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Numerical Methods of Algebra Assignment.

Ques1

$r = 12.5cm$

$r' = 12.65 cm$.

Area of circular plate (actual length) $= \pi r^2$

$= (12.5)^2 \pi$

$= 156.25\pi cm^2$

Area of circular plate (measured length) $= \pi r'^2$

$= \pi (12.65)^2$.

$= 160.0225 \pi cm^2$

(i) Absolute Error = Measured Value − Actual value.

$= 160.0225\pi - 156.25\pi$

$= 3.5225 \pi$

$\boxed{\text{Absolute Error} = 11.066}$

(ii) Relative Error = $\dfrac{\text{Absolute Error}}{\text{Actual Value}}$

$= \dfrac{\cancel{11.066}\ 3.5225\pi}{156.25\pi}$

$\boxed{\text{Relative Error} = 0.0225 \text{ or } 2.25\%}$

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(iii) Percentage Error = Relative Error × 100
$= 0.0225 \times 100$

$\boxed{\text{Percentage Error} = 2.25\%}$

Que 2

x	$f(x) = \log x$.
4	0.60206
4.5	0.6532125
5.5	0.7403627
6.0	0.778153.

(a) $x_1 = 4$ $x_2 = 6$. (using ~~bisection~~ method)

$x_3 = \frac{x_1 + x_2}{2} = \frac{4+6}{2} = \frac{10}{2} = 5$

$y_3 = 0.60206 +$

At $x = 5$.

(a) $y = y_1 + (x - x_1) \frac{(y_2 - y_1)}{(x_2 - x_1)}$.

$y = 0.60206 + (x - 4) \left(\frac{0.778153 - 0.60206}{6 - 4} \right)$

$y = 0.60206 + (5 - 4) \left(\frac{0.1760913}{2} \right)$

$y = 0.60206 + 1 (0.0880456 5)$

$\boxed{y = 0.6901056 5}$ $\boxed{\log(5) = 0.6901056 5}$

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$\boxed{x = 5}$

(2) $x_1 = 4.5 \quad y_1 = 0.6532125$

$x_2 = 5.5 \quad y_2 = 0.7403627.$

$$y = y_1 + (x - x_1)\frac{(y_2 - y_1)}{(x_2 - x_1)}.$$

$$= 0.6532125 + (5 - 4.5)\left(\frac{0.7403627 - 0.6532125}{5.5 - 5}\right)$$

$$= 0.6532125 + \cancel{0.5}\left(\frac{0.0871502}{\cancel{0.5}}\right)$$

$$\boxed{y = 0.7403627}.$$

Q3 $\quad y = x^4 - x - 10.$

$a = 1.5 \quad b = 2$

$$x_1 = \frac{a+b}{2} = \frac{1.5+2}{2} = \frac{3.5}{2} = 1.75$$

$$\boxed{x_1 = 1.75}.$$

$$y_1 = (1.75)^4 - (1.75) - 10$$

$$\boxed{y_1 = -2.371094}.$$

$$x_2 = \frac{x_1 + b}{2} = \frac{1.75+2}{2} = \frac{3.75}{2} = 1.875$$

$$\boxed{x_2 = 1.875}$$

$$y_2 = (1.875)^4 - 1.875 - 10.$$

$$\boxed{y_2 = 0.48462}$$

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$x_3 = \frac{x_1 + x_2}{2} = \frac{1.75 + 1.875}{2}$

$\boxed{x_3 = 1.8125}$

$y_3 = (1.8125)^4 - 1.8125 - 10$. $\boxed{y_3 = -1.020249}$

$x_4 = \frac{x_1 + x_3}{2} = \frac{1.75 + 1.8125}{2} = 1.78125$

$\boxed{x_4 = 1.78125}$

$y_4 = 1.78125^4 - 1.78125 - 10$.

$\boxed{y_4 = -1.714263}$

$x_5 = \frac{x_4 + x_1}{2} = \frac{1.75 + 1.78125}{2} = 1.765625$

$\boxed{x_5 = 1.765625}$

$y_5 = 1.765625^4 - 1.765625 - 10$

$\boxed{y_5 = -2.047045}$

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Ques 4:

$$f(x) = x^3 - x - 1$$
$$y = x^3 - x - 1 = 0.$$

$$x_1 = x - \frac{f(x)}{f'(x)}$$

$$x_1 = x - \frac{(x^3 - x - 1)}{3x^2 - 1}$$

Let $x = 1$

$$x_1 = 1 - \frac{(1 - 1 - 1)}{3(1) - 1}$$

$$x_1 = 1 - \frac{1}{2}$$

$$= \frac{1}{2} \quad \boxed{x_1 = 0.5}$$

$$y_1 = 0.5^3 - 0.5 - 1$$

$$\boxed{y_1 = -1.375}$$

$$y'_1 = 3(0.5)^2 - 0.5$$
$$= 3(0.25) - 0.5$$
$$y'_1 = 0.75 - 0.5$$
$$\boxed{y'_1 = 0.25}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.5 - \frac{(-1.375)}{0.25}$$

$$= 0.5 + 5.5 \quad \boxed{x_2 = 6}$$

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$f(x_2) = 6^3 - 6 - 1 = 209$

$f'(x_2) \quad 3(6)^2 - 1 = 107$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 6 - \frac{209}{107}$

$x_3 = 6 - 1.95327$

$\boxed{x_3 = 4.04673}$

$\boxed{y_3 = 61.22261}$

Ques 5: $7A - (I + A)^3$

$7A - [I^3 + A^3 + 3I^2A + 3IA^2]$

$= 7A - [I + A^2 \times A + 3A + 3A]$

$= 7A - [I + A + 3A + 3A]$

$= 7A - 7A - I = 0 \quad = -I$

Qu 6. $A = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$

$|A| = 1\begin{vmatrix} 0 & 6 \\ 1 & -1 \end{vmatrix} - 4\begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix}$

$= 1(-6) - 4(1 - 2)$

$= -6 - 4(-1) \quad = -6 + 4 = -2$

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As $|A| \neq 0$

A^{-1} exist.

$$\text{adj } A = \begin{bmatrix} -6 & +4 & 4 \\ +1 & -1 & -1 \\ -6 & +2 & 4 \end{bmatrix}^T$$

$$\text{adj } A = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$= \frac{-1}{2}\begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} 3 & -1/2 & 3 \\ -2 & 1/2 & -1 \\ -2 & 1/2 & -2 \end{bmatrix}}$$

Ques 7

$\vec{A} = 8i - 9j$

$\vec{b} = 7i - 9j$

$(64 - 81)$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$.

$56 = 17 \times 32 \times \cos\theta$

$49 - 81$

7

$\frac{56}{17 \times 32} = \cos\theta$

4

$\cos\theta = 0.10294$; $\theta = \cos^{-1}(0.10294)$

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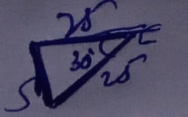
W 

$899+1=n$

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$\boxed{\theta = 84.09143}$ → Required Angle.

Ques 8. $A = 75$ $B = 25$ $\theta = 30°$

R → resultant vector

$R = A^2 + B^2 + 2AB\cos\theta$
$= (75)^2 + (25)^2 + 2(75)(25)\cos 30°$
$R^2 = 5625 + 625 + 3750 \times \frac{\sqrt{3}}{2}$.

$R^2 = 6250 + 3247.5953$

$R^2 = 9497.5952$.

$R =$

$\boxed{R = 97.46\ m}$

Ques 9: 101, 104, 107, - - - - ; 998
$a = 101$; $d = 3$; $n = ?$ $a_n = 998$
$a_n = a + (n-1)d$
$998 = 101 + (n-1)3$
$\frac{998 - 101}{3} = n - 1$

$\frac{\cancel{897}^{299}}{\cancel{3}_1} = n - 1$

$\boxed{n = 300}$

$S_n = \frac{n}{2}[a + a_n] = 150(101 + 998)$

$= 150(1099)$

~~Sn 164899~~ $\boxed{S_n = 164850}$

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Ques 10: $\left.\begin{array}{l} a_6 = 32 \\ a_8 = 128 \end{array}\right]$ G.P

$ar^5 = 32$; $ar^7 = 128$

$$\frac{\cancel{a}r^{7}}{\cancel{a}\cancel{r^5}} = \frac{128}{32}$$

$r^2 = 4$

$\boxed{r = 2}$

Ques 11. $(4+3x)^5$ $\left[(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 \ldots \right.$

$$(4+3x)^5 = 4 + 5(3x) + \frac{5(5-1)}{2!}(3x)^2 + \frac{5(5-1)(5-2)}{3!}(3x)^3$$

$$+ \frac{5(5-1)(5-2)(5-3)}{4!}(3x)^4 + \frac{5(5-1)(5-2)(5-3)(5-4)}{5!}(3x)^5$$

$= 4 + 5x + 90x^2 + 270x^3 + 405x^4 + 243x^5$

$= 4 + x(5 + 90x + 270x^2 + 405x^3 + 243x^4)$.

Ques 12: $A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 2-\lambda & -3 & 0 \\ 2 & -5-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix}$

$|A-\lambda I| = 0.$

$(2-\lambda)\begin{vmatrix} -5-\lambda & 0 \\ 0 & 3-\lambda \end{vmatrix} + 3\begin{vmatrix} 2 & 0 \\ 0 & 3-\lambda \end{vmatrix} = 0$

$-(2-\lambda)(5+\lambda)(3-\lambda) + 3(6-2\lambda) = 0$

$(\lambda-2)(15-2\lambda-\lambda^2) + 18-6\lambda = 0$

$\cancel{15\lambda - 2\lambda^2 - \lambda^3 - 30 + 4\lambda + 2\lambda^2 + 18 - 6\lambda = 0}$

$-\lambda^3 - 13\lambda - 12 = 0$

$\cancel{\lambda^3 + 13\lambda + 12 = 0}$

$\cancel{\lambda^3 + 12\lambda + \lambda + 12 = 0}$

$\cancel{\lambda(\lambda^2 + 2)}$

$\cancel{\lambda^3 + 13\lambda + 12 = 0}.$

$15\lambda - \cancel{2\lambda^2} - \lambda^3 - 30 + 4\lambda + \cancel{2\lambda^2} + 18 - 6\lambda = 0$

$-\lambda^3 + 13\lambda - 12 = 0$

$\lambda^3 - 13\lambda + 12 = 0$

$\cancel{\lambda = 1}$ $\boxed{\lambda = 1}$

$$\begin{array}{r|l} & \lambda^2 + \lambda - 12 \\ \hline \lambda - 1 & \lambda^3 - 13\lambda + 12 \\ & \lambda^3 - \lambda^2 \\ & -\quad + \\ \hline & \lambda^2 - 13\lambda + 12 \\ & \lambda^2 - \lambda \\ & -\quad + \\ \hline & -12\lambda + 12 \\ & -12\lambda + 12 \\ & +\quad - \\ \hline & 0 \end{array}$$

$(\lambda-1)(\lambda^2+\lambda-12)$

$(\lambda-1)(\lambda^2+4\lambda-3\lambda-12)$

$(\lambda-1)[\lambda(\lambda+4)-3(\lambda+4)]$

$(\lambda-1)(\lambda-3)(\lambda+4) = 0$

$\boxed{\lambda = 1, 3, -4}$

↳ Eigen values.

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Eigen vectors

At eigen value 1.

$$\begin{bmatrix} 2-1 & -3 & 0 \\ 2 & -5-1 & 0 \\ 0 & 0 & 3-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} x-3y \\ 2x-6y \\ 2z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x-3y=0$ $\boxed{x=3y}$

$2x-6y=0$
$2x=6y$
$\boxed{y=\frac{x}{3}}$

$2z=0$ $\boxed{z=0}$

$\boxed{\cancel{y=\frac{3y}{3}}}$

At eigen value 3

$$\begin{bmatrix} 2-3 & -3 & 0 \\ 2 & -5-3 & 0 \\ 0 & 0 & 3-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5 & -3 & 0 \\ 2 & -8 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} -5x-3y \\ 2x-8y \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-5x-3y=0$

$5x=-3y$

$\boxed{x=\frac{-3}{5}y}$

$2x-8y=0$

$2x=8y$

$\boxed{y=\frac{x}{4}}$

$\boxed{z=0}$.

At Eigen value, -4.

$$\begin{bmatrix} 2+4 & -3 & 0 \\ 2 & -5+4 & 0 \\ 0 & 0 & 3+4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} 6 & -3 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6x-3y \\ 2x-y \\ 7z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$6x-3y=0$

$6x=3y$

$\boxed{x=\frac{y}{2}}$

$2x-y=0$

$\boxed{y=2x}$

$7z=0$

$\boxed{z=0}$.

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$125 - 7\times 25 + 40 - 3 = 0$ $5 - 8 = 13$

$125 - 175 + 37 = 0$

$-50 + 37 = 0$

-13

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Q.13 $f(x) = x^3 - 7x^2 + 8x - 3$

$f(5) = 5^3 - 7\times 5^2 + 8\times 5 - 3 = -13$

$f'(x) = 3x^2 - 14x + 8.$

$f'(5) = 3(5)^2 - 14(5) + 8 = 13$

$x_0 = 5$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 5 - \frac{(-13)}{13}$

$= 5 + 1 = 6$

$\boxed{x_1 = 6}$

$f(x_1) = 6^3 - 7\times 6^2 + 8\times 6 - 3 = 9.$

$f'(x_1) = 3\times 6^2 - 14\times 6 + 8 = 32.$

$x_2 = 6 - \frac{9}{32}$

$= \frac{192 - 9}{32}.$

$x_2 = \frac{183}{32}$

$\boxed{x_2 = 5.71875}$

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$(a+b)^n = \cancel{a^n}\ {}^nC_0 \ldots \; {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2$

$\frac{4!}{3!!}$

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Ques 14: $(1-x+x^2)^4$

$= {}^4C_0 (1-x)^4 + {}^4C_1 (1-x)^3 \times x^2 + {}^4C_2 (1-x)^2 (x^2)^2 + {}^4C_3 (1-x)(x^2)^3$

$+ {}^4C_4 (x^3)^4$

$= {}^4C_0 (1-x^4) + {}^4C_1 (1-x)^3 x^2 + {}^4C_2 (1-x)^2 x^4 + {}^4C_3 (1-x) x^6$

$+ {}^4C_4 x^{12}$

$= (1-x)^4 + 4(1-x)^3 x^2 + 6(1-x)^2 x^4 + 4(1-x) x^6$

$+ x^{12}$

Ans $= (1-x)^4 + 4x^2(1-x)^3 + 6x^4(1-x)^2 + 4x^6(1-x) + x^{12}$

Ques 15: $e^{-x} = 3 \log x$

x	
0.5	
1.5	

(a) (b).

$x_1 = \frac{0.5 + 1.5}{2} = 1$

$y_1 = 0.555$

$x_2 = \frac{x_1 + b}{2} = \frac{\cancel{2.5}}{\cancel{2}} = 1.25$

$\boxed{x_2 = 1.25}$ $\boxed{y_2 = -0.001555}$

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$$x_3 = \frac{x_1 + x_2}{2} = \frac{1.25 + 1}{2}$$

$$\boxed{x_3 = 1.125}$$

$$\boxed{y_3 = 0.063}$$

$$x_4 = \frac{x_3 + x_2}{2} = \frac{1.125 + 1.25}{2}$$

$$\boxed{x_4 = 1.187}$$

$$\boxed{y_4 = 0.014}$$

$$x_5 = \frac{x_4 + x_3}{2} = \frac{1.187 + 1.125}{2}$$

$$x_5 = \frac{2.312}{2}$$

$$\boxed{x_5 = 1.156}$$

$$\boxed{y_5 = 0.125869}$$