CALCULUS

1. $\lim_{h \to 0} \dfrac{2(h-3)^2 - 18}{h}$

$= \lim_{h \to 0} \dfrac{2[(h-3)^2 - 9]}{h}$

$= \lim_{h \to 0} 2\left[\dfrac{(h-3)^2 - (3)^2}{h}\right]$

$= \lim_{h \to 0} 2\left[\dfrac{(h-3-3)\cancel{(h-3+3)}}{\cancel{h}}\right]$

$= \lim_{h \to 0} 2(h-6)$

put limit $h \to 0$

$2(0-6)$

$2(-6) = -12$

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Question 2 ⇒

a) $x = 4$ b $= x = 6$

$$g(x) = \begin{cases} 2x & x < 6 \\ x-1 & x \geq 6 \end{cases}$$

a) at $x = 4$

$g(4) = 2(4)$
$= 8$

RHL

$$g(4+h) = \lim_{h \to 0^+} 2(4+h) = 8$$

LHL

$$g(4-h) = \lim_{h \to 0^-} 2(4-h) = 8$$

It is continious

b) at $x = 6$

$g(6) = 6 - 1$
$= 5$

RHL,

$$g(6+h) = \lim_{h \to 0^+} (6+h) - 1$$
$$= 5$$

LHL,

$$g(6-h) = \lim_{h \to 0^-} 2(6-h)$$
$$= 12 - 2h$$
$$= 12$$

Discontinious

Question 3 $\lim_{x\to 9} \dfrac{x-9}{3-\sqrt{x}}$

$$= \lim_{x\to 9} \frac{x-9}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}}$$

$$= \lim_{x\to 9} \frac{(x-9)(\sqrt{x}+3)}{(9-x)}$$

$$= \lim_{x\to 9} \frac{\cancel{(x-9)}(\sqrt{x}+3)}{-\cancel{(x-9)}}$$

put limit

$$= -(\sqrt{9}+3)$$

$$= -(3+3)$$

$$= -6$$

Ques 4 $\Rightarrow$

a) $x=-1$ b) $= x=0$ c) $x=3$

$$f(x) = \frac{4x+5}{9-3x}$$

at $x=-1$

$$f(-1) = \frac{-4+5}{9+3} = \frac{1}{12} \text{ continuous}$$

at $x = 0$

$f(x) = \frac{5}{9}$

at $x = 3$

$f(x) = \frac{17}{0}$

Its discontinuous.

Ques 5

$$\lim_{x \to \infty} \frac{6e^{4x} - e^{-2x}}{8e^{4x} - e^{2x} + 3e^{-x}}$$

$e^{\infty} = \infty$

$e^{-\infty} = 0$

$$\lim_{x \to \infty} \frac{\cancel{e^{-x}}(6e^{5x} - e^{-x})}{\cancel{e^{-x}}(8e^{5x} - e^{3x} + 3)}$$

put limit

$$\frac{6e^{\infty} - e^{-\infty}}{8e^{\infty} - e^{\infty} + 3}$$

$\Rightarrow \frac{0}{3} = 0$

$$\frac{6 - e^{-6x}}{8 - e^{-2x} + 3e^{-5x}}$$

$\frac{6}{8}$

$= \frac{3}{4}$

Ques 6 $\Rightarrow$

$$g(y) = \begin{cases} y^2 + 5 & \text{if } y < -2 \\ 1 - 3y & \text{if } y \geq -2 \end{cases}$$

a) $\lim_{y \to 6} g(y) = \lim_{y \to 6} 1 - 3y$

put lim

$1 - 3(6) = 1 - 18 = -17$

b) $\lim_{y \to -2} g(y) =$

RHL

$\lim_{y \to -2^+} = \lim_{y \to -2^+} 1-3y$

$1-3(-2)$

$1+6=7$

LHL

$\lim_{y \to -2^-} y^2+5$

$4+5$

9

Ques 7

$$f(t) = (4t^2-t)(t^3-8t^2+12)$$

diff with resp to t

$$f'(t) = (4t^2-t)(3t^2-16t) + (t^3-8t^2+12)(8t-1)$$

$$f'(t) = [12t^4 \underbrace{-64t^3-3t^3}_{-67}+16t^2] + [8t^4 \underbrace{-t^3-64t^3}_{-65}+8t^2+96t-12]$$

$$f'(t) = 20t^4-132t^3+34t^2-12$$

16
18
34

Ques 8

$$R(w) = \frac{3w+w^4}{2w^2+1} \qquad \frac{u}{v}$$

$$R'(w) = \frac{(2w^2+1)(3+4w^3)-(3w+w^4)(4w)}{(2w^2+1)^2}$$

$$= \frac{[6w^2+8w^5+3+4w^3]-[12w^2+4w^5]}{(2w^2+1)^2}$$

$$= \frac{4w^5+4w^3-6w^2+3}{(2w^2+1)^2}$$

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Ques 9 =) Differ

$$f(x) = 2e^x - 8^x$$

$$f'(x) = 2e^x - 8^x \log 8$$

Ques 10

$$y = z^5 - e^z \ln(z)$$

$$\frac{dy}{dx} = 5z^4 - \left[\frac{e^z}{z} + \ln(z) e^z\right]$$

Ques 11

a) $f(x) = (6x^2+7x)^4$

$$f'(x) = 4(6x^2+7x)^3 (12x+7)$$

b) = $f(t) = 5 + e^{4t+t^7}$

$$f'(t) = e^{4t+t^7} (4+7t^6)$$

Ques 12

a) $z = \ln(7-x^3)$

$$z' = \frac{1}{(7-x^3)} (-3x^2)$$

$$z' = -\left[\frac{3x^2}{7-x^3}\right] \begin{matrix} u \\ v \end{matrix}$$

$$z'' = -\left[\frac{(7-x^3)(6x) + (3x^2)(-3x^2)}{(7-x^3)^2}\right]$$

$$z'' = -\left[\frac{42x - 6x^4 + 9x^4}{(7-x^3)^2}\right]$$

$$z'' = -\left[\frac{42x - 3x^4}{(7-x^3)^2}\right]$$

6) =

$$Q(v) = \frac{2}{(6+2v-v^2)^4}$$

$$Q'(v) = -\frac{-2\times 4(6+2v-v^2)^3(2-2v)}{(6+2v-v^2)^4}$$

$$Q'(v) = \frac{-16(1-v)}{6+2v-v^2}$$

$$Q''(v) = \frac{(6+2v-v^2)\times 0 + 16(1-v)(2-2v)}{6+2v-v^2}$$

$$Q''(v) = \frac{32(1-v)^2}{6+2v-v^2}$$

c) =

$$f(t) = \ln(1+t^2)$$

$$f'(t) = \frac{1}{1+t^2} \times 2t$$

$$f'(t) = \frac{2t}{1+t^2}$$

$$f''(t) = \frac{2(1+t^2) - 2t(2t)}{(1+t^2)^2}$$

$$f''(t) = \frac{2+2t^2-4t^2}{(1+t^2)^2}$$

$$f''(t) = \frac{2-2t^2}{(1+t^2)^2}$$

$$f''(t) = \frac{2(1-t^2)}{(1+t^2)^2}$$

Ques 13.

$$f(x) = x^3 - 3x^2 - 9x + 12$$

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0$$

$$3x^2 - 6x - 9 = 0$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$(x+1)(x-3) = 0$$

$$x = -1, 3$$

$f''(x) = 6x - 6$

at $x = -1$ | at $x = 3$

$= 6(-1) - 6$ | $= 6(3) - 6$

$= -6 - 6$ | $= 18 - 6$

$= -12$ | $= 12$

maximum at $x = -1$ | min at $x = 3$

Ques 14 ⇒

$f(x) = 4x^3 - 18x^2 + 24x - 7$

$f'(x) = 12x^2 - 36x + 24$

$f'(x) = 0$

$12x^2 - 36x + 24 = 0$

$x^2 - 3x + 2 = 0$

$x^2 - 2x - x + 2 = 0$

$x(x-2) - 1(x-2) = 0$

$(x-2)(x-1) = 0$

$x = 2, 1$

$f''(x) = 24x - 36$

at $x = 2$ | at $x = 1$

$f''(2) = 48 - 36$ | $= 24 - 36$

$= 12$ | $= -12$

min at $x = 2$ | max at $x = 1$

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Q 15

$$f(x) = x^2(x+3)$$

$$f'(x) = 2x(x+3) + x^2$$

$$f'(x) = 0$$

$$x(2x+6+x) = 0$$

$$x(3x+6) = 0$$

$$x(x+2) = 0$$

$$x = 0, -2$$

$$f''(x) = \frac{d}{dx}(3x^2+6x)$$

$$f(-2) = 4(1) = 4$$

$$f''(x) = 6x+6$$

$$f''(0) = 6x+6$$

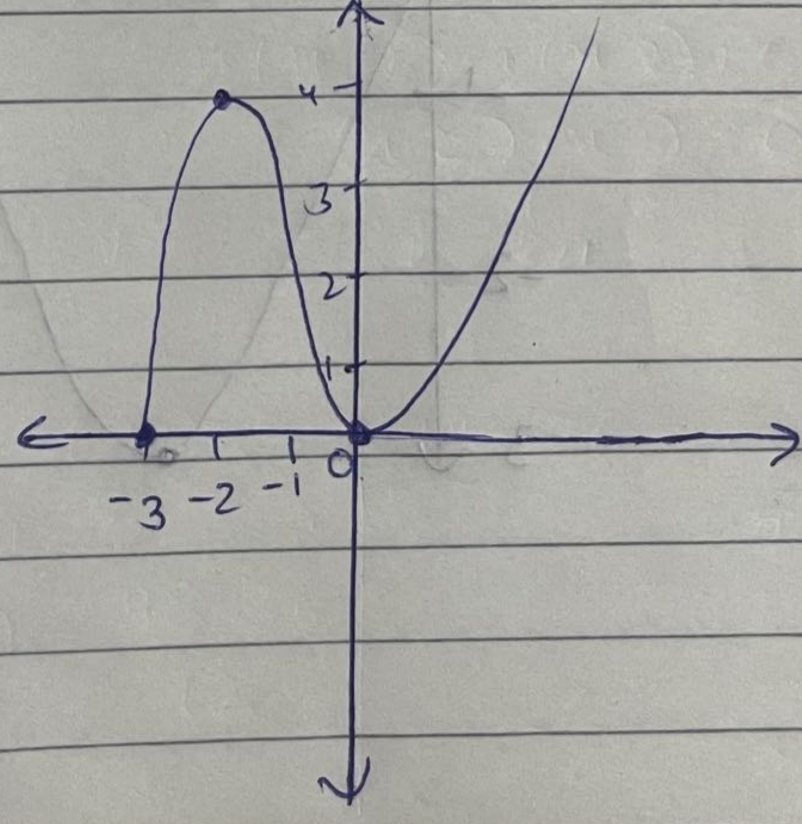
$$= 6$$

min at $x = 0$

$$f''(-2)\ 6x+6$$

$$= -6$$

max at $x = -2$



Ques 16

$$f(x) = 3x^2 - 6x$$
$$f'(x) = 6x - 6$$
$$f'(x) = 0$$
$$6x - 6 = 0$$
$$x = 1$$

$f''(x) = 6$ minimum

$f(1) = -3$

$0 = 3x^2 - 6x$
$6x = 3x^2$
$2 = x$