FM Assignment

① i) $d^{(4)} = 8\%$

$\rightarrow \quad \delta = \ln\left(\frac{1}{1-d}\right)$

$= \quad \delta = P\ln\left(\frac{1}{1-\frac{d^{(P)}}{P}}\right)$

$= \quad \delta = 4\ln\left(\frac{1}{1-\frac{0.08}{4}}\right)$

$= \quad \delta = 4\ln\left(\frac{1}{0.98}\right)$

$= \quad \delta = 0.0808108292\overline{7}$

$\underline{\delta = 8.08108292\overline{7}\,\%}$

ii) $d^{(4)} = 8\%$

$= \quad 1+i = \left(\frac{1}{1-\frac{d4}{4}}\right)^4$

$= \quad 1+i = \left(\frac{1}{1-0.02}\right)^4$

$= \quad 1+i = 1.084165785$

$\underline{i = 8.416578\%}$

(iii) $\frac{d^{(12)}}{12}$

$= \left(1-\frac{d^{(4)}}{4}\right)^4 = \left(1-\frac{d^{(12)}}{12}\right)^{12}$

Raising to $\frac{1}{12}$

$= \left(1-\frac{d^{(4)}}{4}\right)^{1/3} = 1-\frac{d^{(12)}}{12}$

$= (1-0.02)^{1/3} = 1-\frac{d^{12}}{12}$

$\underline{\frac{d^{12}}{12} = 8.053934\%}$

② $PV = ?$

i) $AV = 1000$

$\delta(t) = 0.004t + 0.0002t^2$

$= AV = PV e^{\int_0^{10} \delta(t)}$

$= \therefore PV = 1000\, e^{-\left[\frac{0.004t^2}{2} + \frac{0.0002t^3}{3}\right]}$

$= PV = 1000 \times e^{-0.266667}$

$= 1000 \times 0.765928338$

$\therefore \underline{PV = 765.9283384}$

ii) $AV = PV(1+i)^{10}$

$= 1000 = 765.9283384\,(1+i)^{10}$

$= (1+i)^{10} = 1.305605172$

$= \therefore 1+i = 1.027025404$

$i = 0.027025404 = \underline{2.7025404\,\%}$

③ ii) a) $\frac{d^{(4)}}{4} = 2.25\% = 0.0225$

$d^{(2)} = ?$

$= \left(1 - \frac{d^{(4)}}{4}\right)^4 = \left(1 - \frac{d^{(2)}}{2}\right)^2$

$= (1 - 0.0225)^2 = 1 - \frac{d2}{2}$

$= \frac{d2}{2} = 0.0444937\cancel{5}$

$\underline{d^{(2)} = 8.89875\,\%}$

b) $d^{(1/2)} = 5\% = 0.05$

$d^{(2)} = ?$

$$\left(1 - \frac{d^{(1/2)}}{1/2}\right)^{1/2} = \left(1 - \frac{d^{(2)}}{2}\right)^{2}$$

$$\left(1 - 0.05 \times 2\right)^{1/2} = \left(1 - \frac{d^{(2)}}{2}\right)^{2}$$

$$= \therefore (1 - 0.1)^{1/4} = 1 - \frac{d^2}{2}$$

$$= \text{So } d^{(2)} = 5.19925072\%$$

iii) $i = 5\%$ $n = 91$ days

$\therefore$ Annual SD = ?

lets take an amt of 100

$\therefore$ $PV = AV(1+i)^{91/365}$ — for u 91 days

$= 100(1+i)^{91/365}$

$= 101.2238407$

$\therefore$ $100 \longrightarrow 101.2238407.$

$\therefore$ $d = 4.8494619\%$

④ i) ⓐ $P = 0.08$

To find :- $d^{(12)} = ?$

$$\left[1 - \frac{d^{(12)}}{12}\right]^{12} = \frac{1}{1+0.08}$$

$$\therefore 1 - \frac{d^{(12)}}{12} = \left(\frac{1}{1.08}\right)^{1/12}$$

$$= 1 - \frac{d^{(12)}}{12} = 0.993607102$$

$$= d^{(12)} = 7.671477614\%$$

ⓑ $i^{(365)} = ?$ $\quad i = 0.08$

$$\left(1 + \frac{i^{(365)}}{365}\right)^{365} = 1 + 0.08$$

$$= \therefore 1 + \frac{i^{(365)}}{365} = (1.08)^{1/365}$$

$$= i^{(365)} = 7.69691541\%$$

c) $\delta = \ln(1+i)$

$\delta = \ln(1.08)$

$\delta = 7.6961041 4\%$

d) $i^{(0.5)} = ?$ $\quad i = 0.08$

$$\left(1 + \frac{i^{(0.5)}}{0.5}\right)^{0.5} = (1.08)^{2}$$

$i^{(0.5)} = 8.32\%$

⑤ i) $d^{(4)} = 6\%$

a) $i^{(2)} = ?$

$$\left(1 + \frac{i^{(2)}}{2}\right)^{2} = \frac{1}{\left(1 - \frac{d^{(4)}}{4}\right)^{4}}$$

$$= 1 + \frac{i^{(2)}}{2} = 1.03068876$$

$i^{(2)} = 6.13775516\%$

b) $d^{(12)} = ?$

$d^{(4)} = 6\%$

$$\left(1 - \frac{d^{(4)}}{4}\right)^{4} = \left(1 - \frac{d^{(12)}}{12}\right)^{12}$$

Raising both sides by $1/12$,

$$\left(1 - \frac{0.06}{4}\right)^{1/3} = 1 - \frac{d^{(12)}}{12}$$

$d^{(12)} = 6.0302525 28\%$

(ii)

200,000 —————————————— 2,20,000

15th April 2005 —————————— 15th April 2006

200,000 —————————————— ?

15/04/2005 —————————— 17/7/2015

∴ April → 15 days
May → 31 days
June → 30 days
July → 17 days

93 days

If paid on 15th April 2006

$\frac{220000}{200000} = (1+i)$

$\therefore i = 0.1$ or 10%

∴ Interest paid for 93 days =

$\frac{10}{100} \times \frac{93}{365} = 0.0254794520$

Amount to be paid till 17th July 2005 $= 2,00,000 + \{2,00,000 \times 0.025479452\}$

$= 205905.8904$

(6) (a) let the retail price = Rs 100

Cash payment $= 100 - \frac{30 \times 100}{100}$

$= 70$

(b) Six month credit is 0.25 below the recommend retail price

$= 100 - 25$
$= 75$

∴ effective rate of discount is

$70 = 75(1-d)^{0.5}$

$\frac{70}{75} = (1-d)^{0.5}$

$d = 12.889\%$

$d^{(12)} \Rightarrow$

$1 - d = \left(1 - \frac{d^{12}}{12}\right)^{12}$

$(1-d)^{1/12} = -\frac{d^{12}}{12}$

$= d^{(12)} = 13.7195439$

$\Rightarrow i^{(2)} =$

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = \frac{1}{1-d} \quad ---- \left(\because d = \frac{i}{1+i}\right)$$

$$\Rightarrow 1+\frac{i^{(2)}}{2} = \frac{1}{(1-0.12889)^{1/2}}$$

$$= \frac{i^{(2)}}{2} = 0.07142857143$$

$$= \underline{i^{(2)} = 14.28571429\%}$$

$$= \delta = \ln(1+i)$$

$$= \delta = \ln\left(\frac{1}{1-d}\right)$$

$$= \underline{\delta = 13.7985743\%}$$

⑦ i) ~~26~~ $AV = PV\left(1+\frac{i^{(4)}}{4}\right)^4$

$$\rightarrow \frac{26000}{20000} = \left(1+\frac{i^{(4)}}{4}\right)^{17/3}$$

$$= \left(\left(\frac{13}{10}\right)^{3/17} - 1\right) \times 4 = i^{(4)}$$

$$= \underline{i^{(4)} = 18.95525456\%}$$

(ii) $\left[1-\frac{d^{(2)}}{2}\right]^2 = \left(\frac{1}{1+\frac{i^{(4)}}{4}}\right)^4$

Raising both sides by $1/2$

$$1-\frac{d^{(2)}}{2} = \left(\frac{1}{1.047388133}\right)^2$$

$$\frac{d^{(2)}}{2} = 0.0884417655$$

$$\underline{d^{(2)} = 17.68823531\%}$$

(iii) $26000 = 20000(1+ni)$

$$= 0.3 = \frac{17i}{12}$$

$$= \underline{i = 21.17647059\%}$$

(iv) $d < d^{(12)} < i^{(4)} < i$

Thus, simple interest needs to be higher than the compound discount to maintain the same amount

Date: / /

(10) $$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \le t \le 5 \\ 0.06 & \text{for } 5 \le t \le 10 \\ 0.04 & \text{for } t \ge 10 \end{cases}$$

$\rightarrow$ $v(t) = e^{-\delta}$ or $\dfrac{1}{A(t)}$

$$= \frac{1}{A(0,5) \cdot A(5,10) \cdot A(10,t)}$$

$$= e^{-\int_0^5 0.08dt} \times e^{-\int_5^{10} 0.06dt} \times e^{-\int_{10}^{t} 0.04\,dt}$$

$$= e^{-0.4} \times e^{-0.3} \times e^{-0.04t+0.4}$$

$$= e^{-[0.3+0.04t]} =$$

$$\therefore \boxed{v(t) = e^{-(0.3+0.04t)}}$$

(11) $PV = A v(v(t))$

$\rightarrow$ $PV = C(1-d)^n$

$= 50000\,(1-0.18)^{9/12}$

$= 50000\,(0.82)$

$PV = 43085.42623$

$= \delta = 6\%$. $\quad d^{(12)} = ?$

$\Rightarrow \delta = \ln\left(\frac{1}{1-d}\right)$

$= e^{0.06} = \left(\dfrac{1}{1-\frac{d^{12}}{12}}\right)^{12}$

$= 1 - \dfrac{d^{(12)}}{12} = \dfrac{1}{(e^{0.06})}$

$= \dfrac{d^{(12)}}{12} = 0.004987620$

$= d^{(12)} = 5.985024969$

b) (ii) $d^{(4)} = 6\% = 0.06$

$i^{(2)} = ?$

$= \quad 1+i = \left(1+\frac{i^{12}}{12}\right)^{12}$

$= \quad 1+\frac{d}{1-d} = \left(1+\frac{i^{12}}{12}\right)^{12}$

$= \quad 1+\frac{i^{(12)}}{12} = \left(\frac{1}{1-\frac{d^{(4)}}{4}}\right)^{4}$

$= \quad 1+\frac{i^{(12)}}{12} = \left(\frac{1}{1-\frac{0.06}{4}}\right)^{1/3}$

$= \quad i^{(12)} = 0.0050510721 \times 12$

$= \quad i^{(12)} = 6.060708865\%$

c) $d < \delta < i^{(4)} < i$

d) Answer same as 4 (i)

(12) $AV = 7049\left[A(0,2) \times A(2,4\tfrac{1}{2}) \times A(4\tfrac{1}{2},6)\right]$

$= 7049\left(1+\frac{0.06}{12}\right)^{24} \times (1+0.15) \times \left(\frac{1}{1-\frac{0.06}{12}}\right)^{18}$

$= \quad \boxed{AV = 9999.893613}$

(13) i)

x at 0, $2x$ at 3 $\qquad i = 10\%$

$\rightarrow$

$$2x = x(1+i)^n$$

$$2 = (1+0.1)^n$$

$$2 = (1.01)^n$$

$$\therefore \ln 2 = n \ln(1.01)$$

$$1/n = \frac{\ln(1.01)}{\ln(2)}$$

$$\underline{n = 7.242540902 \text{ years}}$$

ii) $d^{(12)} = 10\%$

$$x = 2x(1-d)^n$$

$$\frac{1}{2} = \left(1 - \frac{d^{(12)}}{12}\right)^{12n}$$

taking log on both sides

$$12n \ln\left(1 - \frac{0.1}{12}\right) = \ln 1/2$$

$$12n = 82.830604713$$

$$\underline{n = 6.90255039 \text{ years}}$$