Date ___/___/___

Asignment - II

1\. Let X be claim size on home insurance
Let Y be claim size on car insurance.

ATQ,

$$X \sim N(800, 100^2)$$
$$Y \sim N(1200, 300^2)$$

$$P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + 800)$$

$$P[Y_1 + Y_2 + Y_3 - (X_1 + X_2 + X_3 + X_4) > 800]$$

$$(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) \sim N(3\times1200 - 4/800), 300^2(3) + 100^2(4) \sim N(400, 310000)$$

$$\approx P\left[Z > \frac{800-400}{\sqrt{310000}}\right]$$

$$P(Z > 0.71842) = 1 - P(Z < 0.71842)$$
$$= 1 - P(Z < 0.72)$$
$$= 1 - 0.76424$$
$$= \boxed{0.23576}$$

Saathi

Date ___ / ___ / ___

2. $N \sim$ Negative Bionomial (K, P)

$$P(N=n) = {}^{n+2}C_n (0.9)^3 (0.1)^n$$

$$= {}^{(n+3)-1}C_{n-1} (0.9)^3 (0.1)^n$$

Hence, $K = 3$

$P = 0.9$

$X \sim$ Gamma $(6, 2)$

i] $M_y(t) = M_n(\log M_x t)$

$$= E\left(\cancel{E}/E/e^{yt} \over N=n\right)$$

$$E\left(\frac{e^{yt}}{N=n}\right) = E\left(e^{x_1 t + x_2 t + x_3 t + x_4 t + \dots x_n t}\right)$$

$$= E(e^{tx_1}) \cdot E(e^{tx_2}) \cdot E(e^{tx_3}) \cdot E(e^{tx_4}) \dots E(e^{tx_n})$$

$$= \left(1 - \frac{t}{2}\right)^{-6n}$$

$$E[E(e^{yt}/N=n)] = E\left[\left(1-\frac{t}{2}\right)^{-6n}\right]$$

$$= E\left[e^{N\log(1-t/2)^{-6}}\right]$$

$$= M_N\left[\log\left(1-\frac{t}{2}\right)^{-6}\right]$$

$$M_N(t) = \left(\frac{Pe^t}{1-qe^t}\right)^K$$

$$= \left[\frac{p e^{\log(1-t/2)^{-6}}}{1 - q e^{\log(1-t/2)^{-6}}} \right]^K$$

$$\frac{p^{(1-t/2)^{-6}}}{1-q^{(1-t/2)^{-6}}} = \left[\frac{(0.9)^{(1-t/2)^{-6}}}{6 - 0.1^{\left(1-\frac{t}{2}\right)^{-6}}} \right] //$$

ii] $V(Y) = E(N) \cdot V(X) + [E(X)]^2 \cdot V(N)$

$$= \left[\frac{3}{0.9} \times \frac{6}{4}\right] + \left[\left(\frac{6}{2}\right)^2 \times \frac{[3(0.1)]}{(0.9)^2}\right]$$

$$= \frac{25}{3}$$

Standard deviation $= 2.887$ //

3.

i] $f(x) = \lambda e^{-\lambda(x-\alpha)}$, $x \geq \alpha$ and $\lambda, \alpha > 0$

$$MGF = E(e^{tx})$$

$$= \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)} \cdot dx$$

$$M_x t = \lambda \int_{\alpha}^{\infty} e^{tx} e^{-\lambda x} \cdot e^{\lambda \alpha} \cdot dx$$

$$= e^{\lambda\alpha} \lambda \int_{\alpha}^{\infty} e^{(t-\lambda)x} \cdot dx$$

$$= e^{\lambda\alpha} \cdot \lambda \left[\frac{e^{(t-\lambda)x}}{t-\lambda} \right]_{\alpha}^{\infty}$$

Date __/__/__

$$= \frac{e^{\lambda\infty}}{t-\lambda}\lambda . \left[\frac{e^{(t-\lambda)}}{t-\lambda}\right]_{\alpha}^{\infty}$$

$$\frac{e^{\lambda\alpha}\lambda}{t-\lambda}\left[-e^{-(\lambda-t)\alpha}\right]$$

$$\frac{e^{\lambda\alpha}\lambda}{\lambda-t} \quad \cancel{e^{-\lambda\alpha}} \; e^{t\alpha} \Rightarrow e^{t\alpha}\frac{\lambda}{\lambda-t}$$

ii) $M_x(t) = \frac{e^{t\alpha}\lambda}{\lambda-t}$

$$= \lambda e^{t\alpha}(\lambda-t)^{-1}$$

$$M_x(t) = \lambda\left[e^{t\alpha}(\lambda-t)^{-1}\right]$$

$$= \lambda\left[(\lambda-t)^{-1}e^{t\alpha}(\alpha) + e^{t\alpha}(\lambda-1)\right.$$

$$= \lambda\left[e^{t\alpha}(\lambda-t)^{-x}\left[\alpha(\lambda-t)+1\right]\right]$$

$$= \frac{\cancel{\lambda}\,1}{\lambda}\left[\lambda\alpha+1\right]$$

$$M''_x(t) = \frac{1}{\lambda}\left[\lambda(1)+0\right]$$

$$= 1$$

$$V(x) = 1 - \frac{1}{\lambda}(\lambda\alpha+1)$$

$$= \frac{\lambda-\lambda\alpha-1}{\lambda}$$

//

Saathi

Date __/__/__

4. $G_Y(t) = \left(\frac{p}{1-qt}\right)^n$

$G_Y t = \sum t^y \times P(Y=y)$

$P(Y=4) \Rightarrow \frac{\sqrt{N+4}}{\sqrt{5}\ \sqrt{N}} p^N q^M$

$= \frac{\sqrt{N+4}}{4!\ \sqrt{N}} p^N q^M$

$= \frac{(N+3)!}{(N-1)!\ 4!} p^N (1-p)^N$ //

6. $X \sim Poi(\lambda)$ $Y \sim Poi(\lambda')$

$M_X(t) = e^{\lambda(e^t-1)}$ $M_Y(t) = e^{\lambda'(e^t-1)}$

Hence, $Z = X - Y$

$M_Z(t) = E(e^{tz}) = E(e^{t(x-y)}) = E(e^{tx} \cdot e^{-ty})$

$= E(e^{tx}) \times E(e^{-ty}) \Rightarrow \frac{E(e^{tx})}{E(e^{ty})}$

$= \frac{e^{\lambda(e^t-1)}}{e^{\lambda'(e^t-1)}}$

$= e^{(\lambda-\lambda')(e^t-1)}$

Date __ / __ / ____

Now Let $Z = X + Y$

$$E(e^{tZ}) = e^{\lambda(e^t - 1)} \times e^{\mu(e^t - 1)}$$

$$= e^{(\lambda + \mu)(e^t - 1)}$$

By uniqueness $Z \sim \text{Poi}(\lambda + \mu)$ //

8. $X \sim \text{Gamma}(3, 2)$

$E(X) = \frac{3}{2}$ $V(X) = \frac{3}{4}$

$E(Y/X=x) = 3x + 1$ $\text{Var}(Y/X=x) = 2x^2 + 5$

$E(Y) = E[E(Y/X=x)] \Rightarrow E(3x+1)$

$$= 3E(X) + 1$$

$$= 3\left(\frac{3}{2}\right) + 1$$

$$= \frac{9}{2} + 1$$

$$= \frac{11}{2}$$

//

$V(Y) = E[V(X/Y)] + V[E(X/Y)]$

$$= E(2x^2 + 5) + V(3x + 1)$$

$$= 2E(X^2) + 5 + 9V(x)$$

$V(X) = E(X^2) - [E(X)]^2$

$$\frac{3}{4} + \frac{9}{4} = E(x^2)$$

$$\frac{12}{4} = E(x^2)$$

$$\boxed{E(x^2) = 3}$$

$$V(Y) = 2(3) + 5 + 9\left(\frac{3}{4}\right)$$

$$= 11 + \frac{27}{4}$$

$$= \frac{44+27}{4}$$

$$= \boxed{\frac{71}{4}}$$

9. $E(X) = 3$ $\quad V(X) = 4$

$E(Y) = 4$ $\quad V(Y) = 1$

Correlation coeff $= -0.3\sqrt{4} \Rightarrow Cov(X, Y)$

$= -0.6$

$Z = X + Y$

$Cov(X, X+Y) = cov(x, X) + Cov(x, y)$

$= V(X) + cov(X, Y)$

Saathi

Date __/__/__

$= 7 + (-0.6)$

$= 3.6$

10. $f(x,y) = K x^{-\alpha} e^{-y/\beta}$

$$K \int_1^\infty \int_1^\infty x^{-\alpha} e^{-y/\beta}$$

$$K \int e^{-y/\beta} \left[\frac{x^{-(\alpha-x)}}{-\alpha+1} \right]_1^\infty = 1$$

$$\frac{K}{1-\alpha} \int e^{-y/\beta} [-1]$$

$$\frac{K}{\alpha-1} \int e^{-y/\beta} \, dy = 1$$

$$\frac{K}{\alpha-1} \left[\frac{e^{-y/\beta}}{-1/\beta} \right]_1^\infty$$

$$\frac{-K}{\alpha-1} (\beta) \left(e^{-1/\beta} \right) = 1$$

$$K = \frac{\alpha-1}{\beta\left(e^{-1/\beta}\right)}$$