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Assignment - 2

1) X = Claim size on a home insurance

Y = Claim size on a Car insurance

$X \sim \text{Normal}(800, 10000)$

$Y \sim \text{Normal}(1200, 90000)$

$P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + 800)$

$P((Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) > 800)$

$\therefore (Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) \sim N(3(1200) - 4(800), (3 \times 300^2 + 4 \times 100^2))$

$\therefore (Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) \sim N(400, 310000)$

$$P\left(Z > \frac{800 - 400}{\sqrt{310000}}\right)$$

$P(Z > 0.718)$

$= \cancel{1 - P(Z >}$

$= 1 - P(Z < 0.718)$

$\therefore \simeq 1 - 0.76424$

$\therefore \simeq 0.23576$

2) N = number of claims

X = claim amount

Y = Total claim amount

$N \sim$ Negative Binomial $(3, 0.9)$

$X \sim$ Gamma $(6, 2)$

$$M_Y(t) = M_N(\log M_X(t))$$

$$M_X(t) = \left(1 - \frac{t}{2}\right)^{-6}$$

$$M_Y(t) = M_N\left(\log\left(1 - \frac{t}{2}\right)^{-6}\right)$$

$$M_Y(t) = \left(\frac{p e^{\log(1 - t/2)^{-6}}}{1 - q e^{\log(1 - t/2)^{-6}}}\right)^3$$

$$M_Y(t) = \left(\frac{p\left(1 - \frac{t}{2}\right)^{-6}}{1 - q\left(1 - \frac{t}{2}\right)^{-6}}\right)^3$$

$$M_Y(t) = \left(\frac{0.9\left(1 - \frac{t}{2}\right)^{-6}}{1 - (0.1)\left(1 - \frac{t}{2}\right)^{-6}}\right)^3$$

$$E(N) = \frac{3}{0.9} = \frac{10}{3} = 3.33$$

$$V(N) = \frac{k(0.1)}{(0.9)^2} = \frac{3}{0.9}\left(\frac{1}{9}\right) = \frac{10}{27} = 0.37$$

$$E(X) = \frac{\alpha}{\lambda} = \frac{6}{2} = 3$$

$$V(X) = \frac{\alpha}{\lambda^2} = \frac{6}{4} = 1.5$$

$$V(Y) = E(N)V(X) + E(X)^2 V(N)$$

$$= 5 + \frac{10}{3} = \cancel{9}\ \frac{25}{3} = 8.33$$

$$SD(Y) = \sqrt{\frac{25}{3}} = 2.8868$$

3)i) $f_x(x) = \lambda e^{-\lambda(x-\alpha)}$

$$M_x(t) = E(e^{tx})$$
$$= \int_\alpha^\infty e^{tx} . \lambda e^{-\lambda(x-\alpha)} \, dx$$
$$= \lambda \int_\alpha^\infty e^{tx} . e^{-\lambda x} . e^{\lambda\alpha} \, dx$$
$$= \lambda e^{\lambda\alpha} \int_\alpha^\infty e^{x(t-\lambda)} \, dx$$
$$= \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left[e^{(t-\lambda)x} \right]_0^\infty$$
$$= \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left[e^{-(\lambda-t)x} \right]_0^\infty$$
$$M_x(t) = \frac{\lambda e^{\lambda\alpha}}{t-\lambda} \left(0 - e^{-\lambda\alpha} . e^{t\alpha} \right)$$
$$M_x(t) = \frac{\lambda e^{t\alpha}}{\lambda - t}$$

3)ii) $$M'_x(t) = \lambda(e^{t\alpha})(\lambda - t)^{-2}(-1)(-1) + \lambda(\lambda-t)^{-1}(e^{t\alpha})(\alpha)$$
$$= \frac{\lambda e^{t\alpha}}{(\lambda-t)^2} + \frac{\lambda e^{t\alpha}\alpha}{\lambda - t}$$

$$E(x) = M'x(0) = \frac{\lambda(1)}{\lambda^2} + \frac{\lambda\alpha}{\lambda}$$

$$= \frac{1 + \lambda\alpha}{\lambda}$$

3) ii) $M''_x(t) = \lambda e^{t\alpha}(-2)(\lambda-t)^{-3}(-1) + \lambda(\lambda-t)^{-2}(e^{t\alpha})(\alpha)$

$+ \lambda\alpha e^{t\alpha}(\lambda-t)^{-2}(-1)(-1) + \lambda\alpha(\lambda-t)^{-1}(e^{t\alpha})(\alpha)$

$$= \frac{2\lambda e^{t\alpha}}{(\lambda-t)^3} + \frac{\lambda e^{t\alpha}\alpha}{(\lambda-t)^2} + \frac{\lambda\alpha e^{t\alpha}}{(\lambda-t)^2} + \frac{\alpha^2\lambda e^{t\alpha}}{(\lambda-t)}$$

$$= \frac{2\lambda e^{t\alpha}}{(\lambda-t)^3} + \frac{2\lambda e^{t\alpha}\alpha}{(\lambda-t)^2} + \frac{\lambda\alpha^2 e^{t\alpha}}{(\lambda-t)}$$

$$E(x^2) = M''_x(0) = \frac{2\lambda(1)}{\lambda^3} + \frac{2\lambda\alpha}{\lambda^2} + \frac{\lambda\alpha^2}{\lambda}$$

$$= \frac{2}{\lambda^2} + \frac{2\alpha}{\lambda} + \alpha^2$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \left(\frac{1 + \lambda^2\alpha^2 + 2\alpha\lambda}{\lambda^2}\right)$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \frac{1}{\lambda^2} - \frac{\lambda^2\alpha^2}{\lambda^2} - \frac{2\alpha\lambda}{\lambda^2}$$

$$= \alpha^2 + \frac{2\alpha}{\lambda} + \frac{2}{\lambda^2} - \frac{1}{\lambda^2} - \alpha^2 - \frac{2\alpha}{\lambda}$$

$$V(x) = \frac{1}{\lambda^2}$$

4) $G_V(t) = \left(\frac{p}{1-qt} \right)^k \quad , p+q=1$

$\therefore$ The given distribution is a negative binomial

$P(V=4) = \frac{\sqrt{k+4}}{\sqrt{5}\sqrt{k}} \cdot p^4 \cdot q^4$

5) $f(x,y) = ax^2 + axy \quad \begin{cases} 0 < x < 5 \\ 10 < y < 20 \end{cases}$

$$1 = \int_{10}^{20} \int_0^5 ax^2 + axy \, dx \, dy$$
$$1 = \int_{10}^{20} \left[a \cdot \frac{x^3}{3} \right]_0^5 + \left[ay \cdot \frac{x^2}{2} \right]_0^5 dy$$
$$1 = \int_{10}^{20} \frac{125a}{3} + \frac{25ay}{2} \, dy$$
$$1 = \frac{125a}{3} \left[y \right]_{10}^{20} + \frac{25a}{2} \left[\frac{y^2}{2} \right]_{10}^{20}$$
$$1 = \frac{1250a}{3} + 1875a$$
$$a = \frac{3}{6875}$$

$$\therefore f_x(x) = \int_{10}^{20} (x^2 + xy) \, dy$$
$$= a \left[x^2 (y)_{10}^{20} + x \left(\frac{y^2}{2} \right)_{10}^{20} \right]$$
$$= \frac{3}{6875} (10x^2 + 150x)$$
$$= \frac{30x^2 + 450x}{6875}$$

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5) $E(Y/x) = y f(x/x)$

$$= y \int_{10}^{20} \frac{f(x,y)}{f_X(x)}$$

$$= \int_{10}^{20} \frac{y\left(\frac{3}{6875}\right)(x^2 + xy)}{\left(\frac{3}{6875}\right)(10x^2 + 150x)} dy$$

$$= \frac{1}{10} \int_{10}^{20} \frac{y(x+y)}{(x+15)} dy$$

$$= \frac{1}{10(x+15)} \left(\frac{xy^2}{2} + \frac{y^3}{3} \right)_{10}^{20}$$

$$= \frac{1}{10(x+15)} \cdot \left(150x + \frac{7000}{3} \right)$$

$$= \frac{1}{x+15} \cdot \left(15x + \frac{700}{3} \right)$$

$$= \frac{45x + 700}{3(x+15)}$$

$$\therefore E(Y/x) = \frac{45x + 700}{3(x+15)}$$

$$P\left(Y > \tfrac{1}{3}x + 10\right) = f_Y(y)$$

$$= \int_0^5 \frac{3}{6875}(x^2 + xy)\, dx$$

$$= \frac{3}{6875} \cdot \left[\frac{x^3}{3} + \frac{x^2}{2} y \right]_0^5$$

$$= \frac{3}{6875} \cdot \left[\frac{125}{3} + \frac{25y}{2} \right]$$

$$= \frac{1}{6875} \cdot \left[\frac{250 + 75y}{2} \right]$$

5) $P\left(Y > \frac{1}{3}x + 10\right) = \int_{\frac{1}{3}x+10}^{20} \frac{1}{6875} \cdot \left(\frac{250 + 75y}{2}\right) dy$

$$= \frac{1}{6875(2)} \left[250 \left(y \right)_{\frac{1}{3}x+10}^{20} + 75 \left(\frac{y^2}{2} \right)_{\frac{1}{3}x+10}^{20} \right]$$

$$= \frac{1}{6875(2)} \left[(250)\left(20 - \frac{x}{3} - 10\right) + 75 \left(200 - \frac{x^2}{9} - 100 - \frac{20x}{3} \right) \right]$$

$$= \frac{1}{6875(2)} \left[5000 - \frac{250x}{3} - 2500 + 15000 - \frac{75x^2}{9} - 7500 - \frac{1500x}{3} \right]$$

$$= \frac{1}{6875(2)} \left[10000 - \frac{1750x}{3} - \frac{25x^2}{3} \right]$$

$$= \frac{1}{13750} \left[10000 - \frac{1750x}{3} - \frac{25x^2}{3} \right]$$

6) $X \sim \text{poisson}(\lambda)$

$Y \sim \text{Poisson}(\mu)$

$M_X(t) = e^{\lambda(e^t - 1)}$

$M_Y(t) = e^{\mu(e^t - 1)}$

$$M_{X-Y}(t) = E\left(e^{t(X-Y)}\right)$$

$$= E\left(e^{Xt - Yt}\right)$$

$$= \frac{E(e^{Xt})}{E(e^{Yt})}$$

$$= \frac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}}$$

$$= e^{(\lambda-\mu)(e^t-1)}$$

$\therefore\ X - Y \sim \text{poisson}(\lambda - \mu)$

$\therefore$ It is proved

6) $M_{X+Y}(t) = E\left(e^{t(X+Y)}\right)$

$= E\left(e^{tX+Yt}\right)$

$= E(e^{tX}) \cdot E(e^{tY})$

$= e^{\lambda(e^t-1)} \cdot e^{\mu(e^t-1)}$

$= e^{(\lambda+\mu)(e^t-1)}$

$\therefore$ $X+Y \sim \text{Poisson}(\lambda+\mu)$

$\therefore$ It is proved

7) i) $Var(X/Y=2) = E(X^2/Y=2) - \cancel{E(X/Y)} \left(E(X/Y=2)\right)^2$

$= \sum_x x^2 \dfrac{P(X=x, Y=2)}{P(Y=2)} - \left(\sum_x x \cdot \dfrac{P(X=x, Y=2)}{P(Y=2)}\right)^2$

$\cancel{= 1^2(2) + 2^2(0.3) + 3^2(0.1) + 4^2(2)}$

$= \dfrac{1^2(0) + 2^2(0.3) + 3^2(0.1) + 4^2(0.2)}{0.6} - \left[\dfrac{1(0) + 2(0.3) + 3(0.1) + 4(0.2)}{0.6}\right]^2$

$= \dfrac{53}{6} - \dfrac{289}{36}$

$= \dfrac{29}{36}$

$E(U/V=v) = \int_0^1 u \cdot f(U/V=v)\, du$

$= \int_0^1 \dfrac{u \cdot f(U, V=v)}{f(V=v)}\, du$ - ①

$f_V(V=v) = \int_0^1 \dfrac{48}{67}(2uv - u^2)\, du$

$= \dfrac{48}{67}\left[\dfrac{2u^2v}{2} - \dfrac{u^3}{3}\right]_0^1$

$= \dfrac{48}{67}\left[v - \dfrac{1}{3}\right]$

$= \dfrac{48(3v-1)}{67(3)} = \dfrac{16(3v-1)}{67}$

7) ii) $E(U/V=v) = \int_0^1 \frac{u \cdot \frac{48}{67} \cdot (2uv - u^2)}{\frac{16}{67} \cdot (3v-1)} \, du$

$= \frac{3}{3v-1} \int_0^1 2u^2v - u^3 \, du$

$= \frac{3}{3v-1} \cdot \left[\frac{2u^3v}{3} - \frac{u^4}{4} \right]_0^1$

$= \frac{3}{3v-1} \cdot \left[\frac{2v}{3} - \frac{1}{4} \right]$

$E(U/V=v) = \frac{8v-3}{4(3v-1)}$

8) $x \sim \text{Gamma}(3,2)$

$E(Y/X=x) = 3x+1$

$Var(Y/X=x) = 2x^2+5$

$E(E(Y/X=x)) = E(Y)$

$\therefore \; E(Y) = E(3x+1)$

$= 3E(x)+1$

$= 3\left(\frac{3}{2}\right)+1$

$= \frac{9}{2}+1$

$= \frac{11}{2}$

8) $Var(Var(Y/x=x)) = V(Y)$

$\therefore V(Y) = E(Var(Y/x=x)) + V(E(Y/x=x))$

$= E(2x^2+5) + V(3x+1)$

$= 2E(x^2)+5 + 9V(x)$

$= 2(3)+5+9\left(\frac{3}{4}\right)$

$= 6+5+6.75$

$= 17.75$

9) $Z = x+Y$

i) $Cov(x,z) = cov(x, x+y)$

$= cov(x,x) + cov(x,y)$

$= V(x) + (-0.6)$

$= 4 - 0.6$

$= 3.4$

ii) $Var(z) = Var(x+y)$

$= V(x) + V(x) + 2cov(x,y)$

$= 4+1+2(-0.6)$

$= 5-1.2$

$= 3.8$

10) $f(x, y) = k x^{-\alpha} e^{-y/\beta}$

$$1 = k \int_1^\infty x^{-\alpha}\, dx \int_1^\infty e^{-y/\beta}\, dy$$

$$1 = k \left[\frac{x^{-\alpha+1}}{-\alpha+1} \right]_1^\infty \left[\frac{e^{-y/\beta}}{-\frac{1}{\beta}} \right]_1^\infty$$

$$1 = k \left[\frac{1}{-\alpha+1} \right] \left[-\beta e^{-1/\beta} \right]$$

$$k = \frac{-1+\alpha}{\beta e^{-1/\beta}}$$

$$k = \frac{\alpha - 1}{\beta e^{-1/\beta}}$$