Probability Assignment

Sol ①

$X \sim Poi(\theta)$

$\frac{X-\theta}{\sqrt{\theta}} \sim N(0,1)$

So, $P(X \geq 200) = 0.025 \Rightarrow P(X > 199.5) = 0.025$

$P(X \leq 200) = 0.025 \Rightarrow P(X < 200.5) = 0.025$

$P\left(Z > \frac{199.5-\theta_1}{\sqrt{\theta_1}}\right) = 0.025$

$P\left(Z < \frac{200.5-\theta_2}{\sqrt{\theta_2}}\right) = 0.025$

So, $\frac{200.5-\theta_2}{\sqrt{\theta_2}} = -1.96$ | $\frac{199.5-\theta_1}{\sqrt{\theta_1}} = 1.96$

$200.5-\theta_2+1.96\sqrt{\theta_2} = 0$ | $199.5-\theta_1-1.96\sqrt{\theta_1} = 0$

$\theta_2 = 230.24$ | $\theta_1 = 173.67$

So, required 95% CI $\Rightarrow (173.67, 230.24)$

Sol ② (i) $\bar{x} = \frac{\sum x_i}{n}$. $\quad x_i \sim N(\mu, \sigma^2)$

So, $E(\bar{x}) = \frac{1}{n} E(\sum x_i) \Rightarrow \frac{n\mu}{n} = \mu$.

$V(\bar{x}) = V\left(\frac{\sum x_i}{n}\right) = \frac{1}{n^2}(V(\sum x_i)) = \frac{n\sigma^2}{n^2}$

$= \sigma^2/n$

Sol (2)

So, $\bar{x} \sim N(\mu, \sigma^2/n)$

(ii) $E(s^2) = E\left(\frac{1}{n-1} \sum (x_i - \bar{x})^2\right)$

$= \frac{1}{n-1} E\left(\sum x_i^2 - n\bar{x}^2\right)$.

$V(x_i) = E(x_i^2) - [E(x_i)]^2$

$\sigma^2 + \mu^2 = E(x_i^2)$.

$V(\bar{x}) = E(\bar{x}^2) - [E(\bar{x})]^2$

$\frac{\sigma^2}{n} + \mu^2 = E(\bar{x}^2)$

$E(s^2) = \frac{1}{n-1} E\left(n\sigma^2 + \cancel{n\mu^2} - \sigma^2 - \cancel{n\mu^2}\right)$.

$= \frac{1}{\cancel{n-1}} \sigma^2 (\cancel{n-1}) = \sigma^2$

So, $\boxed{E(s^2) = \sigma^2}$

(iii) $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$

$V\left(\frac{(n-1)s^2}{\sigma^2}\right) = 2(n-1)$

$\frac{(n-1)^{\cancel{2}}}{\sigma^4} V(s^2) = 2(\cancel{n-1})$

$\boxed{V(s^2) = \frac{2\sigma^4}{n-1}}$

(iv) $P(50 \le s^2 \le 150)$. $n=10$.

$P\left(\frac{50(n-1)}{100} \le \frac{(n-1)s^2}{100} \le \frac{150(n-1)}{100}\right)$

$P(4.5 \le \chi^2_9 \le 13.5)$

-(1-

$P(\chi^2_9 \le 13.5) - P(\chi^2_9 \le 4.5)$

~~$1 - P(\chi^2_9 > 13.5) - 1 + P(\chi^2_9 > 4.5)$~~

$\Rightarrow +0.8587 - 0.1245$

$\Rightarrow 0.7342$

Sol ③ (i) $L = [P(X=0)]^{86} \times P[X=1]^{75} \times P[X=2]^{16} \times P(X=3)^{2} \times P(X=4)^{1}$

$L = \left[{}^4C_0\, p^0 (1-p)^4\right]^{86} \times \left[{}^4C_1\, p(1-p)^3\right]^{75} \times \left[{}^4C_2\, p^2(1-p)^2\right]^{16} \times \left[{}^4C_3\, p^3(1-p)^1\right]^{2} \times$

$\left[{}^4C_4\, p^4 (1-p)^0\right]^1$

$L = \text{const} \times (1-p)^{344} \times p^{75} \times (1-p)^{225} \times p^{32} (1-p)^{32} \times p^6 (1-p)^2 \times p^4.$

$L = \text{const.}\ (1-p)^{603}\ p^{117}$

$\log L = \log(\text{const}) + 603 \log(1-p) + 117 \log p.$

$\frac{\partial L}{\partial p} = \frac{-603}{1-p} + \frac{117}{p} = 0$

$\Rightarrow -603p + 117 - 117p = 0$

$\hat{p} = \frac{117}{720} = 0.1625.$

so, $\boxed{\hat{p} = 0.1625}$

(ii) H_0 : this model is a good fit of Binomial distn.

H_a : this model is not a good fit.

x	0	1	2	3	4
O_i	86	75	16	2	1
E_i	88.55	68.729	20.003	2.587	0.1255

we know : $\cancel{\frac{(O_i - E_i)^2}{E_i}} \sim \chi^2$

x	0	1	≥ 2
O_i	86	75	19
E_i	88.55	68.729	22.7155

we know: $\sum \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_1$

χ^2_1 : $0.0734331 + 0.57218 + 0.6077.$

$\chi^2_{1\,cal}$: $\boxed{1.253}$

χ^2_1 Tab at 5% $= 3.841$

so, $\chi^2_{1\,cal} < \chi^2_{1\,Tab}$

Hence, we have insufficient evidence to reject H_0.

(i)

Sol (4) $f(x) = \frac{c\beta^3}{(x+\beta)^4}$

$\Rightarrow c\int_0^\infty \frac{\beta^3}{(x+\beta)^4}\,dx = 1$

$\Rightarrow c\beta^3 \left[\frac{(x+\beta)^{-3}}{-3}\right]_0^\infty = 1$

$\Rightarrow \frac{c\beta^3}{-3}\left[\frac{1}{(x+\beta)^3}\right]_0^\infty = 1 \Rightarrow \frac{c\beta^3}{-3}\left[0 - \frac{1}{\beta^3}\right] = 1$

$\Rightarrow \boxed{c = 3}$

(ii) So, $f(x) = \frac{3\beta^3}{(x+\beta)^4}$

Comparing → we get to know it's Pareto dist.

$E(x) = \frac{\alpha}{\alpha - 1} = \frac{\beta}{2}.$

$V(x) = \frac{\alpha\lambda^2}{(\alpha-1)^2(\alpha-2)} = \frac{3\beta^2}{2^2} = \frac{3}{4}\beta^2.$

(iii) $\bar{X} = \mu$

$\mu = \frac{\beta}{2}$ — (i)

$\bar{X} = \frac{\Sigma x_i}{n}$ — (ii)

So, $\frac{\beta}{2} = \frac{\sum x_i}{n}$

$$\boxed{\hat{\beta} = 2\bar{x}_n}$$

$\rightarrow$ $E(\hat{\beta}) = E(2\bar{x}_n) = \cancel{2E(2\bar{x}_n)}\ 2E(\bar{x}_n) \Rightarrow 2\left(\frac{\sum x_i}{n}\right)$

$= \cancel{\frac{2}{n}\left(\frac{\sum \beta}{2}\right)}\ \frac{2}{n} \times n\frac{\beta}{2} = \beta.$

$E(\hat{\beta}) = \beta$ hence, this is unbiased estimator.

$\rightarrow$ ~~Since~~: $MSE(\hat{\beta}) = V(\hat{\beta}) + Bias^2(\hat{\beta})$

$MSE(\hat{\beta}) = V(2\bar{x}_n) \Rightarrow V\left(2\frac{\sum x_i}{n}\right) \Rightarrow \frac{4}{n^2}(V\sum x_i)$

$= \frac{4}{n^2}\, n\frac{3\beta^2}{4} \Rightarrow \frac{3\beta^2}{n}$

Since: $n \to \infty$

$MSE \to 0$

Hence: it's consistent.

(iii) $Bias(b\bar{x}_n) = E(b\bar{x}_n) - \beta.$

$b\,E(\bar{x}_n) = \frac{b}{n}(E\sum x_i) \Rightarrow \frac{b}{n}\frac{\beta n}{2}$

$Bias(b\bar{x}_n) = \frac{\beta b}{2} - \beta$

$= \beta\left(\frac{b}{2} - 1\right)$

$MSE(b\bar{x}_n) = V(b\bar{x}_n) + \left[\beta\left(\frac{b}{2}-1\right)\right]^2$

$= \frac{3b^2\beta^2}{4n} + \left[\beta\left(\frac{b}{2}-1\right)\right]^2$

$\frac{\partial MSE}{\partial b} \Rightarrow \frac{6b\beta^2}{4n} + \left[\beta^2\left(\cancel{2}\left(\frac{b}{2}-1\right)\left(\frac{1}{\cancel{2}}\right)\right)\right] = 0$

$$\frac{6b\beta^2}{4n} = \beta^2\left(\frac{b}{2}-1\right)$$

$$\Rightarrow \frac{6b}{2n} = \frac{b-2}{2}$$

$$6b = 2nb - 4n$$

$$4n = 2nb - 6b$$

$$4n = b(2n-6)$$

$$\frac{4n}{2n-6} = b \Rightarrow \boxed{\frac{2n}{n-3} = b}$$

(iv) It is seen (ii) that $\hat{\beta} = 2\bar{x}_n$ is an unbiased estimator. The estimator $b\bar{x}_n$ in (iii) when b $\Rightarrow \frac{2n}{n-3}$ is negatively biased when $b<2$ for $n\geq 1$

$n \to \infty$, $b \to 2$ So, estimator in (iii) is also consistent in view of (ii).

Sol (5) $f(x) = \frac{1}{b-a}$

(i) $E(x) = \frac{b+a}{2}$ $\qquad V(x) = \frac{(b-a)^2}{12}$

$\bar{x} = \frac{b+a}{2}$ $\qquad$ (ii) $s^2 = \frac{(b-a)^2}{12}$

$2\bar{x} - a = \hat{b}$ $\qquad s = \frac{b-a}{2\sqrt{3}}$

$2\bar{x} - \bar{x} + \sqrt{3}s = \hat{b}$ $\qquad 2\sqrt{3}s = b-a$

$\boxed{\bar{x} + \sqrt{3}s = \hat{b}}$ $\qquad 2\sqrt{3}s = 2\bar{x} - a - a$

$2\sqrt{3}s = 2\bar{x} - 2a$

$\sqrt{3}s = \bar{x} - a$

$\boxed{\bar{x} - \sqrt{3}s = \hat{a}}$

(iii) Let the sample be : 1, 2, 3, 4, 50.

So, here : $S = 21.272$

$\bar{x} = 12$

$\hat{a} = -24.844$; $\hat{b} = 48.84$.

Since : $\hat{b}$ shouldn't be ~~less~~ than 50 ~~more~~ less

hence : it show the weakness. of the same.

(iv) $f(x) = \frac{1}{b}$.

$L = \prod_{i=1}^{n} \frac{1}{b} \Rightarrow \frac{1}{b^n}$.

Now : $0 < x_1 \cdots x_n \leq b$.

So, to maximize 'L' we want the minimum value of 'b'

So, $b \geq X_{Max}$

Hence : required MLE is X_{max}.

Sol (6) $H_0 : p = 0.1$ H_a :

Type I error : $P($ H_0 is rejected / H_0 is true $)$

$\Rightarrow P(X=0)\,P(X' \geq 5) + P(X=1)\,P(X' \geq 4) + P(X=2)\,P(X' \geq 3) + P(X \geq 3)$

$\Rightarrow (0.2824)(1 - 0.9957) + (0.6590 + 0.2824)(1 - 0.9744)$
$+ (0.8891 - 0.69590)\,(1 - 0.8891) + (1 - 0.8891)$

$\Rightarrow 0.14727 = 15\%$

(ii) Probability of rejecting the null hypo when $p = 0.3$

$\Rightarrow P(X \geq 3 | P=0.3) + P(X=0 / P=0.3)\ P(X' \geq 5 / P=0.3)$ ...

$P(X=2 | P=0.3)\ P(X' \geq 3 / P=0.3)$

$\Rightarrow 0.91253 = 91\%$

(iii) P(Type II error) = P(accepting Ho | Ho = false)

$\Rightarrow (1 - 0.91253) \Rightarrow 0.08747 \simeq 9\%$

(iv) $n = 120$; $\hat{p} = \frac{40}{120} = 0.333$

$$\frac{\hat{p} - p}{\sqrt{\frac{\hat{p}\hat{q}}{n}}} \sim N(0,1)$$

$\Rightarrow$ $Z_{10\%} = 1.2816$

$\hat{p} - 1.2816 \sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow 0.333 - 1.2816 \sqrt{\frac{0.333(1-0.333)}{120}}$

so, 0.2782 is required lower bound.

(v) $H_0: p = 0.27$ $\quad H_a: p > 0.278$.

for one sample binomial model :

$$\frac{n\frac{x}{n} - p_0}{\sqrt{\frac{pqn}{n}}} \sim N(0,1)$$

$$\frac{39.5 - 120(0.278)}{\sqrt{\frac{1}{120}(120)(0.278)(0.722)}} = 1.2511.$$

since : $Z_{cal} = 1.2511$

$Z_{tab} = 1.2816$

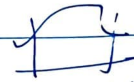
So, $Z_{cal} < Z_{tab}$ we have insufficient evidence to reject H_0 at 10% significance level.

(vi) lower bound CI implies that there is only 10% chance of $p \leq 0.2782$ whereas from hypothesis test, 'p' could be less than or equal to 0.2780 with prob more than 10%.

(vii) $H_0: \mu_1 = \mu_2$ $\quad H_a: \mu_1 \neq \mu_2$

$$Z_c = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2}.$$

$$\frac{65 - 70 - 0}{63.459\sqrt{\frac{1}{12} + \frac{1}{15}}} \simeq 0.2034$$



-2.060 $\quad$ 2.060

So, we have insufficient evidence to reject H_0.

Sol (7) (i) $\bar{x}_{Public} = 30$ | $\bar{x}_{Private} = 35.055$

$s^2_{Public} = 64.3125$ | $s^2_{Private} = 67.4027$.

(ii) The primary condition that should be true for testing equal mean ~~variances~~ is that population variances must be equal for both ~~the~~ private as well as public hospital claims.

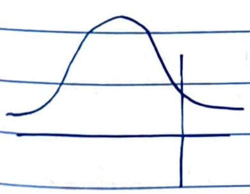
Clearly sample variances of given samples is close enough we can conclude the fact the population variances are same.

(iii) $H_0 : \mu_1 = \mu_2$ $H_a : \mu_2 > \mu_1$

$$Z_c = \frac{\bar{x}_2 - \bar{x}_1 - (\mu_1 - \mu_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$S_p = 8.1153$ $\Rightarrow S_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$

$$Z_{cal} = \frac{5.055 - 0}{8.1153\sqrt{\frac{1}{9}+\frac{1}{9}}} \Rightarrow \frac{5.055}{8.1153\sqrt{\frac{2}{9}}} = 1.321.$$



1.746

Since: $Z_{tab} > Z_{cal}$

So, we have insufficient evidence to reject.

sol 8 Parameter = 'θ'
estimator = '$\hat{\theta}$'

(i) unbiased estimator = $\boxed{E(\hat{\theta}) = \theta}$

(ii) Bias = $E(\hat{\theta}) - \theta$

(iii) $MSE(\hat{\theta}) = Var(\hat{\theta}) + Bias^2(\hat{\theta})$.

$= E(\hat{\theta} - \theta)^2$.

(iv) Clearly: $\hat{\theta}$ is more efficient, no doubt it has some +ve bias but relatively lower MSE which is the main factor in deciding the efficiency of estimator.

(v) If by increasing the sample size i.e 'n' the MSE is decreasing, then estimator would be termed as 'consistent'.

(vi) Two methods are as follows:
→ Methods of moments
→ Maximum likelihood Estimate.

sol 9 $t_k = \frac{\bar{x} - \mu}{s\sqrt{n}}$. is defined as: $\frac{N(0,1)}{\sqrt{\chi^2_k / k}}$

so, $\bar{x}$ ⇒ sample mean
μ ⇒ Population mean
s → sample standard deviation
n → sample size.

i) $\mu = 0$ for $k > 1$

variance $= \dfrac{t}{t-2}$, $t > 2$

i) $X \sim N(\mu, \sigma^2)$

$n = 10$, $\bar{x} = 50$, $s^2 = 48.667$

(a) $$-3.250 \le \frac{\bar{x} - \mu}{s/\sqrt{n}} \le 3.250$$

$$-3.250 \le \frac{50 - \mu}{\sqrt{48.667/10}} \le 3.250$$

$$50 - 3.250\sqrt{\frac{48667}{10}} \le \mu \le 50 + 3.250\sqrt{\frac{48.667}{10}}$$

$(42.8303, 57.1696)$ is CI for 'μ'

(b) $X \sim N\left(0, \frac{9}{7}\right)$

$$\cancel{Z_c = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$$

$$t_{n-1} \sim N\left(0, \sqrt{\frac{9}{7}}\right)$$

$$\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim N\left(0, \sqrt{\frac{9}{7}}\right)$$

$$\frac{\frac{\bar{x} - \mu}{s\sqrt{n}} - 0}{\sqrt{\frac{9}{7}}} \Rightarrow \frac{\bar{x} - \mu}{s\sqrt{7n/9}} \sim N(0,1)$$

critical value $\Rightarrow$ 2.58

CI for 'μ' : $\left[50 - 2.58\sqrt{\frac{4.8667 \times 9}{7}}, 50 + 2.58\sqrt{\frac{4.8667}{\cancel{10}} \times \frac{9}{7}}\right.$

CI for μ : $(43.54, 56.45)$

Sol (10) $H_0 : \lambda = 3$ $\quad X \sim Poi(\lambda)$ $\quad H_a : \lambda \neq 3$.

(i) Type I error : $P(\text{rejecting} \mid H_0 \text{ is true})$

$X \sim Poi(\lambda)$

$\Sigma X_i \sim Poi(n\lambda)$

$\Sigma X_i \sim N(n\lambda, n\lambda)$

$\Rightarrow P(\Sigma X_i > 3100 \mid \lambda = 3)$

$\Rightarrow P\left(\frac{\Sigma X_i - 3000}{\sqrt{3000}} > \frac{3100 - 3000}{\sqrt{3000}}\right)$

$\Rightarrow P(Z > 1.825)$

$\Rightarrow 0.03395$

$\Rightarrow 3.395\%$

(ii) Type II error $= P(\text{accepting} / H_0 \text{ is false})$

$\Rightarrow P(\Sigma X_i < 3100 / H_0 \text{ is false})$

$\Rightarrow P\left[\frac{\Sigma X_i - \cancel{3000}\,1000\lambda}{\sqrt{\cancel{3000}\,1000\lambda}} < \frac{3100 - \cancel{3000}\,1000\lambda}{\sqrt{\cancel{1000}\,1000\lambda}} \,\middle|\, \lambda \neq 3\right]$.

$P\left[Z < \frac{3100 - 1000\lambda}{\sqrt{1000\lambda}} \,\middle|\, \lambda \neq 3\right]$

(iii) Power of test = 1 − Type II

$$1 - P\left[Z < \frac{3100 - 1000\mu}{\sqrt{1000\mu}} \Big/ \lambda \neq 3 \right]$$

(iv) $X = 2900$ $\hat{\lambda} = \frac{2900}{1000} = 2.9$

$\sum X_i \sim N(1000\lambda, 1000\lambda)$

$\sum X_i - 1000$

CI $\left[\hat{\lambda} \pm 2.5758 \sqrt{\frac{\hat{\lambda}}{n}} \right]$

$\hat{\lambda} = 2.9$, $n \rightarrow 1000$

$2.9 \pm 2.575 \sqrt{\frac{2.9}{1000}}$

2.9 ± 0.1387

$\Rightarrow \hat{\lambda} = (2.7613, 3.0387)$. required CI

sol: $\sum x_0 = 213200$

$\sum x_{New} = 213200 - 11000 - 48000 + 27500 + 31500$

$\sum x_{new} = 213200$

$\bar{x}_{new} = \frac{213200}{12} = 17767$.

$\Sigma x^2_{new} = 4919860000 - 11000^2 - 48000^2 + 27500^2 + 31500^2.$

$\Sigma x^2_{new} = 4233360000.$

Standard deviation (new) $= \dfrac{4233360000 - 12(17767)^2}{11}$

$= 6362.99.$

⇒ We observed that mean remained same after removing two temporary employees & adding two permanent ones but standard deviation reduced.

Sol (12) (i) $X_i \sim U(0, \theta)$

$L = \prod_{i=1}^{n} f(x) = \prod_{i=1}^{n} \dfrac{1}{\theta} = \dfrac{1}{\theta^n}$

Now: since: $0 \le x_1 \ldots x_n \le \theta^n.$

so, $X_{max} \le \theta^n$

to maximise 'L' → 'θ' must be min.

so, $L = X_{max}.$

Since: we can't ~~f~~ double derivate 'L' as we will get: $\dfrac{n}{\theta^2}(\ldots) > 0$ which is +ve

So, Cramer Rao ~~bound~~ lower bound & variance isn't applicable here.

(ii) $\hat{\theta}(c) = cY$ $\quad Y = \max X_i$

$F(y) = \cancel{P(X}$ $P(Y_i \le y)$ $\quad P(X \le 3)$ $\quad F_X(3)$

$\Rightarrow P(X_{max} \le y)$

$\Rightarrow P(X_1 \le y) \times P(X_2 \le y) \; . \; . \; P(X_n \le y)$

$\Rightarrow P(X_i \le y)^n \Rightarrow$

$\cancel{P\left(\frac{1}{\theta^n} \le y\right)}$ $\Rightarrow P(X_i \le y)^n$

$\Rightarrow P\left(\int_0^y \frac{1}{\theta} dx\right)^n$

$\Rightarrow P\left(\left(\frac{x}{\theta}\right)_0^y\right)^n \Rightarrow \left(\frac{y}{\theta}\right)^n$

$$\boxed{f(y) = \frac{n}{\theta^n} y^{n-1}}$$

$$\Rightarrow E(Y^k) = \frac{n}{\theta^n}\int_0^\theta y^k y^{n-1} \Rightarrow \frac{n}{\theta^n}\int_0^\theta y^{n+k-1}\,dy$$

$$\Rightarrow \left(\frac{n}{\theta^n}\right)\left[\frac{y^{n+k}}{n+k}\right]_0^\theta \Rightarrow \frac{n}{\theta^n(n+k)}\left[\theta^{n+k}\right].$$

$$\Rightarrow \frac{n\,\cancel{\theta^n}\,\theta^k}{\cancel{\theta^n}(n+k)} \Rightarrow \boxed{\frac{n\theta^k}{n+k}}.$$

$k > 0$

$\theta = cY.$

(iii) $MSE(\hat{\theta}c) = ?$

Bias $= E(\hat{\theta}c) - \theta$

$\Rightarrow cE(Y) - \theta \Rightarrow c\left[\frac{n\theta}{n+1}\right] - \theta$

$\Rightarrow \theta\left[\frac{cn}{n+1} - 1\right]$

$Var(\hat{\theta}c) = E(Y^2) - E^2(Y)$

$= \frac{c^2 n\theta^2}{n+2} - c^2\left(\frac{n\theta}{n+1}\right)^2$

$MSE(\hat{\theta}c) \Rightarrow Bias^2(\hat{\theta}c) + Var(\hat{\theta}c)$

$\Rightarrow \theta^2\left[\frac{cn}{n+1} - 1\right]^2 + \frac{c^2 n\theta^2}{n+2} - c^2\left(\frac{n\theta}{n+1}\right)^2$

$\Rightarrow \theta^2\left[\frac{c^2n^2}{(n+1)^2} + 1 - \frac{2cn}{n+1}\right] + \frac{nc^2\theta^2}{n+2} - \left(\frac{cn\theta}{n+1}\right)^2$

$\Rightarrow \cancel{\frac{c^2n^2\theta^2}{(n+1)^2}} + \theta^2 - \frac{2cn\theta^2}{n+1} + \frac{nc^2\theta^2}{n+2} - \cancel{\frac{(cn\theta)^2}{(n+1)^2}}$

$\Rightarrow c^2\frac{n\theta^2}{n+2} - c\,\frac{2n\theta^2}{n+1} + \theta^2.$

(iv) for $\hat{\theta}c$ to be unbiased:

$E(\hat{\theta}c) - \theta = 0$

$c_u = \frac{\theta cn}{n+1} = \theta \Rightarrow \boxed{c_u = \frac{n+1}{n}}$

(v) $MSE(\hat{\theta}c) = c^2 \frac{n\theta^2}{n+2} - c\frac{2n\theta^2}{n+1} + \theta^2$

$\frac{\partial \hat{\theta}}{\partial c} \Rightarrow 2c\frac{n\theta^2}{n+2} - \frac{2n\theta^2}{n+1} = 0$

$\Rightarrow \frac{\cancel{2}c\,\cancel{n}\cancel{\theta^2}}{n+2} = \frac{\cancel{2}\cancel{n}\cancel{\theta^2}}{n+1}$

$\Rightarrow \boxed{c_m = \frac{n+2}{n+1}}$

$\frac{1}{\frac{n+2}{n+1}}$

(vi) $\hat{\theta}(c_m)$ is more preferrable since in this case MSE would be minimized & it's always better that MSE is minimized even if it has some bias.

as $n \to \infty$, MSE tends to zero.