \log(1)$

$= 0.60206 + 0.08804650$

$= 0.69010650$

6	x	$\log(x)$
	4.5	0.6532125
	5.5	0.7403627

$5.5 - 4.5 = 0.7403627 - 0.6532125$

$1 = 0.087150200$

0.5 causes change of $= \frac{0.08715020}{2}$

$= 0.04357510$

$x = 5$

$\log(5) = \log(4.5) + 0.43575100$

$0.6532125 + 0.04357510$

$= 0.69678760$

Q. 3 $\rightarrow x^4 - x - 10 = 0$

x	y
$x_0 = 1.5$	-6.487
$x_1 = 2$	4

Date ______
Page ______

→ $x_2 = \frac{x_0 + x_1}{2}$ $y_2 = -2.371093750$
$= 1.75$

→ $x_3 = \frac{x_1 + x_2}{2}$ $y_3 = 0.4846194 1$
$\frac{2+1.75}{2} = 1.875$

→ $x_4 = \frac{x_2 + x_3}{2}$ $y_4 = -1.020268413$

→ $x_5 = \frac{x_3 + x_4}{2}$ $y_5 = -0.28723403$
$= 1.84375$

→ $x_6 = \frac{x_4 + x_5}{2} = 1.8235$
$y_6 = -0.76688625$

(4) $f(x) = x^3 - x - 1$ = roots using Newton Raphson Method

$f'(x) = 3x^2 - 1$

x	y
0	-1
1	-1
2	5

$x_0 = 1$
$f(x_0) = -1$
$f'(x_0) = 2$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$
$= 1.3252$
$y_3 = 0.00205$

$f'(x_3) = 4.26846$
$x_4 = x_3 - \frac{f_1(x_3)}{f(x_3)}$
$= 1.32472$
$y_4 = 0.00008712$

Date ______
Page ______

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$1 - \left(\frac{1}{2}\right)$$

$= 1.5$

$y = 0.875$

$= 1.5$

$y = 0.875$

$f'(x_1) = 5.75$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.5 - \left(\frac{0.875}{5.75}\right)$$

$x_2 = 1.34783$

$f(x_2) = 0.100699$

$f'(x_2) = 4.44994$

(5) $A^2 = A$

$7A - (1+A)^3$

$= 7A - (I^3 + A^3 + 3A^2I + 3AI^2)$

$= 7A - (I + A^2 + 6A)$

$= 7A - I + 7A$

$= -I$

(6) $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 0 - 1(6 - 8) + (-1)(0 + 4)$

$= 2 - 4$

$= -2$

$$\operatorname{adj}(A) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \operatorname{adj}(A)$$

$$= \frac{-1}{2} \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}$$

(7) $\vec{a} = 8\hat{i} - 9\hat{j}$

$\vec{b} = 7\hat{i} - 9\hat{j}$

$\vec{a} \cdot \vec{b} = ab\cos\theta$

$= (8\hat{i} - 9\hat{j})(7\hat{i} - 9\hat{j})$

$= 56 + 81 = 137.$

$a = \sqrt{145}$ $\quad \vec{a}\cdot\vec{b} = ab\cos\theta$

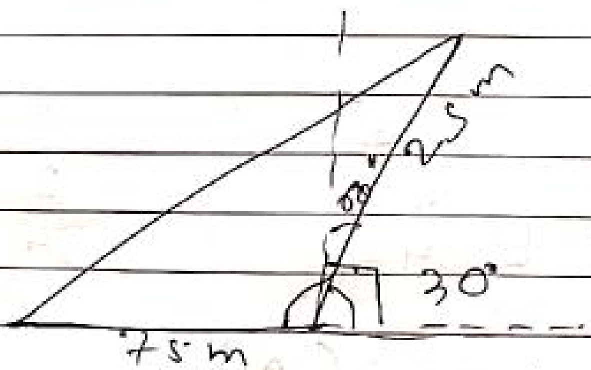
$b = \sqrt{130}$

$\cos\theta = \dfrac{137}{\sqrt{130} \times \sqrt{145}}$

$\cos\theta = 0.997849$

$\theta = 3.758^\circ$

(8)



Displacement vector $= \vec{R}$

$\vec{R} = \vec{a} + \vec{b}$

$R^2 = A^2 + B^2 + 2AB\cos\theta$

$\theta = 80^\circ$

Date
Page

$R^2 = 75^2 + 25^2 + 2(75)(25)\cos\theta$

$R^2 = 9497.595264$

$R = 97.4556$ m

This displacement = 97.4556 meters

(9) $d = 3$

A.P = 101, 104, 107

$a = 101$

$a_n = a + (n-1)d$ $\quad 998 = 101 + (n-1)3$

$n = 300$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{300}{2}(998 + 101)$

$= 150 \times 1099$

$= 164850$

(10) $a_6 = 32$

$ar^5 = 32$ $\qquad \frac{ar^7}{ar^5} = \frac{128}{32}$

$a_8 = 128$

$ar^7 = 128$

$r^2 = 4$

$r = 2$ or -2

$\boxed{r = 2}$

(11) $(3x+4)^5$

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

a
$(a+b)$
$(a+b)^2$
$(a+b)^3$
$(a+b)^4$
$(a+b)^5$

$(3x+4)^4 = (3x)^5 + (5(3x)^4 \times 4)$
$+ (10 \times (3x)^3 \times 4^2) + (10 \times (3x^2) \times 4^3)$
$+ 5(3x \times 4^4) + 4^5$

$= 243x^5 + 1620x^4 + 4320x^3 + 5760x^2$
$+ 3840x + 1024$

(12) $\begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$\begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

Date ______
Page ______

$|A - \lambda I| = (0 - 0 + (3-\lambda)[(2-\lambda)(-5-\lambda)$, $+6]$

$= (3-\lambda)\{-10 - 2\lambda + 5\lambda + \lambda^2 + 6]$

$= [\lambda^2 + 3\lambda - 4][3-\lambda]$

$= 9\lambda - 12 - \lambda^3$

$(A - \lambda I) = -\lambda^3 + 13\lambda - 12$

Comparing coefficients

$a+b+c+d = 1 - 13 + 12$
$= 0.$

Thus one root $= 1$

$$\begin{array}{c|ccc} 1 & 1\ 0 & -13 & 12 \\ & 1 & 1 & -12 \\ \hline & 1 & -12 & 0 \end{array}$$

$\lambda^2 + \lambda - 12 = 0$
$\lambda = 3$ or -4

Eigen values $= -4, 1, 3$

(13) $f(x) = x^3 - 7x^2 + 8x - 3$
$f'(x) = 3x^2 - 14x + 8$

$x_0 = 5$
$f(5) = -13$
$f'(5) = 13$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 5 - \left(\frac{-13}{13}\right)$$

$= 6$

$f(x_1) = 9$
$f'(x_1) = 32$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 6 - \frac{9}{32}$$

$x_2 = 5.71875$
$f(x_2) = 0.84707$

Q.14) $(x^2 - x + 1)^4 = ((x-1)^2 + x)^4$

Using Pascal

1	a
1 1	$a+b$
1 2 1	$(a+b)^2$
1 3 3 1	$(a+b)^3$
1 4 6 4 1	$(a+b)^4$

$(x^2-x+1)^4 = (x-1)^8 + 4x(x-1)^6 + 6x^2(x-1)^4 + 4x^3(x-1)^2 + x^4$

(15) $e^{-x} = 3\log(x)$

$3\log(x) - e^{-x} = 0$

x	y
0.1	-2.07511
0.2	-1.27818
0.3	-0.82782
0.4	-0.52335
0.5	-0.29656
0.6	-0.11673
0.7	0.03188

$x_0 = 0.6$, $y_0 = 0.11673$

$x_1 = 0.7$ $y_1 = 0.03188$

$x_2 = \frac{x_0 + x_1}{2}$

$= 0.65$

$y_2 = 0.03921$

$y_3 = \frac{x_2 + x_1}{2}$

$= 0.675$

$\rightarrow x_3 = \frac{x_2 + x_1}{2}$

$= 0.675$

$y_3 = -0.00293$

$y_4 = 0.01461$

$\rightarrow x_4 = \frac{x_3 + x_1}{2}$

$= 0.6875$

$\rightarrow x_5 = \frac{x_4 + x_3}{2}$

$= 0.68125$

$y_5 = 0.0059$.