Kshiti Vartak Roll No: 86 Section B

DATE

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Calculus Assignment 2

1]

$$\int_{\frac{1}{50}}^{2} \frac{e^{\frac{2}{\omega}}}{\omega^2}\, d\omega$$

Evaluating indefinite integration

$$= \int \frac{e^{\frac{2}{\omega}}}{\omega^2}\, d\omega$$

Substituting $p = \frac{2}{\omega}$

$$= \int -\frac{e^{p}}{2}\, dp$$

$$= -\frac{1}{2}\int e^{p}\, dp$$

$$= -\frac{1}{2} e^{p}$$

$$= -\frac{1}{2} e^{\frac{2}{\omega}} \quad \ldots \text{ Resubstituting } p = \frac{2}{\omega}$$

$$= -\frac{e^{\frac{2}{\omega}}}{2}$$

$$= \left[\frac{-e^{\frac{2}{\omega}}}{2} \right]_{\frac{1}{50}}^{2}$$

$$= -\frac{e^{\frac{2}{2}}}{2} - \left(-\frac{e^{\frac{2}{\frac{1}{50}}}}{2} \right)$$

$$= \frac{-e + e^{100}}{2}$$

2] $\int \frac{2t^3+1}{(t^4+2t)^3}\,dt$

Sabstitute $u = t^4 + 2t$

$= \int \frac{1}{2u^3}\,du$

$= \frac{1}{2} \times \int \frac{1}{u^3}\,du = \frac{1}{2} \times \left(-\frac{1}{2u^2}\right)$

$= \frac{1}{2} \times \left(-\frac{1}{2(t^4+2t)^2}\right) = -\frac{1}{4(t^4+2t)^2}$

$= -\frac{1}{4(t^4+2t)^2} + C$

3] $f'(x) = x^4 + 3x - 9$

$f(x) = \int x^4 + 3x - 9\,dx$

$= \int x^4\,dx + \int 3x\,dx - \int 9\,dx$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x$

$= \frac{x^5}{5} + \frac{3x^2}{2} - 9x + C.$

5] $\int 3(8y-1)\, e^{4y^2-y}\, dy$

Taking $t = e^{4y^2-y}$

$= 3 \times \int 1\, dt$

$= 3t$

$= 3e^{4y^2-y}$ ... (Resubstituting)

$= 3e^{4y^2-y} + C$

6] $f'(x) = \frac{d}{dx}(15\sqrt{x} + 5x^3 + 6)$

$= \frac{15}{2\sqrt{x}} + 15x^2$

$f(x) = \frac{d}{dx} \frac{15}{2\sqrt{x}} + 15x^2$

$= \frac{-15}{4x\sqrt{x}} + 15x^2$

$$\boxed{F(x) = \frac{-15}{4x\sqrt{x}} + 15x^2}$$

7]

2]

$f = x^3 - 10x^2 + 6$

$f' = 3x^2 - 20x$

$f'' = 6x - 20$

$f''' = 6$ and higher order derivative are all 0.

We calculate the derivatives at $x = 3$:

$f(3) = 3^3 - 10 \cdot 3^2 + 6 = 27 - 90 + 6 = -57$

$f'(3) = 3 \cdot 3^2 - 20 \cdot 3 = 27 - 60 = -33$

$f''(3) = 6 \cdot 3 - 20 = 18 - 20 = -2$

$f'''(3) = 6$

so the Taylor Series is $f(x) = \text{sum}(f(n)(x_0)(x - x_0)^n/n!, n = 0, \infty)$

$= -57 + -33(x-3)/1! + -2(x-3)^2/2! + 6(x-3)^3/3! + 0 =$

$-57 - 33(x-3) - (x-3)^2 + (x-3)^3$

and expanding the last expression out gives $x^3 - 10(x^2) + 6$ as required, since the ~~Talay~~ Taylor Series is exact for a finite polynomial

14] $f(x) = \sqrt{1+x}$

The Maclaurin Series is given by

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots \frac{f^{(n)}(0)}{n!}x^n + \cdots$$

The binomial series tell us that:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

$$f(x) = (1+x)^{1/2}$$

$$= 1 + \left(\frac{1}{2}\right)x + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}x^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}x^3 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{4!}x^4 + \cdots$$

$$= 1 + \frac{1}{2}x + \frac{1/4}{2}x^2 + \frac{3/8}{6}x^3 - \frac{15/16}{24}x^4 + \cdots$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \cdots$$

10] $ty' + 2y = t^2 - t + 1$

for $t > 0$ and $y(1) = \frac{1}{2}$

Dividing equation by t

$$y' + \frac{2}{t} y = t - 1 + \frac{1}{t}$$

Let $u(t) = \int \frac{2}{t} dt = 2 \ln t$ -- (Ignore c

$$y(t) = e^{2\ln t} \int e^{-2\ln t} \left(t - 1 + \frac{1}{t}\right) dt$$

$$= t^2 \int \frac{1}{t^2} \left(t - 1 + \frac{1}{t}\right) dt$$

$$= t^2 \int \left(\frac{1}{t} - \frac{1}{t^2} + \frac{1}{t^3}\right) dt$$

$$= t^2 \left(\ln t + \frac{1}{t} - \frac{1}{2t^2} + c\right)$$

Taking $y(1) = \frac{1}{2}$

$$= 1 - \frac{1}{2} + c = \frac{1}{2}$$

$$c = 0$$